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Macroeconomic Effects of Collateral Requirements and Financial Shocks

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Abstract

This article explores the implications of borrower's side collateral constraints have on the real economy. The novel element in this model relative to the industry standard model is that I model the entrepreneurs, which are crucial for investment, as collateral constrained. The model is estimated using Bayesian methods and can be employed to measure the role of collateral. Regarding the results, I document that collateral requirements are highly volatile during the period of 2007–2012, and I find that the effect of an increase in collateral requirements is highly significant. Interestingly, the model assigns an important role for collateral in the shock decomposition, and the contribution of financial shocks is much marked during the financial crisis and substantially shapes macroeconomic fluctuations.

Keywords: Business Loan, Collateral, Financial Shocks.

JEL Codes: E32, E44, G21.

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1 Introduction

The role of collateral in the financial system has attracted the attention of economists more than anytime. Lack of collateral during the global banking crisis of 2008 caused a considerable effect on banks' lending activities, which depend not only on interest rates but on collateral as well. Certainly, the objective of banks' collateral policies is to control and avoid damages that may come from unsecured loans and to improve bank stability. Banks may place stricter conditions, ask for higher collateral, and lend less. The share of business loan supply as a percentage of gross domestic product (GDP) fell dramatically from 11% in 2009 to 8% in 2011 for the United States (see Figure 1). Throughout the same period, nonperforming loans rose considerably, reaching their peak in 2010, while total business loans dropped continuously that same year but rose again after 2011 (see Figure 1). The movement of these three indicators is due to both low profitability in the business sector and the relaxation of collateral requirements prior to 2010 in the United States. In this paper, I argue that the contraction and expansion of loans over the past decade is due to the change in collateral policy and that the collateral channel exerts an influence on the economy.

What is sorely lacking in the studies of financial frictions is empirical evidence on the role of collateral. Entrepreneurs typically offer assets as collateral to borrow from banks which they lose if they default on their loans. The decision of banks to lend will depend not only on the quantity of pledged assets but also on their prices. The last 20 years have seen large movement in asset prices, the volatility of the Russel 3000 price index was high and range from -0.25 in 2008 to 0.14 in 2010. While the variation in total assets of nonfinancial businesses was much smaller. The collateral price, both the fundamental and speculative components, which is an important determinant of firm value is typically omitted. Many macroeconomic models abstract from these facts and assume that prices are uniquely based on fundamentals. The present paper aims to fill this gap in the literature.

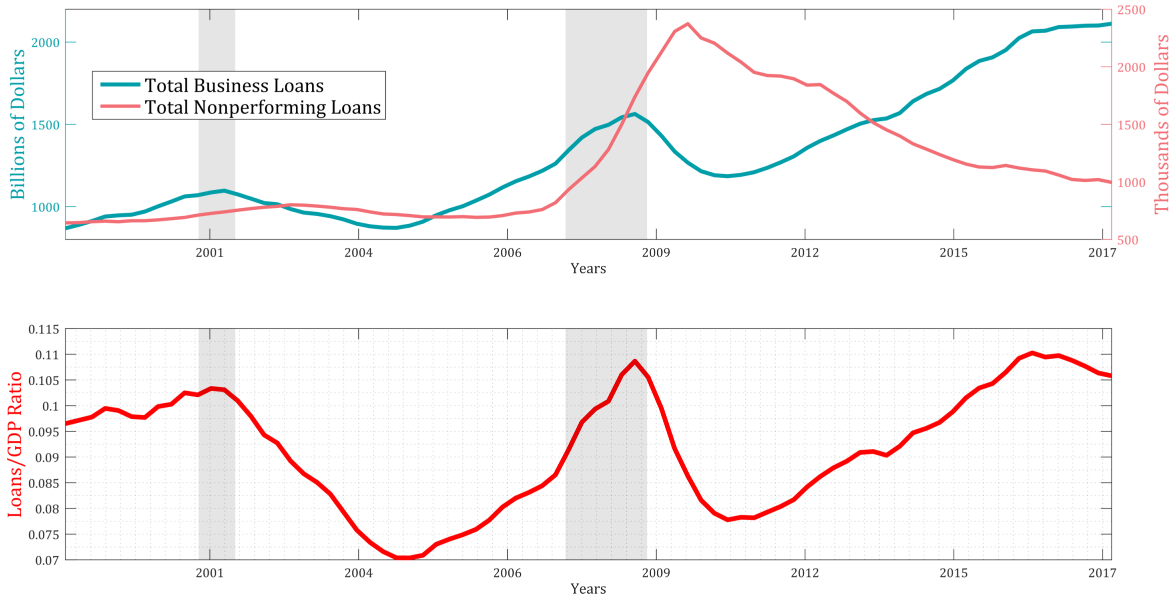

Figure 1: Total Loans, Nonperforming Loans, and Loan/GDP Ratio

Note: Data from Federal Reserve Economic Data (FRED) (01/04/1998–01/01/2018): Total nonperforming loans for commercial banks (USNP). Total stock of commercial and industrial loans issued by all commercial banks (BUSLOANSNSA) divided by gross domestic product (GDP).

Despite the major step of incorporating financial frictions into structural models, there is limited evidence on the impact of collateral on the economy. In this work, I focus on the role of collateral as a driver of aggregate fluctuations in the context of a dynamic stochastic general equilibrium (DSGE) model. In this particular context, entrepreneurs own specific collateral and bank will select assets that are easy to liquidate. The two novel elements are that I introduce a time-varying collateral disturbance into the model, and I consider volatile capital prices to identify the firm value. I show that collateral represents an important source of macroeconomic fluctuations. This result contrast with the well-known results of Justiniano et al. (2015), who show that shocks to collateral have negligible effects on the macroeconomy. To support my claim, I estimate the model over the period 1998–2018 using real aggregate data in addition to financial data.

First, I show that the effect of collateral constraint on the economy is perceptible. I observe that a positive collateral requirement shock implies a decrease in loan supply; a reduction in entrepreneurial net worth, output, and consumption; and a decline in capital and investment. Another aspect of these responses is the persistence of a collateral requirement shock, which appears to be long-lived for capital, consumption, and investment. The intuition is as follows. The bank lending channel propagates a positive collateral requirement through a decline in real activity as capital becomes more scarce and decreases lending activity, thus leading to a contraction in output. Second, I find that a collateral shock plays an important role in the shock decomposition and accounts for 48% of the fluctuations in external financing. This is crucial because bank lending decisions depend mainly on the borrower's collateral, while investment specific technical change (ISTC) plays no role in macroeconomic movement, which is consistent with Schmitt-Grohe and Uribe (2012). The entrepreneur risk shock in this model is less powerful in magnitude, whereas it emerges as the most important shock in Christiano et al. (2014). Third, incorporating financial data into the model reduces considerably the role of productivity shocks in macroeconomic fluctuations. Finally, financial shocks significantly shape macroeconomic fluctuations, especially during the economic downturn of 2008.

A vast literature has introduced financial intermediation into mainstream macroeconomic models. Some major early contributions include Bernanke and Gertler (1989) and Bernanke et al. (1999). In their work, the authors introduced the financial accelerator mechanism into business cycle framework and studied the role of credit market frictions.

Several influential research contributions on New Keynesian models with financial frictions include Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Christiano et al. (2014), Del-Negro and Schorfheide (2013), Del-Negro et al. (2015), Iacoviello (2015), Gertler et al. (2017), and Guerrieri and Iacoviello (2017).[1] I contribute to this literature by analyzing the impact of a collateral disturbance, which can be interpreted as a change in the willingness of banks to accept capital as collateral.

This paper is also related to the literature that focuses on financial frictions and

[1] See also Christiano et al. (2005) for the role of nominal rigidities in the context of the standard New Keynesian model.

banking intermediation. Related studies include Brunnermeier and Sannikov (2014), who consider the effect of various policies on financial stability and the efficiency of financial regulations using a continuous time dynamic model[2]; Curdia and Woodford (2016), who study the effect of monetary policy under the assumption of endogenous variation in the efficiency of the banking system; and He and Krishnamurthy (2012), who address the effect of intermediary equity shock with the presence of nonlinearity. This line of research include Jermann and Quadrini (2012), who provided more detailed discussion about the contribution of financial shocks to macroeconomic fluctuations; and Jakab and Kumhof (2015), who study the effect of financial shocks to illustrate the differences between intermediation of loanable funds and financing through money creation models of banking. The main difference is that these studies restrict their analyses to financial shocks and mute the asset prices volatility, this implies that collateral disturbance will play a modest role. While in this paper the volatility of asset prices deemed crucial to uncover the role of collateral.

In a related study, Becard and Gauthier (2021) consider the impact of collateral shocks on the economy. The approach in this paper is different in two important features. First, I assume that only entrepreneurs are financially constrained by banks' collateral requirements; therefore, bank lending is limited to business loans. Second, I assess the role of collateral by considering collateral price as an important determinant of firm value, and I identify two components: a fundamental component and a speculative component. The main advantage of this assumption is that it allows me to shed light on the economic interaction between the financial sector and the rest of economy, [3] and evaluate the extent of the collateral impact. A recent paper Berger et al. (2020) finds a negative relationship

[2] DiTella and Kurlat (2017) consider the behavior of the yield spreads and propose an explanation for the exposure risk of banks in the context of a dynamic hedging strategy, emphasizing the interaction between a bank's interest rate exposure and its balance sheet.

[3] The key assumption is that the market price of capital differs from the capital's fundamental value. This makes sense because price volatility is not well understood by macroeconomic models, which are based uniquely on fundamentals. This assumption about price volatility is related to the earlier work of Bernanke and Gertler (2000).

between volatility (changes in stock prices) and real activity, while uncertainty shocks are generally uncorrelated with realized volatility.

This paper also focuses on the macroeconomic consequences of financial frictions and the amplifications of shocks in the spirit of Christiano et al. (2014). Furthermore, I add two important components to collateral price that implicitly identify the firm value. By characterizing the aggregate firm value as in Schmitt-Grohe and Uribe (2012), I can provide insight into how the bank lending channel propagates a positive collateral shock using real aggregates and financial dimensions.[4]. As a great deal of the macroeconomics literature abstracts the aggregate flows of financing, this paper fills the gap by including financial data to estimate the model.

Moreover, this paper is related to recent research into financial intermediaries and credit constraint that deserve further discussions. In particular, Justiniano et al. (2015) study the macroeconomic implications of leveraging cycle. Their model accounts for the housing sector and heterogeneity among households, and the authors find that household leveraging and deleveraging has a small macroeconomic effect.[5] Curdia and Woodford (2016) allow for the variability of spreads between the lending rate and deposit rates and the efficiency of financial intermediation and also analyze the responses under the optimal policy. Geanakoplos and Zame (2007) claim that market outcomes depend on collateral through many channels. On details, the scarcity and amount of collateral that an agent must hold to secure their debt can imply restrictions on financing activities, including borrowing and lending, and consequently make collateral scarce in a way that induces significant social inefficiency.

I should emphasize that the key distinction between these papers and the present paper is that I exclusively focus on the implications of changes in collateral requirements in economies with financially constrained entrepreneurs. Specifically, I embed a collateral

[4] See Appendix A for more details about data construction.

[5] See related work by Guerrieri and Lorenzoni (2017), who evaluate the channel by which the shock to the borrowing limit propagates to the economy in an incomplete market framework. For the propagation of credit supply shock in the Euro area see Gerali et al. (2010).

disturbance into a standard macro model and focus on understanding the interactions between a change in collateral requirements and macroeconomic fluctuations. I also introduce an expression for firm value because the magnitude of collateral disturbance will depend on the change of capital price (Tobin Q^k). Most macroeconomic models do not allow for changes in capital price and remain almost unchanged in response to economic shocks.

In this paper, I document new results on collateral shock that are useful for the literature studying the macroeconomic implications of financial shocks. In particular, I find that collateral plays an important role in driving fluctuations in the model. Collateral accounts for 26% of the variance of net worth growth, and 36% of the variance of capital price. It is useful to contrast these results with Christiano et al. (2014), whose main result is that entrepreneur risk shock accounts for 60% of fluctuations in output growth. This is one important difference with the present paper, where entrepreneur risk shock accounts for only 1%. Another result is that investment-specific technical change (ISTC) is muted, which is consistent with Schmitt-Grohe and Uribe (2012), while the marginal efficiency of investment (MEI) plays an important role in driving fluctuations in the model. In contrast, in Greenwood et al. (1997) and Justiniano et al. (2011), the ISTC is seen as one of the most important drivers of economic growth in business cycle frameworks. I also find that the financial sector was the main driver of the economic downturn of 2008. The model captures the behavior of macroeconomic aggregates, for instance, financial shocks including collateral shock led to a decline in output and investment growth. Iacoviello (2015) provides evidence about how the limited borrowing capacity of entrepreneurs will reduce investment levels and lessen output. This is also consistent with the recent work of Del-Negro et al. (2017) who find that a liquidity shock led to a general decline in funding for investment and output during the Lehman episode.

Layout. Section 2 details the collateral collection process in the United States, which motivates the inclusion of collateral constraint in the model. Section 3 presents the basic model. Section 4 explains the estimation results. Section 5 examines the transmission mechanism of collateral shocks and the main empirical finding regarding the propagation of financial shocks. Section 6 analyzes the contribution of collateral shocks to economic fluctuations and discusses the role of financial shocks. Section 7 concludes.

2 Motivating Evidence

Here I document the importance of collateral for business lending and economic activity and the potential interaction between firm collateral constraints and macroeconomic fluctuations, which supports the inclusion of collateral into macroeconomic models.

Collateralized Business Loans. The panic of 2007 highlighted the various aspects of financial activities and other dangers that originated in certain segments of financial markets. Figure 2 shows the percentage of collateral requirements and the volume of loans. The share of secured loans increases continuously to reach more than 60% of total loans in 2017. For example, there are two events in 2008 and 2014 with an uptick in loan supply, while the level of collateralized loans remains the same. Between 2010 and 2013, 40% to 60% of loans approved by banks are secured by collateral. This tendency does not necessarily lead to a drop in lending activity; however, it can be considered an indicator of the tightness of collateral requirements.

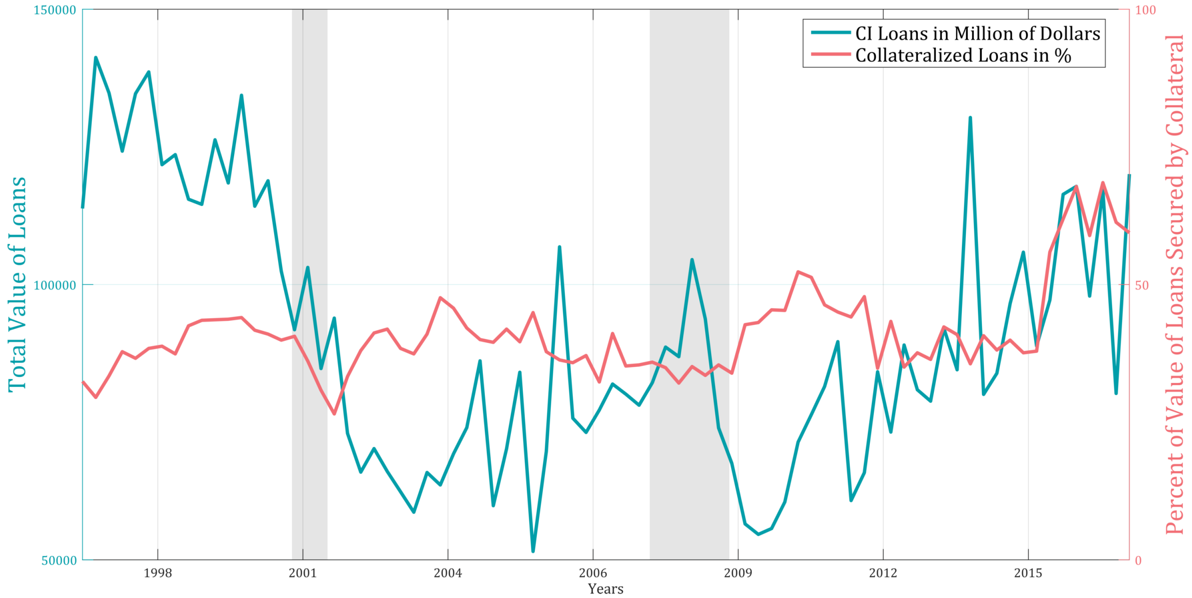

Figure 2: Loans with Pledged Collateral

Note: This survey data is retrieved from FRED, Federal Reserve Bank of St. Louis. The "Survey of Terms of Business Lending" reports a quarterly collection of quantitative and qualitative information on bank lending to small businesses. I focus on trends in lending activity and collateral requirements (for all commercial and industry loans and commercial banks).

The Collateral Collection Process in the US. US banks typically proceed to the collection of collateral after clearly documenting a delinquent loan. Based on the Federal Deposit Insurance Corporation (FDIC) rules, banks should classify all loans that involve some risks by degree of nonpayment risk. These risky loans must be classified into three categories: substandard, doubtful, and loss. This paper focuses on loans classified as loss and considered uncollectible debt. Even if the loan is classified as a loss, banks are able to recover all of part of the amount providing that the loan is secured by collateral. The estimated credit loss of a bank is given by the following equation:

$$\text{net charge-offs} = \text{gross charge-offs} - \text{subsequent recoveries of delinquent debt}$$

Gross charge-offs are the total amount of uncollectible debt, the amount of unpaid loans, and interest. The bank may have some recovery value on delinquent debt, which is determined by the value of collateral minus the cost of selling. The value of net charge-offs can be either positive or negative, reflecting the case that banks can make loss or gain from collateral liquidation. According to the Federal Deposit Insurance Act, banks should remove the delinquent debt from books and the loan should be charged off when the loss occurs within the delinquency time frame adopted by the FDIC. Once the collateral dependent loan is classified as a loss, banks can take legal action and proceed to sell this collateral to repay the loan, such that the value of collateral minus the cost of selling it is sufficient to recover.

The Evolution of Collateral Liquidation and Economic Growth. Entrepreneurs with financial needs may offer collateral to borrow from banks, which they lose if they default on their loans. Table 1 depicts the annual average growth rate of gross domestic product, the annual average growth rate of commercial and industrial loans, the annual average growth rate of uncollectible (CI) debt, and the annual average growth rate of CI loan recoveries.

Table 1: Annual Average Growth Rates of Output, Commercial, and Industrial Loan Charge-offs, Commercial and Industrial Loan Recoveries, and Commercial and Industrial Loans.

Years	Δ % GDP	Δ % CI Loans Charge-offs	Δ % CI Loans Recoveries	Δ % CI Loans
1984-1988	7.24	8.62	5.41	0.72
1989-1998	5.53	2.84	0.29	0.98
1999-2008	4.94	6.74	2.61	1.31
2009-2018	3.71	-1.53	1.64	0.95

The growth rate of uncollectible debt between 1999-2008 of 6.7% reflects the height of the financial crisis. Around the same time, the level of recoveries on the delinquent debt reaches a 2.6% average growth rate, while the growth rate of business lending drops from 1.31% to 0.95% following the Lehman episode. Based on the high level of uncollectible debt and collateral liquidation, this leads to tighter lending conditions for firms, which can be translated into the slowdown of economic growth.

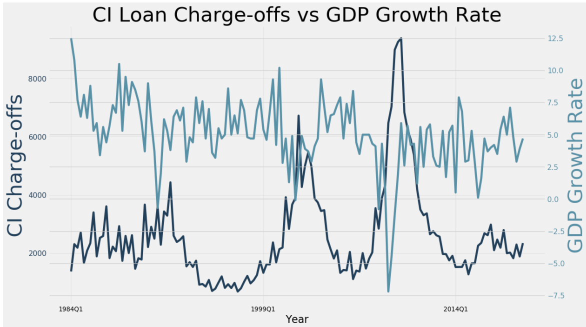

Figure 3: Uncollectible Commercial and Industrial (CI) Loans vs. GDP Growth Rate

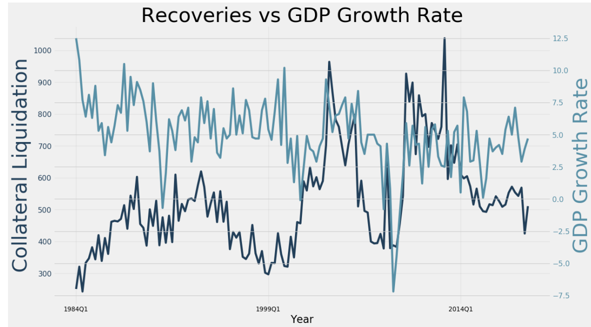

Figure 4: Liquidation of Collateral Commercial and Industrial (CI) Loans vs. GDP Growth Rate

Note: Evolution of uncollectible commercial and industrial loans, liquidation of collateral for uncollectible commercial and industrial loans, and gross domestic product in US. This data is retrieved from the FDIC's financial data and the Bureau of Economic Analysis. Annual data Between 1984-2018.

Figure 4 shows that the level of uncollectible debt reaches its peak between 2008 and 2009. Around the same time, the level of seized collateral also reaches a relatively high level. One interpretation of the high level of liquidated collateral is that the economy experiences tight collateral conditions, as shown in Figure 3. These figures also exhibit a potential relationship between collateral liquidation and economic growth, and between delinquent debt and economic growth. This raises questions as to the impact of collateral on firm lending, economic activity, and the relevance of this channel in explaining quantitatively the movement of macroeconomic aggregates. The main conclusions from the data as described so far constitute the motivation for writing this paper and focusing on the macroeconomic impact of collateral.

3 A Model with Collateral Constraint

General Assumptions

The model developed in this paper is based on Christiano et al. (2014), and augmented with collateral constraint to study the impact of collateral requirement variations on banking decisions and the macroeconomy. I start by considering some general assumptions in this framework. First, the baseline model accounts for nominal rigidities in prices and wages as the price adjustment is less frequent, and where multiple agents maximize their utility and profits. I also assume that good retailers operate under monopolistic competition. Second, in this economy, time is discrete, and there is a continuum of agents who live infinitely. They are rational, forward-looking, and maximize their profits.

Finally, the main agents in this framework are households, bankers, entrepreneurs, firms, capital producers, labor contractors, and the government. In each period t, firms maximize their profits by converting an intermediate good to a final good, setting prices and wages under nominal inertia. Entrepreneurs maximize profits by offering capital services and borrowing from banks. Households maximize their lifetime utility by providing labor services, consuming the final good, and making deposits. Banks intermediate the flow of deposits and loans, and the government sets taxes and nominal interest rates. In equilibrium, all households, firms, and banks behave optimally, and all markets clear.

Model Structure

Next I present the main feature of a model with the banking sector. The key ingredient is a collateral shock that represents a change in the willingness of banks to accept capital as collateral. As entrepreneurs are financially constrained, they are willing to use capital as collateral to obtain new loans. The quality of capital as collateral fluctuates over time. As a result, banks assess their collateral policy and adopt a tight or a loose policy.

This framework will be used to study the macroeconomic consequences of collateral requirements. Model derivations are reported in Appendix D.

Goods Production. The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final good stock, written as:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}. \tag{3.1}$$

The final good is indexed by $j \in [0,1]$, and $1 \leq \nu_p < \infty$ is a shock that follows a standard AR(1) process $\nu_{p,t} = \rho_{\nu_p}\nu_{p,t-1} + \epsilon_t^{\nu_p}$, with $\epsilon_t^{\nu_p} \sim \mathcal{N}(0, \sigma^2_{\nu_p})$. A higher price markup ν_p implies that firms have market power and that the good is less substitutable for other goods. The monopolist produces the intermediate good j using two inputs, labor and capital, to produce a differentiated good according to the following intermediate good production function that relates outputs to inputs:

$$Y_{j,t} = \gamma_t(u_t K_{j,t-1})^{\alpha}(z_t L_{j,t})^{1-\alpha} - \Phi z_t^*, \tag{3.2}$$

where $L_{j,t}$ is the labor input, z_t is the exogenous productivity shock, and $K_{j,t-1}$ is the capital input and that is proportional to u_t, the utilization rate of capital. $\alpha \in (0,1)$ is the intermediate good elasticity that measures the responsiveness of $Y_{j,t}$ to changes in the utilization of capital and variations in labor. γ_t is a covariance stationary technology shock that evolves according to the following process: $\gamma_t = \rho_\gamma \gamma_{t-1} + \epsilon_t^\gamma$. Υ_t is investment specific technology change and Φ is a fixed cost proportional to $z^* = z_t \Upsilon_t^{\left(\frac{\alpha}{1-\alpha}\right)}$, a process introduced to ensure we have a balanced growth path.

The nonstationary productivity shock z_t is assumed to have a growth rate of $\mu^*_{z,t} = \frac{z_t}{z_{t-1}}$. The technological trend $\mu^*_{z,t}$ adheres to an AR(1) process $\mu^*_{z,t} = \rho_{\mu^*_z}\mu^*_{z,t-1} + \epsilon_t^{\mu^*_z}$, where $\epsilon_t^{\mu^*_z}$ has mean zero and standard deviation $\sigma_{\mu^*_z}$. Following Justiniano et al. (2011), I assume that nonstationary investment has a growth rate of $\mu_{\Upsilon,t} = \frac{\Upsilon_t}{\Upsilon_{t-1}}$. Investment-specific technical change shock also adheres to an AR(1) process $\mu_{\Upsilon,t} = \rho_{\mu_\Upsilon}\mu_{\Upsilon,t-1} + \epsilon_t^{\mu_\Upsilon}$, where $\epsilon_t^{\mu_\Upsilon}$ represents an innovation to the growth rate of investment specific productivity, has mean zero, and standard deviation σ_{μ^Υ}.

Price stickiness is incorporated into the model, and this is drawn from the monopolistic power of the producer. The monopolist sets the price P_t of the good by adopting a variant of Calvo-type frictions. In any given period, the producer can reoptimize the price with probability $1-\zeta_p$; otherwise, it cannot reoptimize with probability ζ_p. Afterward, the price level à la Calvo is given by:

$$P_t = \left[(1-\zeta_p)(\tilde{P}_t)^{\frac{\nu_p}{1-\nu_p}} + \zeta_p(\tilde{\pi}P_{t-1})^{\frac{\nu_p}{1-\nu_p}}\right]^{\frac{1-\nu_p}{\nu_p}}.$$

If the producer cannot reoptimize, then it sets the price as $P_{j,t} = \tilde{\pi}_t P_{j,t-1}$, where $\tilde{\pi} = (\pi_{trg,t})^i(\pi_{t-1})^{1-i}$. Letting P_t denote the price of Y_t, the actual inflation is $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$, and $\pi_{trg,t}$ is the target inflation. After setting the prices and given the quantity demanded, the monopolist minimizes the production cost:

$$W_j L_{j,t} + \tilde{r}_t^k P_t u_t K_{j,t-1},$$

subject to production function (3.2), to determine its demand for labor and capital inputs.

Labor Market. In this environment, labor services are provided by households to intermediate firms via labor contractors. The organization of the labor market adopted in the paper is similar to the one presented in Christiano et al. (2014), which is a variant of Erceg et al. (2000). Labor contractors combine differentiated labor inputs $L_{i,t}$ $i \in [0,1]$ that they convert into homogeneous labor L_t. I assume that labor services have the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 (L_{i,t})^{\frac{1}{\nu_l}} di\right]^{\nu_l}$$

subject to

$$\int_0^1 W_{i,t} L_{i,t} di = W_t L_t.$$

Here, labor contractors determine the demand for labor type, where ν_l denotes the fixed markup, L_t is the quantity of homogenous labor, and $W_{i,t}$ is the wage rate.

The first-order condition with respect to labor $L_{i,t}$ can be written as:

$$L_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\frac{1+\nu_l}{\nu_l}} L_t.$$

To define the aggregate wage W_t, I take the total labor supply expression $W_t L_t = \int_0^1 W_{i,t} L_{i,t} di$, then simplify to obtain the expression for the aggregate wage:

$$W_t = \left[\int_0^1 W_{i,t}^{-\frac{1+\nu_l}{\nu_l}} di \right]^{-\frac{\nu_l}{1+\nu_l}}.$$

Then to determine the optimal wage I assume that there is a monopoly union that represents all workers and sets their wage. The monopoly union faces Calvo-style frictions,

$$W_t = \left[(1 - \zeta_l)(\tilde{W}_t)^{\frac{1}{1-\nu_l}} + \zeta_l (\tilde{\pi}_{l,t} (\mu^*_{z,t})^{i_\mu} (\mu^*_{z,t})^{1-i_\mu} W_{i-1})^{\frac{1}{1-\nu_l}} \right]^{1-\nu_l},$$

with probability $1 - \zeta_l$. The monopoly can reoptimize the wage, while with probability ζ_l, the monopoly cannot reoptimize.

If the monopoly cannot reoptimize, then it sets the wage according to:

$$W_{w,t} = (\mu_{z^*,t})^{i_l} (\mu^*_z)^{1-i_l} \tilde{\pi}_{l,t} W_{w,t-1},$$

where $\tilde{\pi}_{l,t} = (\pi^*_{t,t})^{i_l} (\pi^*_{l,t-1})^{1-i_l}$. The wage must be equal to the previous wage level at time $t - 1$ adjusted by the inflation and growth rates.

Households. All households in this economy have identical preferences, which take the form:

$$\begin{aligned} &\text{maximize} \quad E_0\Sigma_{t=0}^{\infty}\beta^t\left\{\zeta_{c,t}(\log(C_t - bC_{t-1})) - \psi_l\frac{(L_t)^{1+\sigma_l}}{1+\sigma_l}\right\} \\ &\text{subject to} \quad (1+\tau^c)P_tC_t + T_t \leq (1-\tau^l)W_tL_t + (1+R_t)T_{t-1}, \end{aligned} \tag{3.3}$$

where $0 < \beta < 1$ is the discount factor, C_t is per capita consumption, T_t is the deposit, and b is the internal habit in consumption. The parameter σ_l is the curvature on disutility of labor, and ψ_l is the disutility weight on labor. $\zeta_{c,t}$ is the preference shock and is assumed to evolve as follows:

$$\zeta_{c,t} = \rho_{\zeta_c}\zeta_{c,t-1} + \epsilon_t^{\zeta_c}.$$

Capital Producers. In this environment, there is a representative producer of capital that operates the following technology:

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\big(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\big)\right)I_t,$$

where capital decays at the fixed rate $0 < \delta \leq 1$. According to this equation, the new capital depends on the existing capital and investment good I_t. The quantity of investment at period t is proportional to the adjustment cost function S [6], and $\zeta_{I,t}$ denotes the shock to the MEI, which is assumed to obey an AR(1) process $\zeta_{i,t} = \rho_{\zeta_i}\zeta_{i,t-1} + \epsilon_t^{\zeta_i}$, where $\rho_{\zeta_i} \in (0,1)$. The idea is simple: the capital producer combines the previous capital with the investment goods to produce a new capital, which is supplied to entrepreneurs.

Entrepreneurs. The entrepreneur defines the utilization rate of capital with the user cost function, which equals the return on renting capital services:

$$\frac{P_t}{Y_t}a(u_t)\omega K_t = \tilde{r}_t^k P_t u_t \omega K_t,$$

[6] As in Smets and Wouters (2007) and Christiano et al. (2014), $S(x_t) = \frac{1}{2}\{\exp[\sqrt{S''}(x_t - x)] + \exp[-\sqrt{S''}(x_t - x)] - 2\}$, where $x_t = \zeta_{I,t}\frac{I_t}{I_{t-1}}$, and x is the steady state of x_t.

where Υ_t denotes the investment specific technical change shock, and the adjustment cost function is specified as $a(u) = r^k(\exp[\sigma_a(u-1)]-1)\frac{1}{\sigma_a}$.

Then, the return on capital is given by:

$$\{(1-\tau^k)[\tilde{r}^k_{t+1}u_{t+1} - \frac{a(u_{t+1})}{\Upsilon_{t+1}}]P_{t+1} + (1-\delta)Q^k_{t+1} + \tau^k\delta Q^k_t\}K_{t+1}\omega.$$

This expression can be simplified to $(1+R^k_{t+1})Q^k_tK_{t+1}\omega$. In period $t+1$, the entrepreneur enjoys the average gross nominal rate of return on capital:

$$(1+R^k_{t+1}) = \frac{(1-\tau^k)[\tilde{r}^k_{t+1}u_{t+1} - \Upsilon^{-1}_{t+1}a(u_{t+1})]P_{t+1} + (1-\delta)Q^k_{t+1} + \tau^k\delta Q^k_t}{Q^k_t}.$$

Each entrepreneur purchases capital good K_{t-1} at price Q^k_{t-1} using loans M_{t-1} obtained from banks and net worth N_{t-1}. Then,

$$K_{t-1}Q^k_{t-1} = M_{t-1} + N_{t-1}.$$

Entrepreneurs are hit by an idiosyncratic shock ω, which describes the case when entrepreneurs are unable to pay their debt. Letting $\sigma_{e,t}$ denote the standard deviation of $\log\omega$, which obeys an AR(1) process $\sigma_{e,t} = \sigma_{e,ss}\left(1+\epsilon^{\sigma_e}_t\right) + \rho_{\sigma_e}\left(\sigma_{e,t-1} - \sigma_{e,ss}\right)$; thus, the risk level increases when σ_e goes up. The default threshold for entrepreneurs can be defined as:

$$\bar{\omega}_t = \frac{(1+R^e_{t-1})M_{t-1}}{(1+R^k_t)\kappa_{t-1}Q^k_{t-1}K_{t-1}}.$$

Entrepreneurs go bankrupt when $\omega \leq \bar{\omega}_{t+1}$ as they are unable to pay the interest and principal. As a result, the pledged assets are seized by the bank. The coefficient of collateral κ obeys to an AR(1) process $\kappa_t = \rho_\kappa\kappa_{t-1} + \epsilon^\kappa_t$.

Entrepreneurs' expected net worth [7] is given by:

$$E_t\left\{\int_{\bar{\omega}_{t+1}}^{\infty}[(1+R^k_{t+1})\omega Q^k_t K_t-(1+R^e_t)M_t]dF_t(\omega)\right\},$$

where R^k_{t+1} is the rate of return on capital, and R^e_{t+1} is the net interest rate paid by entrepreneurs on their debt M_t. Thus, the problem for entrepreneurs is to maximize their earnings, which can be rewritten as:

$$E_t[1-\kappa_t\Gamma_t(\bar{\omega}_{t+1})](1+R^k_{t+1})Lev_tN_t,$$

subject to participation constraint: [8]

$$\begin{aligned} & E_t\Bigg\{[1-F_t(\omega_{t+1})](1+R^e_t)M_t \\ +(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF_t(\omega)(1+R^k_{t+1})\kappa_t Q^k_t K_t \geq (1+R_{t+1})M_t & \Bigg\}. \end{aligned} \tag{3.4}$$

The term ω captures the idiosyncratic risk in businesses. The threshold value of the idiosyncratic risk in businesses $\bar{\omega}$ is defined such that if $\omega > \bar{\omega}$, the borrower retains the collateral and pay $(1+R^e_t)M_t$. If $\omega < \bar{\omega}$, the bank will seize the collateral $\kappa_t(1+R^k_{t+1})Q^k_t K_t$ and pay a monitoring cost μ associated with the contract. R^e_t is the contractual interest rate, $\bar{\omega}$ is the threshold level under which an entrepreneur declares bankruptcy and cannot pay back the debt, and R_{t+1} is the risk free interest rate.

Note that the equation of entrepreneur bank participation (3.4) must hold with strict equality in every state of nature; it cannot be violated because that would mean that the bank would make profits or loss. The first term on the left-hand side of the

[7] See Appendix D for detailed computations.

[8] The participation constraint can be simplified to:

$$E_t\left\{\kappa_t[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]=\frac{Lev_t-1}{Lev_t}\frac{(1+R_{t+1})}{(1+R^k_{t+1})}\right\}.$$

equation corresponds to the returns from non-defaulting entrepreneurs. The second term corresponds to the returns from defaulting entrepreneurs whose collateral is seized by banks and net of monitoring cost μ. The right-hand side describes the return on loans given the interest rate demanded by banks R_{t+1}.

The intuition behind this condition is as follows. The financial intermediary suffers no loss when providing loans to entrepreneurs. That is to say that in equilibrium, banks earn an expected return of $(1 + R_{t+1})M_t$ at the riskless interest rate R_{t+1}. Either the entrepreneur pays the debt $(1 + R^e_t)M_t$ with probability $[1 - F_t(\omega_{t+1})]$ and keeps the collateral, or declares bankruptcy with probability $\int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$ and the bank will seize the collateral $\kappa_t Q^k_t K_t$ net of monitoring cost μ that the bank pays to monitor defaulting entrepreneurs.

For the surviving entrepreneurs, their net worth can be written as:

$$N_t = \gamma^e \big([1 - \kappa_{t-1}\Gamma_{t-1}(\bar{\omega}_t)](1 + R^k_t) Q^k_{t-1} K_{t-1} \big) + w.$$

Entrepreneurs obtain the aggregate profit $[1 - \kappa_{t-1}\Gamma_{t-1}(\bar{\omega}_t)](1 + R^k_t) Q^k_{t-1} K_{t-1}$ at the end of the period. γ^e is the percentage of entrepreneurs who survive and receive w a transfer payment when new entrepreneurs enter in the next period. Financial wealth describes the case where only a fraction of entrepreneurs survives and accumulates their wealth in addition to the wealth belonging to defaulting entrepreneurs. Thus, defaulting entrepreneurs exit the market.[9]

Following Schmitt-Grohe and Uribe (2012), I characterize the aggregate firm value given by:

$$V_t = Y_t - W_t L_t - \Upsilon I_t + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} Q^k_{t+1} K_{t+1},$$

[9] This assumption about free entry and exit of entrepreneurs from the market is frequently used in the literature; see Bernanke et al. (1999).

which equals the current dividend and present discounted value of future dividends. Notice that the relative price of capital $Q_t^k = \frac{V_t}{K_t} + s_t^*$ includes two components: average capital price and speculative component. These components can be interpreted as a price bubble that explains the fluctuations in capital market and macroeconomic aggregates.

Aggregation. In general, the labor market, the consumption good market, and the loan market clear. It is convenient now to set K_t and N_t, which denote, respectively, the aggregate capital services, and the aggregate net worth of entrepreneurs, and can be written as:

$$N_t = \int_0^1 N_{j,t} dj \qquad \text{(net worth clearing)} \tag{3.5}$$

$$K_t = \int_0^1 K_{j,t} dj \qquad \text{(capital market clearing).} \tag{3.6}$$

Monetary Policy Rule. I assume that monetary policy obeys a standard Taylor rule. The linearized form of monetary policy rule is given by:

$$R_t - R = \rho_p(R_{t-1} - R) + (1 - \rho_p)[a_\pi(E_t\pi_{t+1} - \pi_{trg,t}) + a_{\Delta y}(Y_t - Y)],$$

where ρ_p is a smoothing parameter, a_π is the policy weight on inflation, and $a_{\Delta y}$ is the policy weight on output growth. This rule states that changes in interest rate R_t depend on the deviation between a central bank's inflation target and expected inflation rate and also on the deviation between the output and its steady state.

Resource Constraints. I complete the setup of the model by specifying the main aggregate resource constraint, which can be written as:

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\frac{K_t}{\Upsilon \mu_{z,t}^*} + \theta\frac{1-\gamma^e}{\gamma^e}(N_{t+1} - w).$$

The first term on the right-hand side of the resource constraint equation $D_t^{w,e}$ represents banks' spending on monitoring entrepreneurs with

$$D_t = \kappa_{t-1}\mu G(\bar{\omega})(1+R_t^k)\frac{Q_{k,t-1}K_t}{\pi_t \mu_{z,t}^*},$$

while G_t denotes the government consumption, which is given by

$$G_t = z_t^* g_t,$$

where g_t is a shock that follows an AR(1) process $g_t = g_{ss}\left(1+\epsilon_t^g\right) + \rho_g\left(g_{t-1} - g_{ss}\right)$. C_t is the aggregate consumption. The term $\frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}}$ defines the aggregate investment, and $a(u_t)\frac{K_t}{\Upsilon \mu_{z,t}^*}$ is the aggregate capital utilization costs of entrepreneurs.

Stochastic Processes The model is subject to nine structural exogenous shocks. The shock to price markup is defined as:

$$\nu_{p,t} = \rho_{\nu_p}\nu_{p,t-1} + \epsilon_t^{\nu_p}, \tag{3.7}$$

where the shock captures changes in price markup, the disturbance $\epsilon_t^{\nu_p}$ has mean zero, and the standard deviation σ_{ν_p}.

The nonstationary investment shock Υ_t, which is a second source of aggregate fluctuations, is assumed to have a growth rate of $\mu_{\Upsilon,t} = \frac{\Upsilon_t}{\Upsilon_{t-1}}$. Investment-specific technical change shock is assumed to obey to the following law of motion:

$$\mu_{\Upsilon,t} = \rho_{\mu_\Upsilon}\mu_{\Upsilon,t-1} + \epsilon_t^{\mu_\Upsilon}, \tag{3.8}$$

where $\epsilon_t^{\mu_\Upsilon}$ represents an innovation to the growth rate of investment specific productivity, has mean zero and the standard deviation σ_{μ_Υ}.

I assume that government spending evolves according to the following law of motion:

$$g_t = \rho_g g_{t-1} + \epsilon_t^g, \tag{3.9}$$

where the disturbance ϵ_t^g is an i.i.d process with mean zero and the standard deviation σ_g.

The nonstationary productivity shock z_t, which is an important source of aggregate fluctuations, is assumed to have a growth rate of $\mu_{z,t}^* = \frac{z_t}{z_{t-1}}$. The technological trend $\mu_{z,t}^*$ is assumed to obey to the following law of motion:

$$\mu_{z,t}^* = \rho_{\mu_z^*} \mu_{z,t-1}^* + \epsilon_t^{\mu_z^*}, \tag{3.10}$$

where $\epsilon_t^{\mu_z^*}$ has mean zero and the standard deviation $\sigma_{\mu_z^*}$.

The stationary technology shock is assumed to follow the autoregressive process:

$$\gamma_t = \rho_\gamma \gamma_{t-1} + \epsilon_t^\gamma, \tag{3.11}$$

where the innovation ϵ_t^γ has mean zero and the standard deviation σ_γ.

I assume that the default probability is given by:

$$F_{t-1}(\bar{\omega}_t) = \frac{log(\bar{\omega}_t) + \frac{(\sigma_{e,t-1})^2}{2}}{\sigma_{e,t-1}},$$

where $\bar{\omega}_t$ is an idiosyncratic shock, with $\bar{\omega}_t \in [0, \infty)$ and $E(\bar{\omega}_t) = 0$. The standard deviation of log $(\bar{\omega}_t)$ is given by $\sigma_{e,t-1}$. The risk shock $\sigma_{e,t-1}$ evolves over time according to the following law of motion:

$$\sigma_{e,t} = \rho_{\sigma_e} \sigma_{e,t-1} + \epsilon_t^{\sigma_e}, \tag{3.12}$$

the disturbance $\epsilon_t^{\sigma_e}$ has a mean of zero and the standard deviation σ_{σ^e}.

The consumption preference evolves over time according to the law of motion:

$$\zeta_{c,t} = \rho_{\zeta_c}\zeta_{c,t-1} + \epsilon_t^{\zeta_c}, \tag{3.13}$$

where current consumption preferences are explained by past consumption preferences and distrubted by an error term time $\epsilon_t^{\zeta_c}$, which has a mean of zero and the standard deviation σ_{ζ^c}.

The law of motion of marginal efficiency of investment can be written as:

$$\zeta_{i,t} = \rho_{\zeta_i}\zeta_{i,t-1} + \epsilon_t^{\zeta_i}, \tag{3.14}$$

where $\zeta_{i,t}$ represents a disturbance to the process that transforms an investment good into productive capital. The term $\epsilon_t^{\zeta_i}$ has a mean of zero and the standard deviation σ_{ζ^i}.

The shock to collateral requirement evolve over time according to the following law of motion:

$$\kappa_t = \rho_\kappa \kappa_{t-1} + \epsilon_t^\kappa, \tag{3.15}$$

where the current collateral requirement depends on past realizations of itself. The disturbance ϵ_t^κ has a mean of zero and the standard deviation σ^κ.

Equilibrium Definition

Household's optimality conditions. The first order conditions with respect to C_t, T_t, L_t are as follows:

$$\lambda_{z,t}(1+\tau_c)P_t - \frac{\zeta_{c,t}}{C_t - bC_{t-1}} + b\beta E_t \frac{\zeta_{c,t+1}}{C_{t+1} - bC_t} = 0 \tag{3.16}$$

$$\lambda_{z,t} - \beta E_t \lambda_{z,t+1}(1 + R_{t+1}) = 0 \tag{3.17}$$

$$- \psi_l (L_t)^{\sigma_l} - \lambda_{z,t} W_t = 0. \tag{3.18}$$

For each period, the representative household chooses how much to consume C_t, to save T_t, and decides on labor supply L_t. This choice maximizes the utility function using the budget constraint. The household takes P_t, R_{t+1}, W_t, $\lambda_{z,t}$ as given.

Labor contractors optimality conditions. Labor services are provided by households to intermediate firms via labor contractors. Labor contractors are introduced into the model to buy the differentiated labor from households and sell it to intermediate good producers. They set the wage rate W_t subject to calvo frictions, with probability $1 - \zeta_w$ contractors can optimize wages and with probability ζ_w they can keep the wage unchanged. The calvo wage auxilary variables $K_{w,t}$ and $F_{w,t}$, the aggregate wage index W_t, and the first order condition with respect to the optimized wage $\tilde{W}_t$ are as follows:

$$K_{w,t} = (L_t)^{1+\sigma_L} + \beta \zeta^l E_t \left[\left(\frac{\tilde{\pi}_{w,t+1} (\mu^*_{z,t+1})^{\iota_\mu} (\mu^*_z)^{1-\iota_\mu}}{\pi_{w,t+1}} \right)^{\frac{\nu_l}{1-\nu_l}(1+\sigma_L)} K_{w,t+1} \right] \tag{3.19}$$

$$F_{w,t} = \frac{L_t}{\nu_l} P_t \lambda_{z,t} + \beta \zeta^l E_t \big(\tilde{\pi}_{w,t+1} (\mu^*_{z,t+1})^{\iota_\mu} (\mu^*_z)^{1-\iota_\mu} \big)^{\frac{1}{1-\nu_l}} \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\nu_l}{1-\nu_l}} \frac{1}{\pi_{t+1}} F_{w,t+1} \Big] \tag{3.20}$$

$$W_t = \big[(1-\zeta_l)(\tilde{W}_t)^{\frac{1}{1+\nu_l}} + \zeta_l \big(\tilde{\pi} (\mu^*_{z,t})^{\iota_w} (\mu^*_z)^{1-\iota_w} W_{t-1} \big)^{\frac{1}{1+\nu_l}} \big]^{1+\nu_l} \tag{3.21}$$

$$\frac{K_{w,t}}{F_{w,t}} = \left(\frac{\tilde{W}_t}{W_t} \right)^{\frac{1-\nu_l(1+\sigma_L)}{1-\nu_l}} \frac{W_t}{P_t} \frac{1}{\Psi_L}. \tag{3.22}$$

The labor contractor solves for $F_{w,t}$, $\tilde{W}_t$, W_t, $K_{w,t}$ and takes L_t, P_t, $\lambda_{z,t}$, $\pi_{w,t+1}$ as given.

Producers optimality conditions. The production function is given by:

$$Y_{j,t} = \gamma_t (u_t K_{j,t-1})^{\alpha} (z_t L_{j,t})^{1-\alpha} - \Phi z^*_t.$$

The producer optimality conditions with respect to capital and labor and the euler equation are derived as follows:

$$\begin{aligned}
\lambda_1 &= \frac{W_t}{(1-\alpha)\gamma_t(u_t K_{j,t-1})^{\alpha}(z_t L_{j,t})^{-\alpha}} \\
\lambda_{t,1} &= \frac{r_t^k}{\alpha\gamma_t(z_t L_{j,t})^{1-\alpha}(u_t K_{j,t-1})^{\alpha-1}} \\
\frac{K_{t-1}}{L_t} &= \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{r_t^k}.
\end{aligned} \tag{3.23}$$

The Lagrange multiplier λ_1 associated with the production function is derived as follows:

$$\lambda_{t,1} = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{(r_t^k)^{\alpha}(W_t)^{(1-\alpha)}}{\gamma_t}. \tag{3.24}$$

The producer solves for K_{t-1}, L_t, $\lambda_{t,1}$, and takes W_t, r_t^k as given.

Calvo price setting. The calvo price auxilary variables $K_{p,t}$ and $F_{p,t}$, the aggregate price index P_t, and the first order condition with respect to the optimized price $\tilde{P}_t$ are as follows:

$$K_{p,t} = \lambda_{1,j,t}\nu_{t,p}Y_t + \beta\zeta^p E_t\left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}}\right)^{(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}})}K_{p,t+1}\right] \tag{3.25}$$

$$F_{p,t} = \nu_{t,p}Y_t + \beta\zeta^p E_t\left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}}\right)^{(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}})}F_{p,t+1}\right] \tag{3.26}$$

$$P_t = \left[(1-\zeta_p)(\tilde{P}_t)^{\frac{1}{1+\nu_p}} + \zeta_p(\tilde{\pi}P_{t-1})^{\frac{1}{1+\nu_p}}\right]^{1+\nu_p} \tag{3.27}$$

$$\tilde{P}_t = \nu_p\frac{K_{p,t}}{F_{p,t}}. \tag{3.28}$$

The producer solves for $K_{p,t}$, $F_{p,t}$, P_t, $\tilde{P}_t$ and takes $\lambda_{t,1}$, Y_t, π_{t+1} as given.

Capital producer optimality conditions. Capital producers accumulate capital according to the following expression:

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right) I_t. \tag{3.29}$$

The first order condition of investment is as follows:

$$\begin{aligned}
&\lambda_{z,t} Q_t^k \left[1 - S\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right) - \zeta_{I,t}\frac{I_t}{I_{t-1}} S'\left(\zeta_{I,t}\frac{I_t}{I_{t-1}}\right)\right] \\
&- \frac{\lambda_{z,t} P_t}{\Upsilon_t \mu_{\Upsilon,t}} + \beta \lambda_{z,t+1} Q_{t+1}^k \zeta_{I,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S''\left(\zeta_{I,t+1}\frac{I_{t+1}}{I_t}\right) = 0.
\end{aligned} \tag{3.30}$$

The capital producer solves for investment I_t and capital K_t, and takes $\lambda_{z,t}$, P_t, Y_t, W_t, L_t, Υ_t, Q_t^k as given.

Entrepreneur's optimality conditions. The relative price of capital is given by:

$$Q_t^k = \frac{Y_t - W_t L_t - \Upsilon_t^{-1} I_t}{K_t} + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} Q_{t+1}^k \frac{K_{t+1}}{K_t} + s_t^*.$$

Entrepreneurs set the utlization rate of capital u_t at a rental rate of r_t^k.

$$r_t^k = r_{ss}^k \exp\left(\sigma_a(u_t - 1)\right) \tag{3.31}$$

$$(1 + R_{t+1}^k) = \frac{(1-\tau^k)\left(u_{t+1}\tilde{r}_{t+1}^k - \Upsilon_{t+1}^{-1} a(u_{t+1})\right) P_{t+1} + (1-\delta) Q_{t+1}^k}{Q_t^k} + \tau^k \delta. \tag{3.32}$$

Entrepreneurs solve for the utilization rate u_t and capital rental rate r_t^k and take R_{t+1}^k, Υ_{t+1}, Q_{t+1}^k, P_{t+1} as given.

Each entrepreneur buys capital K_t at price Q_t^k from capital producers, selects the optimal debt contract by choosing the value of the firm $Q_t^k K_t$ and the loan M_t that maximizes the expected net worth N_t:

$$K_{t-1}Q^k_{t-1} = M_{t-1} + N_{t-1}.$$

The first order condition with respect to default threshold is derived as follows:

$$\begin{aligned}
&E_{t-1}\Bigg[\Big[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\Big]\frac{1+R^k_t}{1+R_{t-1}} \\
&+ \frac{\big[G^{e'}_{t-1} + \kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - G^{e'}_{t-1}(\bar{\omega}_t)]\big]}{\kappa_{t-1}\big[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - \mu G^{e'}_{t-1}(\bar{\omega}_t)\big]} \\
&\left(\kappa_{t-1}\big[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\big]\frac{(1+R^k_t)}{(1+R_{t-1})} - 1\right)\Bigg] = 0. \qquad (3.33)
\end{aligned}$$

The bank zero profit condition holds:

$$\kappa_{t-1}\big[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\big]\frac{(1+R^k_t)}{(1+R_{t-1})} = \frac{Q^k_{t-1}K_{t-1} - N_{t-1}}{(Q^k_{t-1}K_{t-1})}. \qquad (3.34)$$

The collateral constraint holds:

$$\bar{\omega}_t\kappa_{t-1}(1+R^k_t)Q^k_{t-1}K_{t-1} = (1+R^e_t)M_{t-1}.$$

The entrepreneurs net worth evolves as follows:

$$\begin{aligned}
N_t =& \gamma^e_t\Big(Q^k_{t-1}K_{t-1}[(1+R^k_{t-1}) - (1+R_{t-1}) + \mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R^k_{t-1})] \\
&+ (1+R_{t-1})N_{t-1})\Big) + w. \qquad (3.35)
\end{aligned}$$

The entrepreneur solves for R^k_t, ω, M_{t-1}, N_{t-1}, takes as given $F_t(\omega)$, R_{t-1}, $\Gamma_{t-1}(\bar{\omega}_t)$, $G_{t-1}(\bar{\omega}_t)$, K_{t-1}, and Q^k_{t-1}.

The Taylor rule is defined as:

$$R_t - R = \rho_p(R_{t-1} - R) + (1 - \rho_p)[a_\pi(E_t\pi_{t+1} - \pi_{trg,t}) + a_{\Delta y}(Y_t - Y)].$$

The resource constraint is given by:

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\frac{K_t}{\Upsilon \mu^*_{z,t}} + \theta\frac{1-\gamma^e}{\gamma^e}(N_{t+1} - w).$$

The dynamic competitive equilibrium is defined by a set of quantities and prices $\{\pi_t, \lambda_{1,t}, r^k_t, I_t, u_t, \bar{\omega}_t, R^k_t, N_{t-1}, \lambda_{z,t}, C_t, \tilde{W}_t, L_t, K_t, R_t, P_t, W_t, F_{p,t}, F_{w,t}, Y_t, M_t, Q^k_t, \tilde{P}_t, s^*_t\}$.

4 Quantitative Implementation

In this section, I first present the calibrated parameters of the model. Afterward, I estimate the unknown parameters and shocks using Bayesian methods. Finally, I report the moments of the model and the data and compare the model predictions with the data.

4.1 Econometric Estimation of Parameters

Once I solve for the model using the Lagrangian methods or substituting out the constraint, I can detrend the model using a specific trend growth for each variable. After that, I can compute nonstochastic steady states (Model derivations are reported in Appendix D). The next step will be estimating the model by proceeding in the following three stages: (i) I calibrate some economic parameters. (ii) I construct empirical data to combine it with the model equation. I estimate the remaining economic and shock parameters using the Bayesian methods and quarterly US data over the period 1998–2018.

Calibrated Parameters. I calibrate some economic parameters of the model following Christiano et al. (2010). First, I start with the following set of parameters:

$$\{\beta, \sigma^L, b, \nu^l, \mu_z^*, \delta, \alpha, \nu^p, \Phi, \gamma^e, \mu, \sigma^e, w, \Theta, \tau^k, \tau^l, \tau^c, \Upsilon, x, g\}$$

The baseline parameters are reported in Table 2. Afterward, I set the steady state value of stocastic process γ_t, $\mu_{\Upsilon,t}$, $\zeta_{I,t}$, $\zeta_{c,t}$, κ_t to one, and the capital utlization rate u_t to one.

Table 2: Calibrated Parameters

Par.	Description	Value
β	Discount rate	0.9966
σ^L	Curvature on disutility of labor	1
b	Habit persistence parameter	0.63
ν^l	Steady state markup suppliers of labor	1.05
μ_z	Growth rate of the economy	1.0036
δ	Depreciation rate on capital	0.025
α	Power on capital in production function	0.4
ν^p	Steady state markup, intermediate good firms	1.2
Φ	Fixed cost, intermediate goods	0.07
γ^e	Percent of entrepreneurs who survive	0.9762
μ	Fraction of realized profits lost in bankruptcy	0.94
$Var(\log \omega)$	Variance of log of idiosyncratic productivity	0.24
w	Transfer from households	0.009
Θ	Fraction of net worth of entrepreneurs exiting the economy	0.1
τ^c	Tax rate on consumption	0.05
τ^k	Tax rate on capital income	0.32
τ^l	Tax rate on labor income	0.24
Υ	Trend rate of ISTC	1.0035
x	Growth rate of monetary base	3.71/400
g	Share of government consumption	0.2

Household: I set the discount factor β, the elasticity of labor supply σ^L, and the consumption habit b to 0.9966, 1, and 0.63, respectively, chosen to yield an annual policy rate R of 5.18%. I fix the wage markup ν^l to 1.05.

Production: I take a standard value of the depreciation rate on capital δ to equal 0.025. I fix the price markup ν^p, the growth rate of the economy μ_z, and the power on capital in production function α to 1.2, 1.0036, and 0.4, respectively, which are in line with the range of values in the literature of business cycle.

Entrepreneurs: I set the fraction of net worth of entrepreneurs exiting the economy Θ to 0.1. I take the value of the standard deviations of the idiosyncratic productivity shock

0.24 to solve for ω. The percentage of entrepreneurs who survive γ^e is set at 0.9762, and the transfer received by entrepreneurs w is fixed at 0.009. The parameter μ is chosen to target the value 1.566, which is close to the ratio of external financing in the data.

Policy and shocks: The growth rate of monetary base x is set to 3.71/400 to obtain central bank inflation. I take the tax rate on consumption τ^c equal to 0.05 to target λ_z. I also set tax rates on capital τ^k and on labor τ^l to 0.32 and 0.24, respectively. The trend rate of investment-specific technological change is set to Y 1.00035. I set $g_t = 0.2$ to match government spending G.

Estimated Parameters. I turn now to describe the procedure to estimate the unknown parameters. For this I use quarterly US data as shown in Figure 5. These time series are transformed such that I obtain nine observables: growth in consumption, growth in output, growth in investment, growth in business net worth, growth in external financing, growth in hours worked, growth in total factor productivity, growth in government spending, and growth in capital price. Appendix A describes in detail how the data is constructed.

Following a Bayesian approach and after some data transformation, I choose prior distributions of selected parameters for estimation. The choice of prior plays a crucial role since it can distort the construction of posterior densities, the strategy I adopt in this paper does not eliminate the risk of weak identification of parameters. The model is very stylized which makes it difficult to discern between endogenous and exogenous sources of persistence in the model and data may produce inaccurate parameter estimates. Typically, I rely on existing empirical literature where the priors are common for some parameters. I use the Metropolis–Hasting algorithm with $10,000$ draws per chain to estimate their posterior distributions over the period 1998–2018 (see Figures 14 and 15). Estimation results are reported in Table 3.

Table 3: Prior and Posterior Distribution of Estimated Parameters

		Prior Distribution			Posterior Distribution [a]			
		Density	Mean	Std. Dev.	Mean	Median	Mode	Std. Dev.
Shock Parameters								
σ_{ν_p}	St. dev. Price Markup Shock	Inv. Gamma	0.0023	0.0033	0.058	0.0581	0.0572	0.0012
σ_{μ^Υ}	St. dev. IST Shock	Inv. Gamma	0.0023	0.0033	0.012	0.0119	0.012	0.0005
σ_g	St. dev. Gov. Spend. Shock	Inv. Gamma	0.0023	0.0033	0.0192	0.0192	0.0192	0.0004
$\sigma_{\mu_z^*}$	St. dev. Tech. Trend Shock	Inv. Gamma	0.0023	0.0033	0.0079	0.0079	0.0078	0.0005
σ_γ	St. dev. Stat. Tech. Shock	Inv. Gamma	0.0023	0.0033	0.0248	0.0249	0.0249	0.0011
σ_{σ^e}	St. dev. Entre. Risk Shock	Inv. Gamma	0.0825	0.1167	0.6789	0.6802	0.6904	0.0271
σ_{ζ^c}	St. dev. Cons. Pref. Shock	Inv. Gamma	0.0023	0.0033	0.1082	0.1082	0.1087	0.0015
σ_{ζ^i}	St. dev. MEI Shock	Inv. Gamma	0.0023	0.0033	0.0709	0.0709	0.0713	0.0027
σ_{κ^k}	St. dev. Coll. Requir. Shock	Inv. Gamma	0.0023	0.0033	0.0793	0.0793	0.0799	0.0017
ρ_{ν_p}	Autoc. Price Markup Shock	Beta	0.5	0.2	0.7247	0.7244	0.7239	0.021
ρ_{μ^Υ}	Autoc. IST Shock	Beta	0.5	0.2	0.9462	0.9461	0.9415	0.0152
ρ_g	Autoc. Gov. Spend. Shock	Beta	0.5	0.2	0.8063	0.8062	0.8093	0.0061
$\rho_{\mu_z^*}$	Autoc. Tech. Trend Shock	Beta	0.5	0.2	0.0547	0.0534	0.053	0.0217
ρ_γ	Autoc. Stat. Tech. Shock	Beta	0.5	0.2	0.7075	0.7044	0.6863	0.026
ρ_{σ^e}	Autoc. Entre. Risk Shock	Beta	0.7	0.2	0.0384	0.0382	0.0461	0.0128
ρ_{ζ^c}	Autoc. Cons. Pref. Shock	Beta	0.5	0.1	0.9906	0.9907	0.9909	0.0014
ρ_{ζ^i}	Autoc. MEI Shock	Beta	0.5	0.2	0.3936	0.3926	0.3964	0.0103
ρ_{κ^k}	Autoc. Coll. Requir. Shock	Beta	0.5	0.2	0.9892	0.9902	0.9915	0.0047
Macroeconomic Parameters								
ζ^l	Calvo Wage Stickiness	Beta	0.75	0.1	0.3534	0.3509	0.3622	0.0192
b	Habit Parameter	Beta	0.5	0.1	0.6691	0.6697	0.6664	0.0043
ι^μ	Wage Indexing Weight on Technology Growth	Beta	0.5	0.15	0.8829	0.8816	0.9103	0.0281
ι^w	Wage Indexing Weight on Inflation Target	Beta	0.5	0.15	0.0914	0.0861	0.096	0.0386
μ	Monitoring Cost	Beta	0.275	0.15	0.9193	0.9211	0.9327	0.0307
σ^a	Utilization Cost Curvature	Normal	1	1	-0.4284	-0.4318	-0.464	0.0375
S	Investment Adjustment Cost Curvature	Normal	5	3	3.8091	3.8124	3.7816	0.0779
ζ^p	Calvo Price Stickiness	Beta	0.5	0.1	0.5544	0.5533	0.5494	0.018
ι^p	Price Indexing Weight on Inflation Target	Beta	0.5	0.15	0.2198	0.2267	0.2138	0.054
α_π	Policy Weight on Inflation	Normal	1.5	0.25	1.2125	1.2015	1.1934	0.0565
$\alpha_{\Delta y}$	Policy Weight on Output Growth	Normal	0.25	0.1	0.4831	0.4738	0.4955	0.0409
ρ_p	Policy Smoothing Parameter	Beta	0.75	0.1	0.8287	0.8275	0.8298	0.0054

[a] The table reports the results of the Bayesian estimation. Posterior statistics are constructed using 10,000 draws per chain.

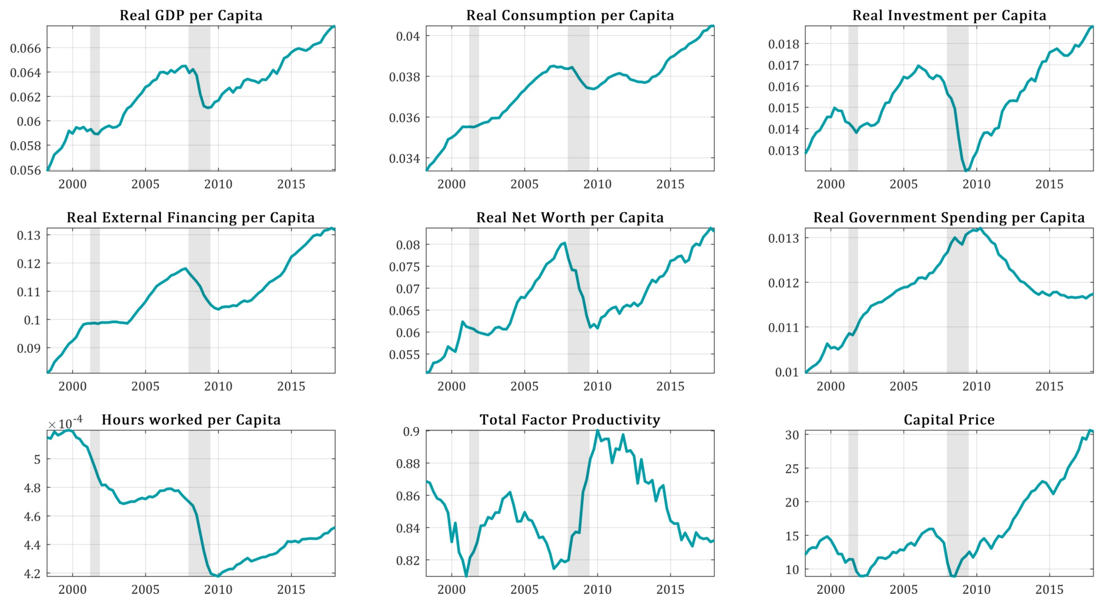

Figure 5: US Data

Note: Quarterly data retrieved from Federal Reserve Bank of St. Louis Database (FRED).

4.2 Model Fit

To assess if the model is reliable to mimic the data, first, I compare the volatility, correlation, and autocorrelation in the model and the data, considering the selected aggregates: consumption, output, investment, business net worth, external financing, hours worked, total factor productivity, government spending, and capital price. Second, I compare the model predictions with data, taking, for example, the ratio of some aggregates and interest rates.

Table 4: Data and Model Moments

	Y	*C*	*I*	*M*	*H*	*N*	*TFP*	*G*	*Q*
				Standard Deviation					
Model	2.12	1.61	8.02	5.02	10	15.98	0.56	0.91	3.04
Data	0.6	0.37	2.3	1.06	0.73	2.2	0.94	0.79	6.55
				Correl. with Δ Output					
Model	1	0.46	0.86	0.26	0.06	-0.04	0.18	0.19	0.29
Data	1	0.66	0.86	0.5	0.6	0.5	0.06	0.28	0.01
				First Order Autocorrel.					
Model	0.56	0.8	0.49	0.6	0.84	-0.28	0.05	-0.1	0.79
Data	0.36	0.67	0.59	0.79	0.73	0.37	-0.07	0.48	0.45

[a] The columns *Y*, *C*, *I*, *M*, *H*, *N*, *TFP*, *G*, and *Q* refer to the growth rates of output, consumption, investment, external financing, net worth, total factor productivity, government spending, and capital price.

In Table 4, I analyze the volatility in the model and the data using the standard deviation statistic. The model shows a high volatility of all aggregates with respect to the data. Notice that investment, hours worked, net worth, external financing, and output are the most volatile aggregates; however, aggregate hours growth, aggregate business net worth growth, and aggregate investment growth perform poorly in mimicking their analogues in the data. When I compare the correlation between output growth and the growth in nine macroeconomic aggregates, the model has a relatively modest performance in fitting the data. Consumption, investment, hours, and external financing are positively correlated to output growth, and only net worth supply is negatively correlated to output growth. A positive serial correlation for consumption, external financing, investment, and hours shows that these aggregates have a positive influence on itself over time. In contrast with the data, the model shows that net worth, government spending, and total factor productivity have upward and downward patterns. In the model, net worth has a negative influence on itself in the 1^{st} lag and similarly for government spending, while in the data, total factor productivity has a negative influence on itself in the 1^{st} lag.

Table 5 shows a good performance of the model in matching data properties for investment, consumption, external financing, and government spending. Interest rate data over the sample period also matches the model predictions.

Table 5: Model versus Data

Variables	Model	Data
Investment-Output Ratio $\left(\frac{I}{Y}\right)$	0.222	0.246
Consumption-Output Ratio $\left(\frac{C}{Y}\right)$	0.562	0.599
External Financing-Output Ratio $\left(\frac{K-N}{Y}\right)$	1.566	1.731
Gov. Spending-Output Ratio $\left(\frac{G}{Y}\right)$	0.2	0.189
Cost of External Finance (R^e)	6.21	5.115
Rate of Return on Capital (R^k)	10.51	10.726

4.3 Collateral Requirement Process

Now I examine the estimated collateral coefficient over the period 1998–2018. The time variation of κ gives an intuition on how collateral policy was conducted, considering the changes in collateral capacity of entrepreneurs and economic conditions.

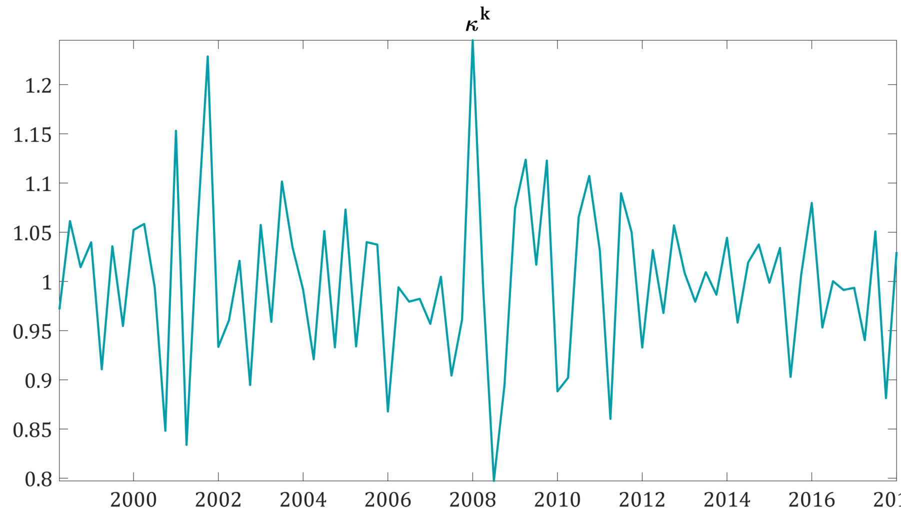

Figure 6: Collateral Requirement

Figure 6 portrays the exogenous innovations to collateral from the estimated model. The collateral requirement coefficient κ shows that there is a sharp increase during the financial crisis of 2008. The model provides an evidence in support of the volatility of collateral requirements during the period 2007–2012. This movement can be interpreted as a continuous change in bank policies regarding collateral requirements, either tightening or easing collateral requirements. Prior to the financial crisis, banks provide loans excessively by conducting a relaxed collateral policy as banks' collateral policies remain stable. The movement of collateral coefficient can be interpreted as a change in banks' collateral policies by increasing their collateral requirements. That is, they are less willing to lend against a collateral.

5 The Effect of Collateral Shocks

To compare different shocks, I look at the responses of the main macroeconomic variables to a collateral requirement shock. I explain how the real activity is affected by financial shocks, in particular, collateral shock. I comment on the impulse responses of the collateral capacity of entrepreneurs to financial shocks. I select the shocks that affect collateral, namely, collateral requirements, entrepreneur risk, ISTC, and MEI.

Collateral Requirements

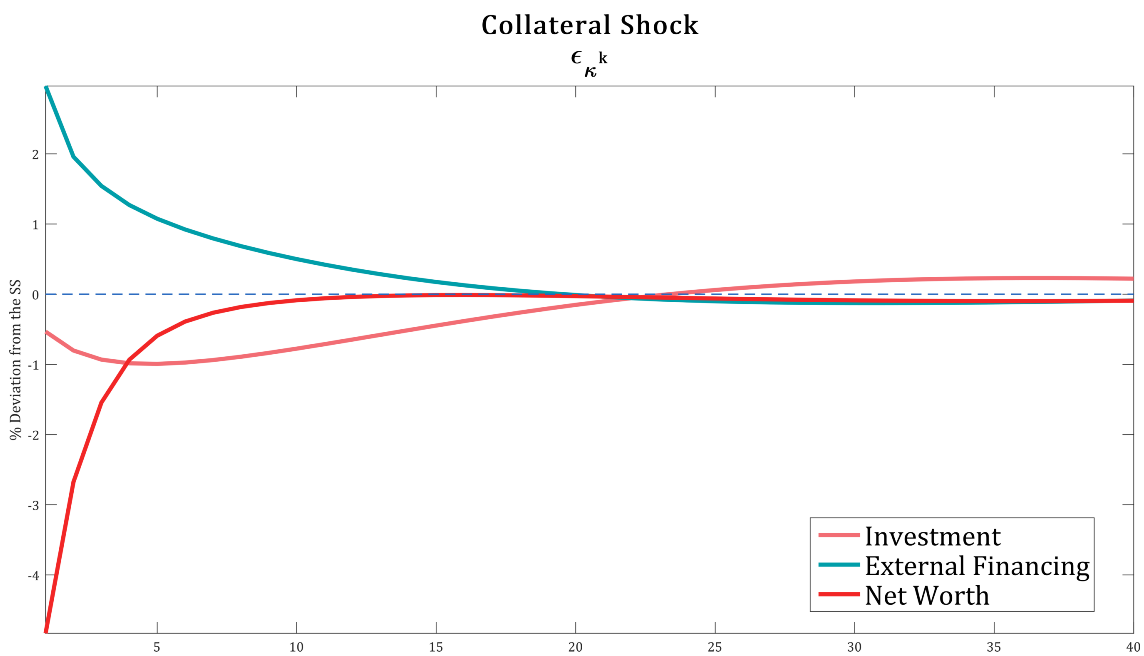

Figure 7: Impulse Responses to Collateral Requirement Shocks

Note: Time horizon is quarter. The figure shows the impulse responses of investment, external financing, and net worth to a positive collateral requirement shock.

A positive collateral requirement shock, as shown in Figure 7, lowers lending volume, entrepreneur net worth, and default threshold. Collateral requirements have another implication: investment and output decline. In fact, the restriction on financing activities will affect the collateral capacity of entrepreneurs, and then the capital stock used as collateral grows scarce. This will lead to temporary drop in investment. The decline in investment will reduce the profitability of entrepreneurs, while the recovery of investment will induce an increase in entrepreneurs' net worth. A collateral requirement shock provokes a long-lived decrease in consumption. This dynamics can be explained by the fact that both aggregates, wages, and hours worked declined in responses to a collateral requirement shock, and given the assumption about wage stickiness in the model, the decrease in aggregate labor will cause a sharp drop in consumption before readjusting again after few periods (see Appendix C for the detailed impulse responses of the estimated model to all shocks).

Output Responses

A number of interesting interpretations emerge from Figure 8. Bank lending channels propagate a positive collateral requirement shock through a decrease in real activity by less than 0.5% because collateral will become scarce, and subsequently will make obtaining loans more difficult and thus affect the economic activity. On the other hand, output declines in response to a positive shock to the MEI. This shock is transmitted through the investment channel. Furthermore, an investment technical change can raise output by close to 2%. In turn, the entrepreneur risk shock leads to a sharp increase in output and quickly declines to reach its steady state. The risk shock implies a countercyclical credit spread in the first four years after the shock. A high level of risk is associated with a high premium in the entrepreneur interest rate over the risk-free interest rate, which explains the sharp increase in output.

Collateral Capacity of Entrepreneurs

Figure 9 depicts the impulse responses of capital to financial shocks. Capital appears to be more responsive to ISTC and the MEI. In turn, the responses of capital stock to the entrepreneur risk shock and collateral shock are long-lived, while an increase in collateral

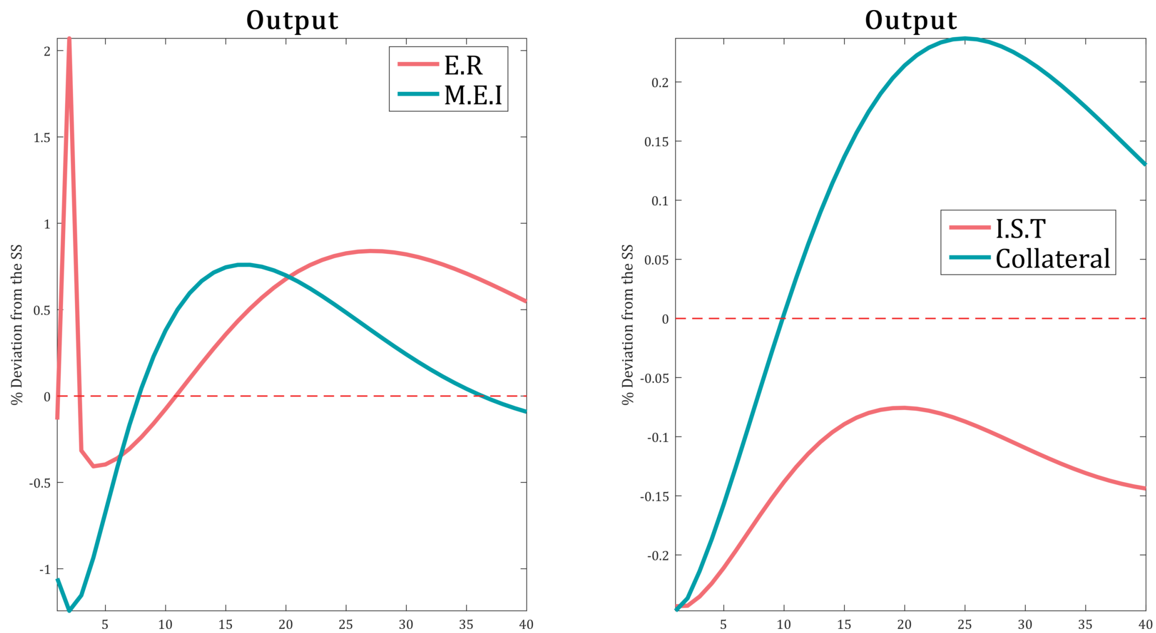

Figure 8: Real Activity Responses to Financial Shocks

Note: Time horizon is quarter. The figure shows the impulse responses of output to a positive collateral requirement shock.

requirements will lead to a decline in capital stock; however, it will quickly recover and decrease to reach its steady state.

6 Measuring the Importance of Collateral Shock

I turn now to analyze the economy model through the shock decomposition of the entire time horizon 1998–2018 with all shocks included. I report the contribution of each shock; one would ideally measure and compare the contribution of a collateral shock. I select the growth in macroeconomic variables, which are output, consumption, investment, external financing, worked hours, net worth, total factor productivity, government spending, and capital price. Table (6) illustrates the contribution of all shocks to growth in nine macroeconomic aggregates.

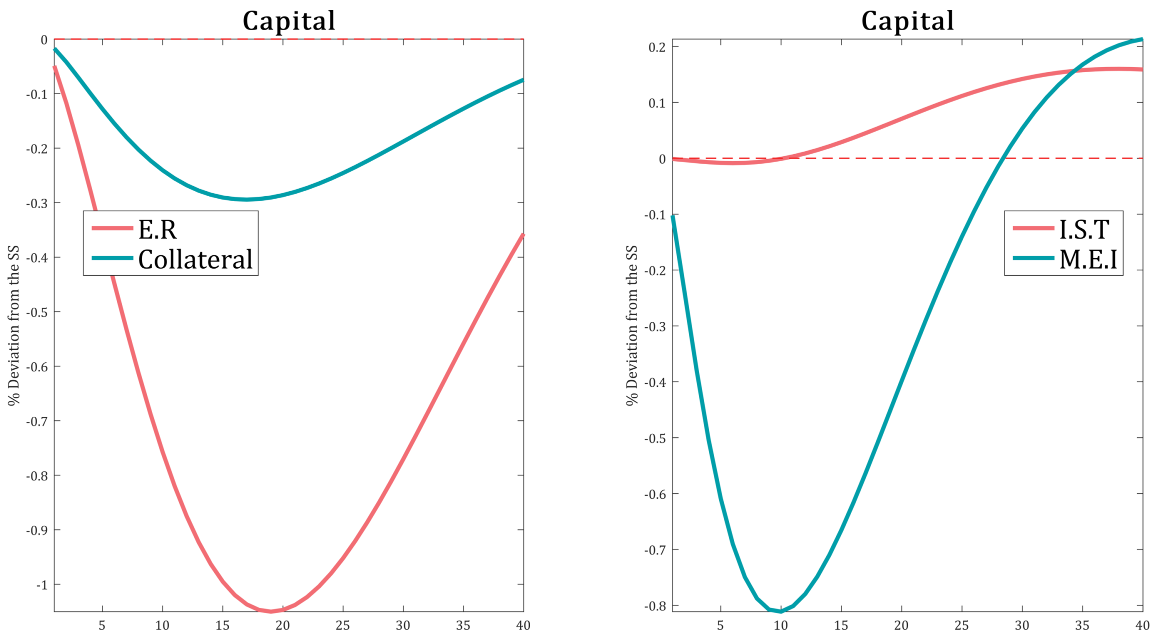

Figure 9: Impulse Responses of Capital to Financial Shocks

Note: Time horizon is quarter. The figure shows the impulse responses of capital to the collateral requirement, entrepreneur risk, ISTC, and MEI shocks.

Price Markup Shocks. The estimation results show that markup disturbance explains 26% of the variance of output growth. This shock also explains a significant fraction of variation in consumption growth (64%), external financing growth (27%), and hours worked growth (14%).

Collateral Requirement Shocks. Collateral requirements are an important source of aggregate fluctuations in this model. The shock accounts for 48% of fluctuations in external financing, 36% of the variance of capital price, and 26% of the variance of net worth.

Marginal Efficiency of Investment and Investment-Specific Technical Change Shocks. I find that the MEI plays a central role in driving fluctuations in the model. More than 17% of investment fluctuations can be attributed to this shock. This disturbance also explains 50% of the unconditional variance of output growth. However, I find that it accounts for

Table 6: Variance Decomposition Predicted by the Model

Shocks [a]	Y	C	I	M	H	N	TFP	G	Q
Price Markup	26	64	7	27	14	4	0	0	12
ISTC [b]	1	0	0	0	0	0	0	0	1
Government Spending	4	0	0	0	0	0	0	100	0
Technology Trend	6	7	1	0	2	0	100	0	4
Stationary Technology	1	6	0	6	22	1	0	0	0
Entrepreneur Risk	1	0	2	17	6	57	0	0	23
Consumption Preference	4	16	0	0	14	0	0	0	1
MEI [c]	50	2	77	1	22	11	0	0	23
Collateral Requirement	6	3	12	48	19	26	0	0	36

[a] Variance decomposition in percentage of the entire time horizon 1998–2018. Parameters are set at posterior mean. Columns refer to the growth rate of output, consumption, investment, external financing, hours, net worth, total factor productivity, government spending, and capital price.
[b] Investment specific technical change.
[c] Marginal efficiency of investment.

22% of the variance of hours worked growth and 23% of the variance of capital price growth. By contrast, ISTC plays no role in macroeconomic movement, consistent with Schmitt-Grohe and Uribe (2012). This shock accounts for only 1% of the variance of output growth.

Entrepreneur Risk Shocks. According to the table, the entrepreneur risk shock accounts for 57% of the variance of net worth growth and 23% of the variance of capital price growth while it explains only 17% of the variance of consumption growth.

Consumption Preference and Government Spending Shocks. It is evident from the estimation results that consumption preference contributes modestly to the fluctuations of aggregate consumption. As discussed above, markup shocks explain a large fraction of the unconditional variance of consumption. This finding differs sharply from the existing literature.[10] This is particularly the case the model implies, that consumption preference disturbance accounts for 16% of consumption growth and 14% of the variance of hours worked growth. Furthermore, I also show that government spending explains only 4% of the variance of output growth.

[10] see, for example, Guerrieri and Iacoviello (2017) and Iacoviello (2015), where aggregate consumption is substantially explained by consumption preference disturbance.

Technology Trend and Stationary Technology Shocks. The model includes two production disturbances: technology trend and stationary technology. These two shocks explain modestly the movement in macroeconomic aggregates. This result differs sharply from the existing literature that assigns an important role to these disturbances. An interesting result that emerges from this analysis is that when financial data, for example, external financing, net worth, and capital price, are incorporated into the model, then productivity shocks vanish in importance. The estimated model technology trend explains 6% of the variance of output growth and 7% of the variance of consumption. Meanwhile, stationary technology accounts for 22% of the unconditional variance hours worked growth and explains only 6% of consumption growth and external financing growth.

Implications for Aggregates. A key assumption in this model allows entrepreneurs to pledge their assets as collateral, risking the loss of this collateral if they default. Incorporating collateral constraints adds more realism to the model because no banks should be willing to accept any type of collateral. Under a collateral-constrained environment, entrepreneurs will use their assets to secure their debt. If they declare bankruptcy, then the bank can seize the pledged assets.

The overall conclusions that stand out from this analysis are the following. First, the main result is that the model assigns a minor role for the ISTC shock in the shock decomposition.[11] Suggesting that, when there is a shock that affects the transformation of investment goods into productive capital, such a shock will have a modest impact on the macroeconomy. I find that collateral disturbance accounts for 26% of the variance of net worth growth and 36% of the variance of capital price. This shock also accounts for 6% of the variance of output growth and 3% of the variance of consumption growth. However, even if the contribution of risk shock is less powerful in magnitude and accounts for only 1% of the variance of output growth, it explains a significant fraction of net worth growth

[11] This contradicts the existing literature where the ISTC shock dominates.

(57%) and capital price growth (23%).[12]

Second, the magnitude of collateral shock is greater than the rest of shocks for external financing and capital price, suggesting that collateral shock plays an important role in the shock decomposition. Accordingly, I find that the MEI shock has a significant effect on growth in output and investment. The contribution of ISTC shock appears to be small; nevertheless, financial shocks tend to dominate in the shock decomposition. This supports the finding of Jakab and Kumhof (2015) in which the contribution of collateral shock is somewhat higher in magnitude. Finally, financial shocks imply different degrees of impact on the macroeconomy. The effects of an entrepreneur risk shock and an ISTC shock are less marked than a MEI shock and a collateral requirement shock. These two shocks appear to be dominant in explaining the movement in output, investment, loan supply, and capital price.

The Role of Financial Shocks To deepen our understanding of the role of financial shocks, I use the model to quantify the extent to which financial shock can drive economic fluctuations. As I have many shocks, I separate them into financial and real shocks.[13] This simple example illustrates how financial and real shocks shape macroeconomic fluctuations in the US economy by providing the historical decomposition of output growth, consumption growth, aggregate investment growth, external financing growth, investment growth, and business net worth growth. This is useful to understand how aggregates commove and to what extent this can be driven by financial shocks.

Figure 10 displays the contribution of economic shocks to output growth, which appears to be driven by financial shocks, especially during the financial crisis. Financial shocks can explain to a large extent the movement in investment growth. Figure 11 shows

[12] The model of Christiano et al. (2014) suggests that the entrepreneur risk shock accounts for 60% of the fluctuations in the output growth and emerges as the most important shock.

[13] Real shocks include price markup, government spending, technology trend, stationary technology, and consumption preference. Financial shocks include ISTC, entrepreneur risk, MEI, and collateral requirement.

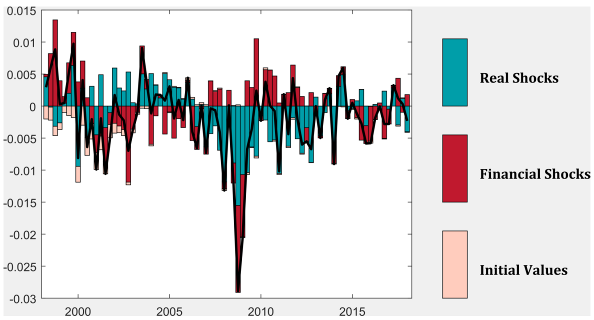

Figure 10: Output

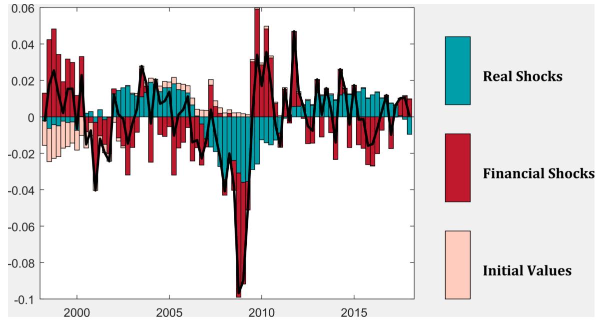

Figure 11: Investment

Note: Shock decomposition of output and investment. The solid line represents the data and colored bar represents the contribution of shocks in the aggregate movements. Real shocks include price markup, government spending, technology trend, stationary technology, and consumption preference. Financial shocks include ISTC, entrepreneur risk, MEI, and collateral requirement. Parameters are set at posterior mean.

that the effects of financial shocks on investment growth are relatively high for the entire period. Another feature of the model is that financial shocks contribute substantially to the movement in investment, especially around 2008. This feature is apparent and is captured fairly well after the financial crisis of 2008.

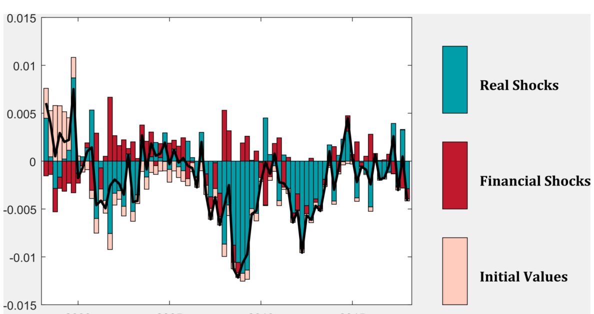

Figure 12: Consumption

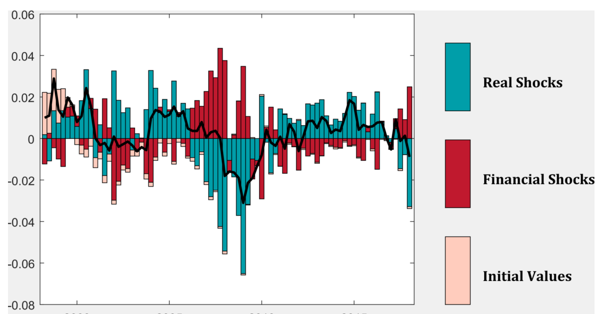

Figure 13: External Finance

Note: Shock decomposition of consumption and external finance. The solid line represents the data and colored bar represents the contribution of shocks in the aggregate movements. Real shocks include price markup, government spending, technology trend, stationary technology, and consumption preference. Financial shocks include ISTC, entrepreneur risk, MEI, and collateral requirement. Parameters are set at posterior mean.

Figure 12 shows the relative contribution of both financial and real shocks to the volatility of consumption growth. This volatility is mainly explained by real shocks. Figure 13 illustrates the sensitivity of external financing to both financial and real shocks.

Financial shocks appear to contribute largely to movements in external financing growth. A natural outcome of this analysis is that financial factors in macroeconomic models are non-negligible at least under the present specification where ISTC is muted. In fact, financial shocks are important in explaining the behavior of macroeconomic variables and broadly predict macroeconomic fluctuations, especially around the period of financial crisis.

7 Conclusion

In this paper, I estimate a DSGE model with collateral-constrained firms to analyze the impact of changes in collateral requirements and quantify the contribution of collateral shocks. I show that a collateral requirement shock contributes significantly to the shock decomposition analysis and explains a large component of the macroeconomic fluctuations in the model. Financial shocks disproportionately affect the macroeconomy; for instance, collateral requirements and marginal efficiency of investment shocks appear to be dominant.

Furthermore, accounting for financial data in the quantitative analysis reduces considerably the contribution of productivity shocks in macroeconomic fluctuations. This analysis shows that incorporating financial shocks is important in shaping macroeconomic fluctuations, even though investment-specific technical change is insufficient in explaining macroeconomic performance. The main result is that collateral matters for business cycle fluctuations, and the model simulations affirm the importance of the presence of collateral constraints in this model. One way to think about collateral disturbance is that when entrepreneurs face tight collateral conditions, it will lead to a contraction in lending and a general decline of investment, thus the lack of collateral can amplify the severity of financial distress. This suggests the need to adopt policies that support collateral markets and can mitigate the consequences of the disruption in the financial system.

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Online Appendix: Macroeconomic Effects of Collateral Requirements and Financial Shocks

Appendix A Data Description

In order to evaluate the model and apply it to data, I use six US macroeconomic time series that cover the period 1998Q1-2018Q1. To make links between the macroeconomic data and the model, I add to the model the following equations:

$$
\begin{aligned}
gdpobs_t &= \frac{(C_t + \frac{I_t}{\mu_{Y,t}} + g_t)}{(C_{t-1} + \frac{I_{t-1}}{\mu_{Y,t-1}} + g_{t-1})} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 && \text{change in output per capita} \\
consumptionobs_t &= \frac{C_t}{C_{t-1}} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 && \text{change in consumption per capita} \\
investobs_t &= \frac{I_t}{I_{t-1}} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 && \text{change in investment per capita} \\
hours\_obs_t &= \frac{L_t}{L} - 1 && \text{change in hours worked per capita} \\
externobs_t &= \frac{Q^k_t * K_t - N_t}{Q^k_{t-1} * K_{t-1} - N_{t-1}} \frac{\mu^*_{z,t}}{\mu^*_z} - 1 && \text{change in external financing per capita} \\
netwobs_t &= N_t / N_{t-1} - 1 && \text{change in business net worth per capita} \\
capriceobs_t &= s^*_t && \text{change in capital price} \\
govobs_t &= \frac{G_t}{G_{t-1}} - 1 && \text{change in government spending per capita} \\
tfpobs_t &= \frac{\mu^*_{z,t}}{\mu^*_{z,ss}}, && \text{change in TFP}
\end{aligned}
$$

Data are mainly obtained from the Federal Reserve Bank of St. Louis. Below, I describe the steps employed to construct the macroeconomic data used for estimation purposes and the data used to construct some plots.

Table 7: Data Sources

	Data Sources
Variables	**Description**
Gross Domestic Product	GDP, Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2018)
Price Deflator for Gross Domestic Product	GDPDEF, Gross Domestic Product Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2018)
Consumption	PCND, Personal Consumption Expenditures: Nondurable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2018)
	PCESV, Personal Consumption Expenditures: Services, Billions of Dollars, Quarterly, Seasonally Adjusted, Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2018)
Price Deflator for Personal Consumption Expenditures: Nondurable Goods	[DNDGRD3Q086SBEA], Personal Consumption Expenditures: Nondurable Goods (Implicit Price Deflator), Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. (01/01/1947-01/01/2018)
Price Deflator for Personal Consumption Expenditures: Services	[DSERRD3Q086SBEA], Personal Consumption Expenditures: Services Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2018)
Investment	GPDI, Gross Private Domestic Investment, Billions of Dollars, Seasonally Adjusted Annual Rate, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2018)
	[PCDG], Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2018)
Price Deflator for Gross Private Domestic Investment	[A006RD3Q086SBEA], Gross Private Domestic Investment Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2018)

Price Deflator for Personal Consumption Expenditures: Durable Goods	[DDURRD3Q086SBEA], Personal Consumption Expenditures: Durable Goods Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1945-01/01/2018)
Hours Worked	HOANBS, Nonfarm Business Sector: Hours of All Persons, Index 2012=100, Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1948-01/01/2018)
Capital Stock	[NNBTASQ027S] Nonfinancial Noncorporate Business; Total Assets, Level (DISCONTINUED), Millions of Dollars, Not Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1948-01/01/2018)
	[TABSNNCB] Nonfinancial Corporate Business; Total Assets, Level, Billions of Dollars, Not Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1948-01/01/2018)
Population	CNP16OV, Civilian Noninstitutional Population, Thousands of Persons, Monthly, Not Seasonally Adjusted, [CNP16OV 20180202], retrieved from FRED, Federal Reserve Bank of St. Louis. (01/01/1947-01/01/2018) Transformed Quarterly Data CNP16OV.
Volume of Business Loans, Survey of Terms of Business Lending	EVANQ, Total Value of Loans for All Commercial and Industry Loans, All Commercial Banks (DISCONTINUED), Millions of Dollars, Quarterly, Not Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1997-01/04/2017)
Percent of Collateralized Business Loans, Survey of Terms of Business Lending	ESANQ, Percent of Value of Loans Secured by Collateral for All Commercial and Industry Loans, All Commercial Banks (DISCONTINUED), Percent, Quarterly, Not Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (04/01/1997-04/01/2017)
Fed Funds Rate	FEDFUNDS, Effective Federal Funds Rate, Percent, Not Seasonally Adjusted, Monthly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/07/1954-01/06/2018) (Transformed into Quarterly Data)
Relative Price Investment	A006RD3Q086SBEA, Gross Private Domestic Investment (Implicit Price Deflator), Index 2012=100, Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2018)

Wages	COMPNFB, Nonfarm Business Sector: Compensation per Hour, Index 2012=100, Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2018)
External Financing	TNWMVBSNNCB, Nonfinancial Corporate Business; Net Worth, Level, Billions of Dollars, Not Seasonally Adjusted, Quarterly, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2018)
Government Spending	GCE, Government Consumption Expenditures and Gross Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2018)
Net Worth	TNWMVBSNNCB, Nonfinancial Corporate Business; Net Worth, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted, retrieved from FRED, Federal Reserve Bank of St. Louis. (01/04/1948-01/01/2018)

Variables	**Data Construction Description**
Series for Change in Output per Capita	I take the Gross Domestic Product [GDP] and divide it by the Implicit Price Deflator [GDPDEF_20180628] and by Population [CNP16OV_20180202]. Then, I take the log difference and remove the mean to match the series with the model equation.
Series for Change in Consumption per Capita	I take Personal Consumption Expenditures: Nondurable Goods [PCND] and divide it by the Implicit Price Deflator [DNDGRD3Q086SBEA]. I take personal Consumption Expenditures: Services [PCES], I divide it by the Implicit Price Deflator [DSERRD3Q086SBEA], and I sum these two series and divide by Population [CNP16OV_20180202]. Then, to combine the series with the model equation, I take the log difference and subtract the mean.
Series for Change in Investment per Capita	I take the Gross Private Domestic Investment [GDPI] and divide it by the Implicit Price Deflator [A006RD3Q086SBEA]. I take Personal Consumption Expenditures: Durable Goods [PCDG] and divide it by the Implicit Price Deflator [DDURRD3Q086SBEA]. I sum these two series and divide by Population [CNP16OV_20180202]. Then, to combine the series with the model equation, I take the log difference and subtract the mean.

Series for Change in Hours Worked per Capita	I take the index of Worked Hours in the Nonfarm Business Sector [HOANBS] and divide it by Population [CNP16OV_20180202]. Then, I take the log difference to combine with the model equation.
Series for Change in External Financing per Capita	I take the sum of Total Assets in Nonfinancial Noncorporate Business [NNBTASQ027S] and Nonfinancial Corporate Business [TABSNNCB], then I subtract the Net Worth of Nonfinancial Corporate Business and divide the result by the Implicit Price Deflator [GDPDEF_20180628] and by Population [CNP16OV_20180202]. Then, I take the log difference and subtract the mean to combine with the model equation.
Series for Change in Business Net Worth per Capita	I take the sum of Net Worth of Nonfinancial Corporate Business [TNWMVBSNNCB] and divide it by the Price Deflator [GDPDEF] and by Population [CNP16OV_20180202]. Then, to combine these series with the model equation, I take the log difference.
Series for Change in Capital Price	I take the Russell 3000 Price Index [RU3000PR] and I transform it to quarterly data. Then, I take the log difference to combine it with the model equation.
Change in Government Spending per Capita	I take the Government Consumption Expenditures and Gross Investment [GCE] and divide it by the Price Deflator [GDPDEF] and by Population [CNP16OV_20180202]. Then, I take the log difference and subtract the mean to combine it with the model equation.
Change in TFP	I multiply [HOANBS] by [COMPNFB] to obtain the aggregate labor factor L, and I compute the aggregate capital factor $K = [NNBTASQ027S] + [TABSNNCB]$. Then, I use the following formula to compute the total factor productivity $TFP = \frac{GDP}{K^{(1-\alpha)}L^{\alpha}}$ where $\alpha = 0.64$. Then I take the log difference to match it with the model equation.

Appendix B Prior Posterior Densities

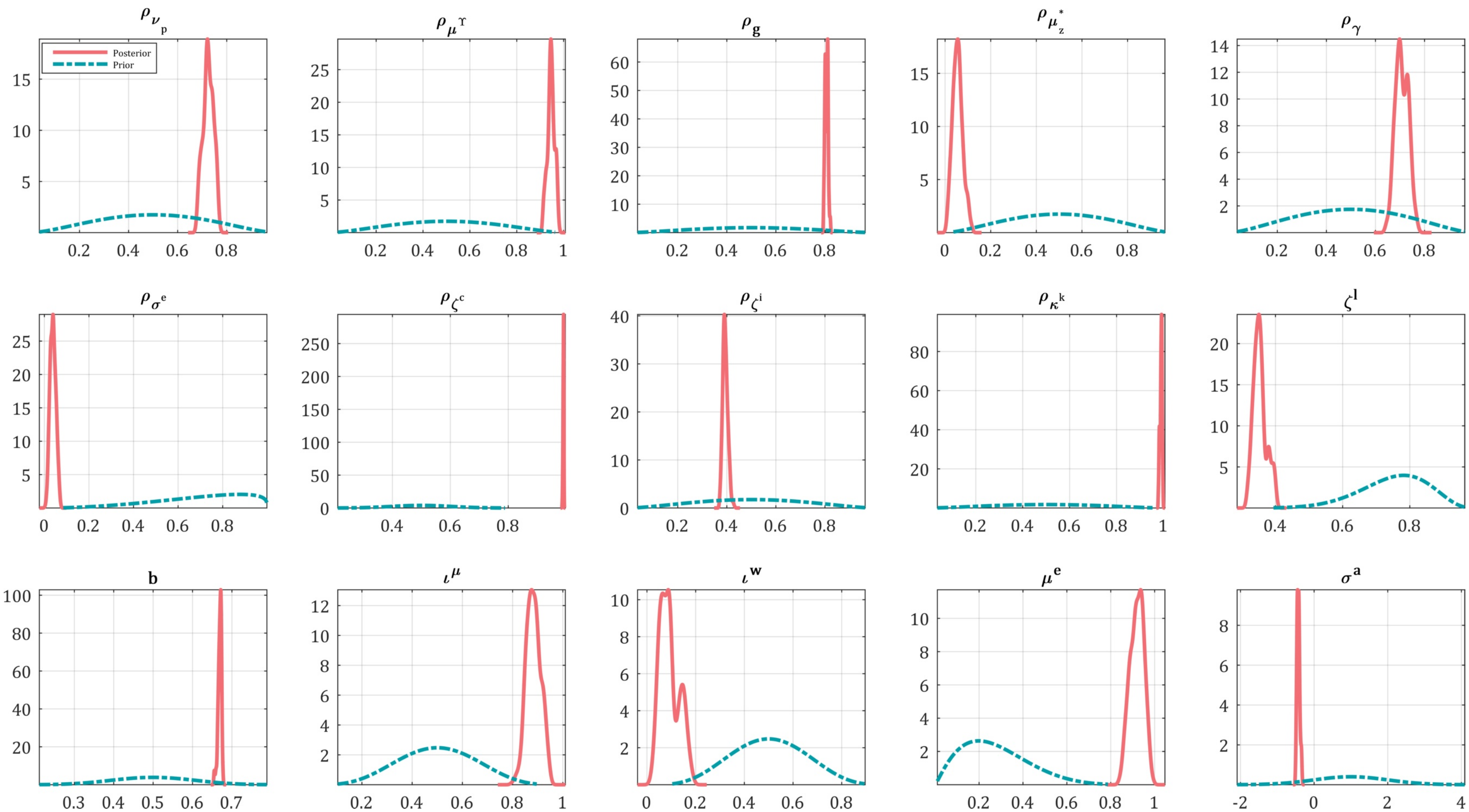

Figure 14: Prior and Posterior

Note: The posterior distribution is constructed over the period 1998–2018 using the Metropolis–Hasting algorithm with 10,000 draws per chain needed to achieve convergence.

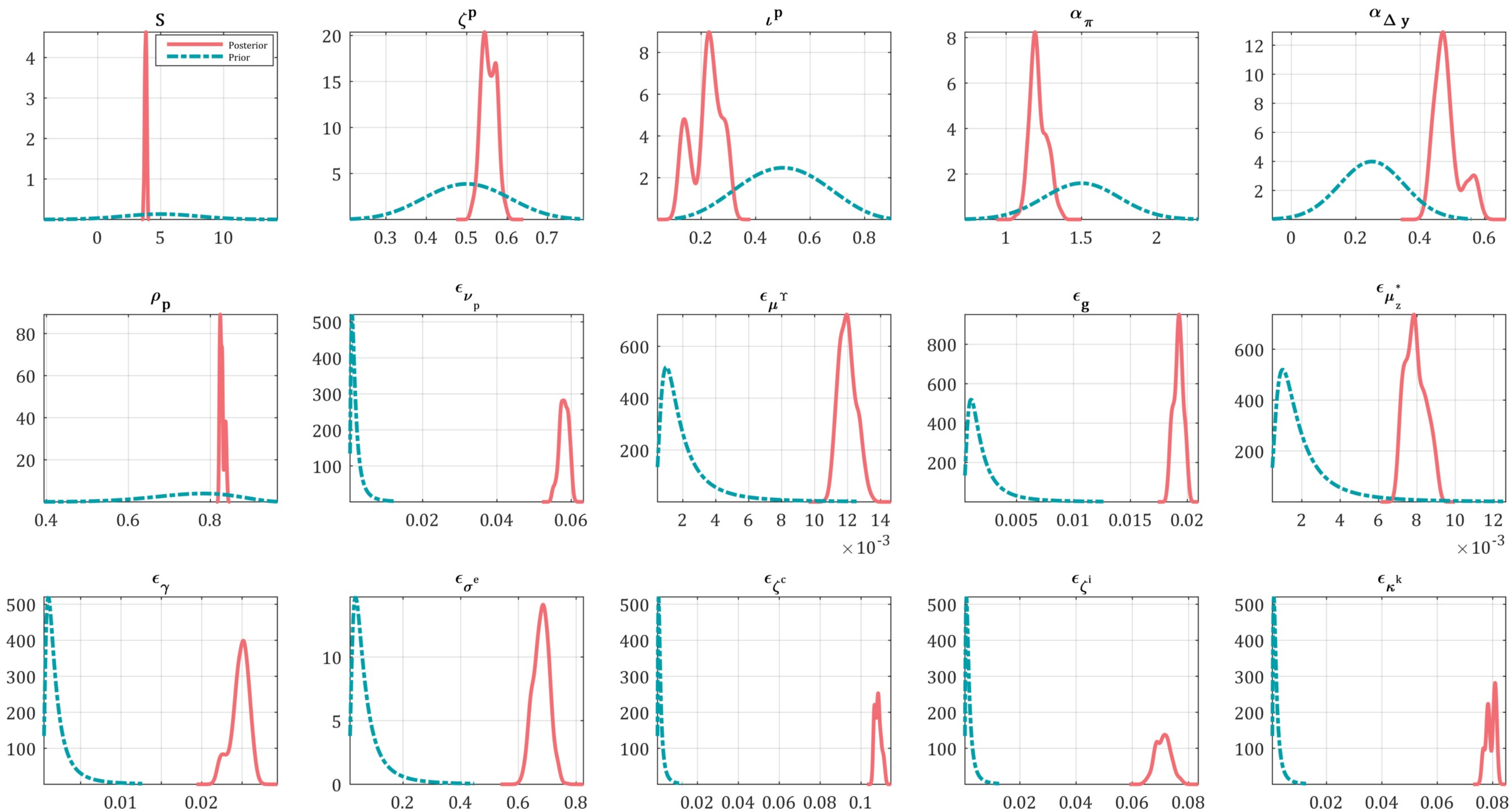

Figure 15: Prior and Posterior

Note: The posterior distribution is constructed over the period 1998–2018 using the Metropolis–Hasting algorithm with 10,000 draws per chain needed to achieve convergence.

Appendix C Model Impulse Responses

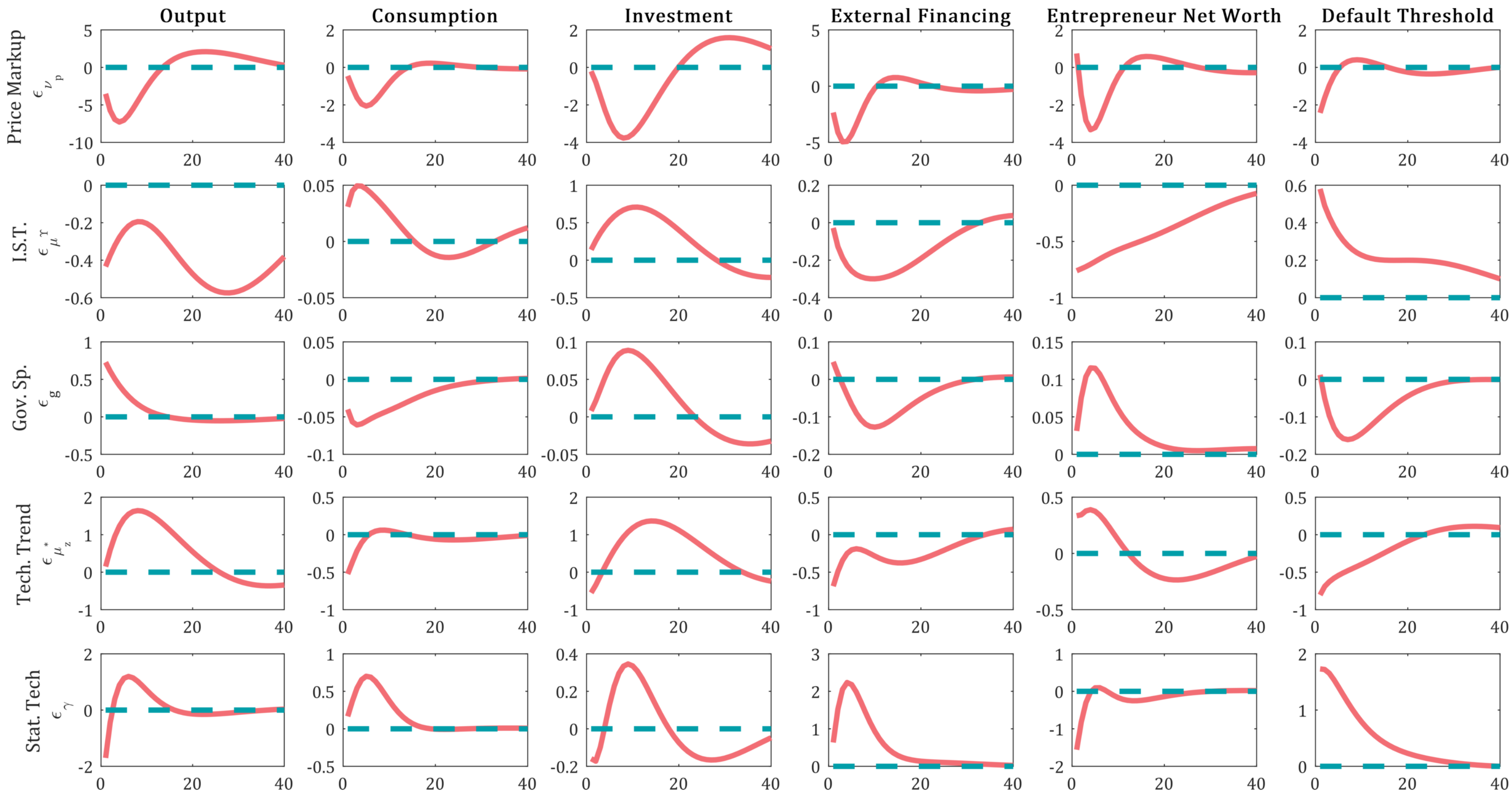

Figure 16: Impulse Responses of the Estimated Model

Note: Time units are quarterly. The simulation shows the responses of output, consumption, investment, external financing, net worth, and default threshold to price markup, investment-specific technical change, government spending, technology trend, stationary technology, entrepreneur risk, consumption preference, marginal efficiency of investment, and collateral requirement shocks.

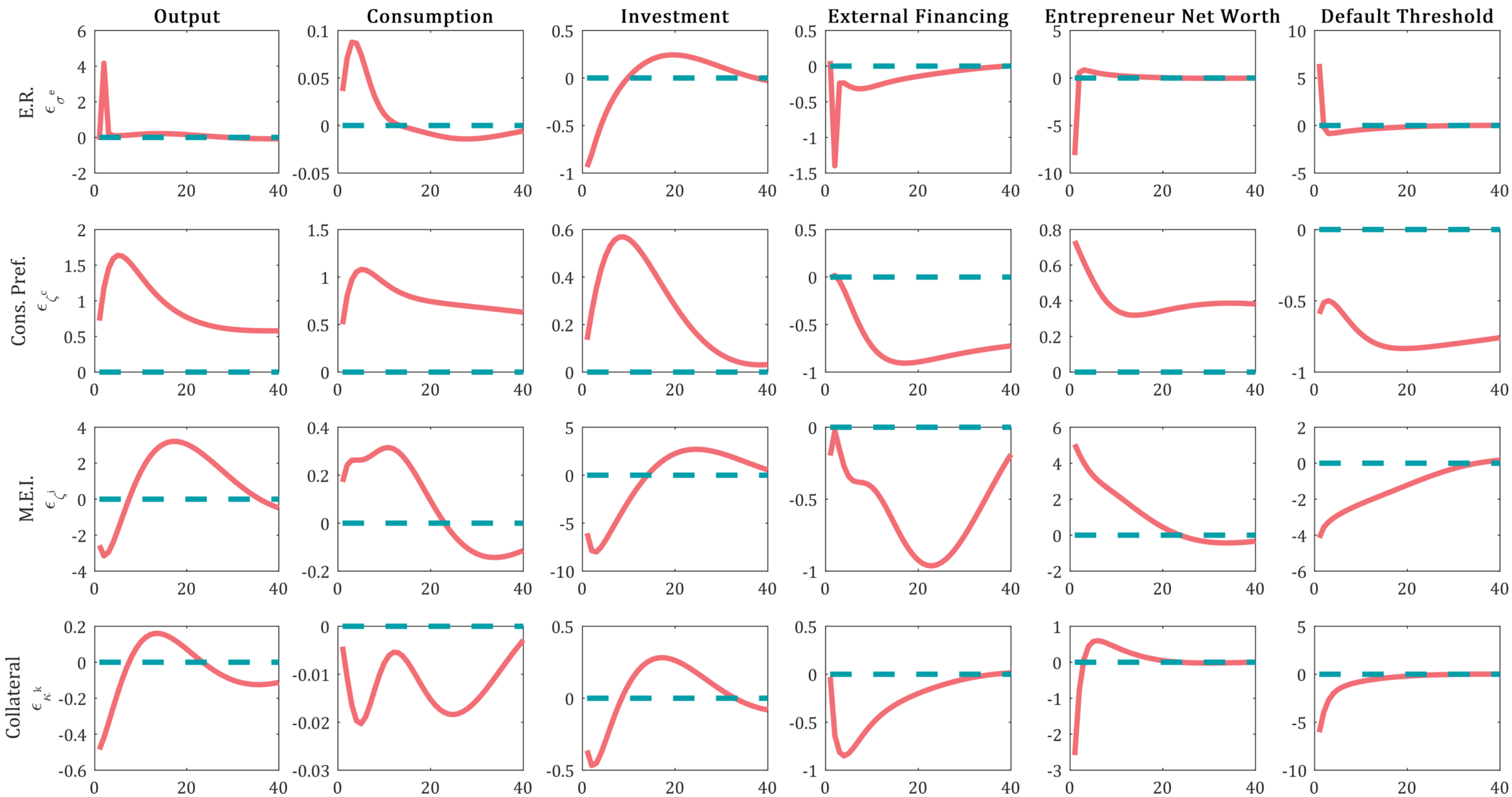

Figure 17: Impulse Responses of the Estimated Model

Note: Time units are quarterly. The simulation shows the responses of output, consumption, investment, external financing, net worth, and default threshold to price markup, investment-specific technical change, government spending, technology trend, stationary technology, entrepreneur risk, consumption preference, marginal efficiency of investment, and collateral requirement shocks.

Appendix D Model Computations

This appendix describes the computation of the equilibrium of a discrete time dynamic model using Lagrangian methods or substituting out the constraint. In section D, I sort all first-order conditions and definition terms. In section E, I detrend the model by using specific trend growth for each variable. In section F, I find the non-stochastic steady state.

D.1 Final Good Producers

The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final good stock, which writes:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}$$

Final good producers purchase the good and resell it to consumers. Their objective is to maximize their profits.

$$\begin{aligned} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \\ P_t \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p} - \int_0^1 P_{j,t} Y_{j,t} dj. \end{aligned}$$

The first-order condition with respect to $Y_{j,t}$ is given by

$$\begin{aligned} (\partial Y_{j,t}) : \qquad & P_t (1+\nu_p) \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p-1} \frac{1}{1+\nu_p} Y_{j,t}^{(\frac{1}{1+\nu_p})-1} - P_{j,t} = 0 \\ & \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{\nu_p} Y_{j,t}^{(\frac{-\nu_p}{1+\nu_p})} = \frac{P_{j,t}}{P_t} \\ & \left[\int_0^1 Y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{-(1+\nu_p)} Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{\frac{1+\nu_p}{\nu_p}} \\ & Y_t^{-1} Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\frac{1+\nu_p}{\nu_p}}. \end{aligned}$$

From the first-order condition, $Y_{j,t}$ is given by

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\frac{1+\nu p}{\nu p}} Y_t.$$

To define the aggregate price of the final good P_t, I use an expression for the output, which is equal to price times quantities $P_t Y_t = \int_0^1 P_{j,t} Y_{j,t} dj$; then, I simplify to obtain the expression for the aggregate price of the final good:

$$P_t Y_t = \int_0^1 P_{j,t} \left(\frac{P_{j,t}}{P_t}\right)^{-\frac{1+\nu p}{\nu p}} Y_t dj$$
$$P_t = \left[\int_0^1 P_{j,t}^{-\frac{1+\nu p}{\nu p}} dj\right]^{-\frac{\nu p}{1+\nu p}}.$$

D.2 Intermediate Good Producers

The problem of the intermediate good producer is to minimize the cost,

$$W_j L_{j,t} + r_t^k u_t K_{j,t-1},$$

subject to the production function,

$$Y_{j,t} = \gamma_t (u_t K_{j,t-1})^{\alpha} (z_t L_{j,t})^{1-\alpha} - \Phi z_t^*. \tag{D.1}$$

I can write the intermediate good producer problem in Lagrangian form as:

$$\mathcal{L} = \quad (W_j L_{j,t} + r_t^k u_t K_{j,t-1}) + \lambda_{1,t}(Y_{j,t} - \gamma_t (u_t K_{j,t-1})^{\alpha} (z_t L_{j,t})^{1-\alpha} + \Phi z_t^*),$$

where λ_1 is the Lagrange multiplier associated with the production function. The first-order conditions with respect to labor and capital yield:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial L_{j,t}}: \qquad & W_t - (1-\alpha)\gamma_t (u_t K_{j,t-1})^{\alpha}(z_t L_{j,t})^{-\alpha}\Big)\lambda_{1,t} = 0 \\
& W_t - (1-\alpha)\gamma_t \left(\frac{u_t K_{j,t-1}}{z_t L_{j,t}}\right)^{\alpha} \lambda_{1,t} = 0 \\
& \lambda_{1,t} = \frac{W_t}{(1-\alpha)\gamma_t (u_t K_{j,t-1})^{\alpha}(z_t L_{j,t})^{-\alpha}} \\
\frac{\partial \mathcal{L}_t}{\partial u_t K_{j,t-1}}: \qquad & r_t^k - \alpha\gamma_t \left(\frac{z_t L_{j,t}}{u_t K_{j,t-1}}\right)^{1-\alpha} \lambda_{1,t} = 0 \\
& \lambda_{1,t} = \frac{r_t^k}{\alpha\gamma_t (z_t L_{j,t})^{1-\alpha}(u_t K_{j,t-1})^{\alpha-1}}.
\end{aligned} \tag{D.2}
$$

I can eliminate the Lagrange multiplier using these two conditions, which imply the following:

$$
\begin{aligned}
\frac{W_t}{(1-\alpha)\gamma_t (u_t K_{j,t-1})^{\alpha}(z_t L_{j,t})^{-\alpha}} &= \frac{r_t^k}{\alpha\gamma_t (z_t L_{j,t})^{1-\alpha}(u_t K_{j,t-1})^{\alpha-1}} \\
\frac{W_t}{(1-\alpha)\gamma_t}\left(\frac{(z_t L_{j,t})}{(u_t K_{j,t-1})}\right)^{\alpha} &= \frac{r_t^k}{\alpha\gamma_t}\left(\frac{(u_t K_{j,t-1})}{(z_t L_{j,t})}\right)^{(1-\alpha)} \\
\frac{u_t K_{j,t-1}}{z_t L_{j,t}} &= \frac{\alpha}{1-\alpha}\frac{W_t}{r_t^k}.
\end{aligned}
$$

Then, if I integrate both $K_{t-1} = \left[\int_0^1 K_{j,t-1} dj\right]$ and $L_t = \left[\int_0^1 L_{j,t} dj\right]$, I obtain :

$$
\frac{u_t K_{t-1}}{z_t L_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{r_t^k}.
$$

I further simplify this expression with $u_t K_{t-1} = K_{t-1}$ and $L_t = z_t L_t$.

$$
\frac{K_{t-1}}{L_t} = \left(\frac{\alpha}{1-\alpha}\right)\frac{W_t}{r_t^k}
$$

Then, the total cost function $W_j L_{j,t} + r_t^k u_t K_{j,t-1}$ is simply rewritten as:

$$W_t L_t + r_t^k K_t$$
$$W_t + r_t^k \frac{K_t}{L_t}.$$

To define the marginal cost,[14] I divide the total cost by the productivity function $Y_t = \gamma_t (K_{t-1})^\alpha (L_{,t})^{1-\alpha}$ and use the expression for $\frac{K_{t-1}}{L_t} = \left(\frac{\alpha}{1-\alpha}\right) \frac{W_t}{r_t^k}$.

$$\begin{aligned}
\text{Marginal cost } = \lambda_{t,1} &= \frac{W_t + r_t^k \frac{K_{t-1}}{L_t}}{\gamma_t (K_{t-1})^\alpha (L_{,t})^{1-\alpha}} \\
\lambda_{t,1} &= \frac{W_t + r_t^k \frac{K_{t-1}}{L_t}}{\gamma_t \left(\frac{K_{t-1}}{L_t}\right)^\alpha} \\
\lambda_{t,1} &= \frac{W_t + r_t^k \left(\frac{\alpha}{1-\alpha} \frac{W_t}{r_t^k}\right)}{\gamma_t \left(\frac{\alpha}{1-\alpha} \frac{W_t}{r_t^k}\right)^\alpha} \\
\lambda_{t,1} &= \frac{W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)}{\gamma_t \left(\frac{\alpha}{1-\alpha} \frac{W_t}{r_t^k}\right)^\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t} \left[W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)\right] \left[\frac{\alpha}{1-\alpha} \frac{W_t}{r_t^k}\right]^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t} \left[W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)\right] \left[\frac{1-\alpha}{\alpha} \frac{r_t^k}{W_t}\right]^{\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t} \left[W_t + \left(\frac{\alpha}{1-\alpha} W_t\right)\right] \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^\alpha (W_t)^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t} \left[W_t \left(1 + \frac{\alpha}{1-\alpha}\right)\right] \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^\alpha (W_t)^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t} \left[W_t \left(\frac{1}{1-\alpha}\right)\right] \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^\alpha (W_t)^{-\alpha} \\
\lambda_{t,1} &= \frac{1}{\gamma_t} \left(\frac{1}{1-\alpha}\right) \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (r_t^k)^\alpha (W_t)^{(1-\alpha)} \\
\lambda_{t,1} &= \frac{1}{\gamma_t} \left(\frac{1}{1-\alpha}\right) \left(\frac{1}{\alpha}\right)^{\alpha} (1-\alpha)^\alpha (r_t^k)^\alpha (W_t)^{(1-\alpha)}
\end{aligned}$$

[14] In equilibrium, marginal cost equals marginal utility $\lambda_{t,1}$.

$$\lambda_{t,1} = \frac{1}{\gamma_t}\left(\frac{1}{1-\alpha}\right)\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{(1-\alpha)}\right)^{-\alpha}(r_t^k)^{\alpha}(W_t)^{(1-\alpha)}$$

$$\lambda_{t,1} = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{(r_t^k)^{\alpha}W^{(1-\alpha)}}{\gamma_t}. \tag{D.3}$$

Considering a firm's pricing problem, I assume that prices are sticky, as in Erceg et al. (2000). For those firms, the problem is to choose the price level that maximizes their profits:

$$\begin{aligned} \underset{P_{j,t}}{\text{maximize}} \quad & E\Sigma_{s=0}^{\infty}\beta^s\zeta^p[Y_{j,t}(P_{j,t}^* - \lambda_{1,j,t+s}P_{j,t+s})] \\ \text{subject to} \quad & Y_{j,t} = Y_t\left(\frac{P_{j,t}}{P_t}\right)^{\frac{\nu_p}{1-\nu_p}}, \end{aligned}$$

I solve the firm problem by substituting $Y_{j,t}$ into the firm's profits function:

$$(\partial P_{j,t}^*): \qquad P_{j,t}^* = \nu_p\frac{E_t\Sigma_{k=0}^{\infty}\beta_k\zeta_k^p\left(\frac{P_{t+k}}{\tilde{\pi}_{t+k}P_t}\right)Y_{j,t+k}\lambda_{1,j,t+k}}{E_t\Sigma_{k=0}^{\infty}\beta_k\zeta_k^pY_{j,t+k}}$$

where the price indexation $\tilde{\pi}_{t+k} = \pi_{trg,t}^{\iota^p}\pi_{t-1}^{1-\iota^p}$. To further simplify this condition, I divide by P_t and obtain:

$$\tilde{P}_t = \nu_p\frac{K_{p,t}}{F_{p,t}},$$

where $\tilde{P}_t = \frac{P_{j,t}^*}{P_t}$.

Then, the recursive form is written as:

$$K_{p,t} = \lambda_{1,j,t}\nu_{t,p}Y_t + \beta\zeta^pE_t\left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}}\right)^{(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}})}K_{p,t+1}\right] \tag{D.4}$$

$$F_{p,t} = \nu_{t,p}Y_t + \beta\zeta^pE_t\left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}}\right)^{(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}})}F_{p,t+1}\right], \tag{D.5}$$

with $\tilde{\pi}_t = \pi_{trg,t}^{\iota^p}\pi_{t-1}^{1-\iota^p}$ and $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$. I can now drop all indexes, since all firms have the same optimum, and given that $P_t = \left[\int_0^1 P_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}$, it follows that:

$$P_t = \left[(1-\zeta_p)(\tilde{P}_t)^{\frac{1}{1+\nu_p}} + \zeta_p(\tilde{\pi} P_{t-1})^{\frac{1}{1+\nu_p}} \right]^{1+\nu_p}.$$

I substitute with $\tilde{P}_t = \nu_p \frac{K_{p,t}}{F_{p,t}}$ such that $P_t = \left[(1-\zeta_p)(\nu_p \frac{K_{p,t}}{F_{p,t}})^{\frac{1}{1+\nu_p}} + \zeta_p(\tilde{\pi} P_{t-1})^{\frac{1}{1+\nu_p}} \right]^{1+\nu_p}$; then, I obtain:

$$P_t = \left[(1-\zeta_p) \left(\frac{1-\zeta_p \left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}} \right)^{\left(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}} \right)}}{1-\zeta_p} \right)^{\frac{1}{1+\nu_p}} + \zeta_p(\tilde{\pi} P_{t-1})^{\frac{1}{1+\nu_p}} \right]^{1+\nu_p} \tag{D.6}$$

D.3 Labor Contractors

Labor services are provided by households to intermediate firms via labor contractors; these services are combined into homogeneous labor with:

$$L_t = \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di \right]^{1+\nu_l}.$$

Labor contractors maximize their profits in a perfectly competitive market:

$$W_t L_t - \int_0^1 W_{i,t} L_{i,t} di$$
$$W_t \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di \right]^{1+\nu_l} - \int_0^1 W_{i,t} L_{i,t} di.$$

The first-order condition with respect to labor $L_{i,t}$ can be written as:

$$\begin{aligned}
(\partial L_{i,t}) : \qquad & W_t(1+\nu_l) \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di \right]^{1+\nu_l-1} \frac{1}{1+\nu_l} L_{i,t}^{\left(\frac{1}{1+\nu_l}\right)-1} - W_{i,t} = 0 \\
& \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di \right]^{\nu_l} L_{i,t}^{\left(\frac{-\nu_l}{1+\nu_l}\right)} = \frac{W_{i,t}}{W_t} \\
& \left[\int_0^1 L_{i,t}^{\frac{1}{1+\nu_l}} di \right]^{-(1+\nu_l)} L_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{\frac{1+\nu_l}{\nu_l}}
\end{aligned}$$

$$L_t^{-1} L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\frac{1+\nu_l}{\nu_l}}.$$

From the first-order condition, I obtain:

$$L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\frac{1+\nu_l}{\nu_l}} L_t.$$

To define the aggregate wage W_t, I take the total labour supply expression $W_t L_t = \int_0^1 W_{i,t} L_{i,t} di$, then simplify to obtain the expression for the aggregate wage:

$$\begin{aligned} W_t L_t &= \int_0^1 W_{i,t} \left(\frac{W_{i,t}}{W_t}\right)^{-\frac{1+\nu_l}{\nu_l}} L_t di \\ W_t &= \left[\int_0^1 W_{i,t}^{-\frac{1+\nu_l}{\nu_l}} di\right]^{-\frac{\nu_l}{1+\nu_l}}. \end{aligned}$$

The problem of wage setting is given as:

$$\begin{aligned} \underset{W_{i,t}}{\text{maximize}} \quad & E\Sigma_{s=0}^{\infty} \beta^s \zeta^{c,t+s} \zeta^l \left[-\Psi_L \int_0^1 \frac{L_{i,t+s}}{1+\sigma_L} di + \lambda_{z,t+s} W_{i,t} \Pi_{i,t+s}^w - L_{i,t+s}\right] \\ \text{subject to} \quad & L_{i,t+s} = \left(\frac{W_{i,t}\Pi_{i,t+s}^w}{W_{t+s}}\right)^{\frac{\nu_l}{1-\nu_l}} L_{t+s} \\ \text{where} \quad & \Pi_{i,t+s} = \Sigma_{k=1}^{s} (\mu_{z,t+k})^{\iota^\mu} L_t (\mu_z)^{1-\iota^\mu} \tilde{\pi}_{w,t+k} \\ \text{and} \quad & W_{t+s} = \pi_{w,t+s} \dots \pi_{w,t+1} W_t \end{aligned}$$

$$\frac{\partial \mathcal{L}_t}{\partial \tilde{W}_{i,t}} : W_{j,t} = \frac{\Psi_L E_t \Sigma_{s=0}^{\infty} \beta_s \zeta_{c,t+s} \zeta_{l,s} \nu_l \left[\left(\frac{\tilde{W}\Pi_{i,t+s}^w}{W_{t+s}}\right)^{\frac{\nu_l}{1-\nu_l}} L_{j,t+s}\right]^{1+\sigma_L}}{E_t \Sigma_{s=0}^{\infty} \beta_s \zeta_{c,t+s} \zeta_{l,s} L_{t+s} \left(\frac{\Pi_{i,t+s}^w}{\pi_{w,t+s} \dots \pi_{w,t}}\right)^{\frac{\nu_l}{1-\nu_l}} \left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{\nu_l}{1-\nu_l}} \lambda_{z,t+s} \Pi_{i,t+s}^w},$$

where $\tilde{W}_t = W_{i,t}$. After some algebra, I obtain:

$$\left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{1-\nu_l(1+\sigma_L)}{1-\nu_l}} \frac{W_t}{P_t}\frac{1}{\Psi_L} = \frac{K_{w,t}}{F_{w,t}}.$$

The recursive form is written as:

$$K_{w,t} = (L_t)^{1+\sigma_L} + \beta\zeta^l E_t\left[\left(\frac{\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}(1+\sigma_L)} K_{w,t+1}\right] \tag{D.7}$$

$$F_{w,t} = \frac{L_t}{\nu_l}P_t\lambda_z + \beta\zeta^l E_t\big(\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}\big)^{\frac{1}{1-\nu_l}}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}}\frac{1}{\pi_{t+1}}F_{w,t+1}\Big]. \tag{D.8}$$

Wages in this economy are sticky and are of the form:

$$W_t = \big[(1-\zeta_l)(\tilde{W}_t)^{\frac{1}{1+\nu_l}} + \zeta_l(\tilde{\pi}(\mu^*_{z,t})^{\iota_w}(\mu^*_z)^{1-\iota_w}W_{t-1})^{\frac{1}{1+\nu_l}}\big]^{1+\nu_l}, \tag{D.9}$$

with $\tilde{\pi} = \pi^{\iota^w}_{trg,t}\pi^{1-\iota^w}_{t-1}$. Then, I substitute the last expression for $\left(\frac{\tilde{W}_t}{W_t}\right)^{\frac{1-\nu_l(1+\sigma_L)}{1-\nu_l}} \frac{W_t}{P_t}\frac{1}{\Psi_L} = \frac{K_{w,t}}{F_{w,t}}$ and I obtain:

$$\frac{1}{\Psi_L}\left[\frac{1-\zeta^l\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}(\mu^*_{z,t})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}\right)^{\frac{1}{1-\nu_l}}}{1-\zeta^l}\right]^{1-\nu_l(1+\sigma_L)}\frac{W_t}{P_t}F_{w,t} - K_{w,t} = 0.$$

D.4 Capital Producers

Capital producers accumulate capital according to the following equation:

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right)I_t. \tag{D.10}$$

Capital producers maximize their profits:

$$\Pi_t = \left[Q^k_t\left[(1-\delta)K_{t-1} + \left(1-S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}}\right)\right)I_t\right] - Q^k_t(1-\delta)K_{t-1} - \left(\frac{P_t}{\Upsilon_t\mu}\right)\right].$$

The Lagrangian representation of the problem is given by:

$$\mathcal{L} = E_0 \Sigma_{t=0}^{\infty} \beta \lambda_z \left[Q_t^k \left[(1-\delta) K_{t-1} + \left(1 - S\left(\zeta_{I,t}, \frac{I_t}{I_{t-1}} \right) \right) I_t \right] - Q_t^k (1-\delta) K_{t-1} - \left(\frac{P_t}{\Upsilon_t \mu_\Upsilon} \right) \right].$$

Given the capital producer profit function, the first-order condition with respect to investment is of the form:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial I_t} : \qquad & \lambda_{z,t} Q_t^k \left[1 - S\left(\zeta_{I,t} \frac{I_t}{I_{t-1}} \right) - \zeta_{I,t} \frac{I_t}{I_{t-1}} S'\left(\zeta_{I,t} \frac{I_t}{I_{t-1}} \right) \right] \\ & - \frac{\lambda_{z,t} P_t}{\Upsilon_t \mu_{\Upsilon,t}} + \beta \lambda_{z,t+1} Q_{t+1}^k \zeta_{I,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S''\left(\zeta_{I,t+1} \frac{I_{t+1}}{I_t} \right) = 0. \end{aligned} \tag{D.11}$$

D.5 Entrepreneurs

The rental income from a unit of capital is equal to the cost of capital utilization:

$$\frac{P_t}{\Upsilon_t} a(u_t) K_t \omega_t = r_t^k P_t K_t \omega_t.$$

This takes the Lagrangian form:

$$\mathcal{L} = \frac{P_t}{\Upsilon_t} a(u_t) K_t \omega_t - r_t^k P_t K_t \omega_t.$$

Given the capital utilization cost function, the first-order condition with respect to capital utilization is:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial u_t K_t} : \qquad & \frac{P_t}{\Upsilon_t} a'(u_t) \omega_t - r_t^k P_t \omega_t = 0 \\ & a'(u_t) = r_t^k \Upsilon_t, \end{aligned} \tag{D.12}$$

where $a'(u_t) = r^k_{ss} \exp\big(\sigma_a(u_t - 1)\big)$. Then, I substitute $a'(u_t)$ in the first-order condition, and it becomes:

$$r^k_{ss} \exp\big(\sigma_a(u_t - 1)\big) = r^k_t \Upsilon_t,$$

or equivalently, with $r^k_t = r^k_t \Upsilon_t$, I obtain $r^k_{ss} \exp\big(\sigma_a(u_t - 1)\big) = r^k_t$.

Entrepreneur revenues take the form:

$$\Big[(1 + R^k_{t+1})Q^k_t\Big]\omega_t K_{t+1},$$

which embodies the revenues from capital, the cost associated with capital, and revenues from selling undepreciated capital to capital producers. This can be rewritten as:

$$\Big[(1 - \tau^k)\big(u_{t+1} r^k_{t+1} - \frac{a(u_{t+1})}{\Upsilon_{t+1}}\big)P_{t+1} + (1 - \delta)Q^k_{t+1} + \tau^k \delta Q^k_t\Big]\omega_t K_{t+1}.$$

Equivalently,

$$\begin{aligned}
(1 + R^k_{t+1})Q^k_t = &\quad (1 - \tau^k)\big(u_{t+1} r^k_{t+1} - \frac{a(u_{t+1})}{\Upsilon_{t+1}}\big)P_{t+1} + (1 - \delta)Q^k_{t+1} + \tau^k \delta Q^k_t \\
(1 + R^k_{t+1}) = &\quad \frac{(1 - \tau^k)\big(u_{t+1} r^k_{t+1} - \Upsilon^{t+1} a(u_{t+1})\big)P_{t+1} + (1 - \delta)Q^k_{t+1} + \tau^k \delta Q^k_t}{Q^k_t} \\
(1 + R^k_{t+1}) = &\quad \frac{(1 - \tau^k)\big(u_{t+1} r^k_{t+1} - \Upsilon^{t+1} a(u_{t+1})\big)P_{t+1} + (1 - \delta)Q^k_{t+1}}{Q^k_t} + \tau^k \delta. \quad \text{(D.13)}
\end{aligned}$$

Entrepreneur capital is equal to total loans and net worth:

$$Q^k_{t-1} K_{t-1} = M_{t-1} + N_{t-1}. \tag{D.14}$$

The debt contract between an entrepreneur and a bank is signed before the realization of shocks. I define the threshold $\bar{\omega}_t$ below which entrepreneurs may default:

$$\bar{\omega}_t \kappa_{t-1}(1+R^k_t)Q^k_{t-1}K_{t-1} = (1+R^e_t)M_{t-1}.$$

The bank zero-profit condition is given by:

$$\begin{aligned}
&[1-F_{t-1}(\bar{\omega}_t)](1+R^e_t)M_{t-1} \\
&+(1-\mu_{t-1})\kappa_{t-1}\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)(1+R^k_t)Q^k_{t-1}K_{t-1} = (1+R_{t-1})M_{t-1}.
\end{aligned}$$

Using the definitions:

$$\begin{aligned}
Lev_t = &\quad \frac{M_t}{N_t} \\
\Gamma_{t-1}(\bar{\omega}_t) = &\quad \bar{\omega}_t[1-F_{t-1}(\bar{\omega}_t)] + G_{t-1}(\bar{\omega}_t) \\
G_{t-1}(\bar{\omega}_t) = &\quad \int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega),
\end{aligned}$$

the bank zero-profit condition can be rewritten as:

$$\begin{aligned}
&[1-F_{t-1}(\bar{\omega}_t)]\bar{\omega}_t\kappa_{t-1}(1+R^k_t)Q^k_{t-1}K_{t-1} \\
&+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\kappa_{t-1}(1+R^k_t)Q^k_{t-1}K_{t-1} = (1+R_{t-1})M_{t-1} \\
&\Bigg[[1-F_{t-1}(\bar{\omega}_t)]\bar{\omega}_t \\
&+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\Bigg]\kappa_{t-1}(1+R^k_t)Q^k_{t-1}K_{t-1} = (1+R_{t-1})M_{t-1} \\
&\Bigg[[1-F_{t-1}(\bar{\omega}_t)]\bar{\omega}_t \\
&+\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega) - \mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\Bigg]\kappa_{t-1}(1+R^k_t)Q^k_{t-1}K_{t-1} = (1+R_{t-1})M_{t-1} \\
&\big[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\big]\kappa_{t-1}(1+R^k_t)Q^k_{t-1}K_{t-1} = (1+R_{t-1})M_{t-1}
\end{aligned}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{M_{t-1}}{Q_{t-1}^k K_{t-1}}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{M_{t-1}}{M_{t-1}+N_{t-1}}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{\frac{M_{t-1}}{N_{t-1}}}{\frac{M_{t-1}+N_{t-1}}{N_{t-1}}}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{\frac{M_{t-1}}{N_{t-1}}}{\frac{M_{t-1}}{N_{t-1}}+1}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{Lev_{t-1}}{Lev_{t-1}+1}$$

$$\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\kappa_{t-1}\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1}) = Lev_{t-1}$$

$$\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1}) = Lev_{t-1}.$$

The entrepreneur profits Π are given by:

$$\begin{aligned}
\Pi = \quad & \int_{\bar{\omega}_t}^{\infty}\left[\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} - (1+R_t^e)M_{t-1}\right]dF_{t-1}\omega_t \\
= \quad & \int_{\bar{\omega}_t}^{\infty}\left[\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} - \bar{\omega}_t\kappa_{t-1}(1+R_t^k)Q_{t-1}^k K_{t-1}\right]dF_{t-1}\omega_t \\
= \quad & \int_{\bar{\omega}_t}^{\infty}\left[\left[\omega_t - \bar{\omega}_t\kappa_{t-1}\right]dF_{t-1}\right]\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} \\
= \quad & \left[\int_{\bar{\omega}_t}^{\infty}\omega_t dF_{t-1} - \int_{\bar{\omega}_t}^{\infty}\bar{\omega}_t\kappa_{t-1}dF_{t-1}\right]\omega_t(1+R_t^k)Q_{t-1}^k K_{t-1} \\
= \quad & \left[\int_{\bar{\omega}_t}^{\infty}\omega_t dF_{t-1}\omega_t - \bar{\omega}_t\kappa_{t-1}[1-F_{t-1}\bar{\omega}_t]\right](1+R_t^k)Q_{t-1}^k K_{t-1} \\
= \quad & \left[[1-G_{t-1}(\bar{\omega}_t)] - \bar{\omega}_t\kappa_{t-1}[1-F_{t-1}\bar{\omega}_t]\right](1+R_t^k)Q_{t-1}^k K_{t-1} \\
= \quad & \left[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\right](1+R_t^k)(N_{t-1}+M_{t-1}) \\
= \quad & \left[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\right](1+R_t^k) \\
& (N_{t-1}+M_{t-1})\frac{N_{t-1}(1+R_{t-1})}{N_{t-1}(1+R_{t-1})} \\
= \quad & \left[[1-G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\right]
\end{aligned}$$

$$
\begin{aligned}
& (1+R_t^k)\left(\frac{N_{t-1}+M_{t-1}}{N_{t-1}}\right)\frac{N_{t-1}(1+R_{t-1})}{(1+R_{t-1})} \\
= \quad & \Big[[1-G_{t-1}(\bar{\omega}_t)]-\kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t)-G_{t-1}(\bar{\omega}_t)]\Big] \\
& \frac{1+R_t^k}{1+R_{t-1}}\Big[1+Lev_{t-1}\Big]N_{t-1}(1+R_{t-1}).
\end{aligned}
$$

Therefore, the entrepreneur's problem is to maximize their profits subject to the bank zero-profit condition defined above. The Lagrangian representation of the problem is:

$$
\begin{aligned}
\mathcal{L}_t = \quad & E_{t-1}\Bigg[\Big[[1-G_{t-1}(\bar{\omega}_t)]-\kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t)-G_{t-1}(\bar{\omega}_t)]\Big]\frac{1+R_t^k}{1+R_{t-1}} \\
& \Big[1+Lev_{t-1}\Big](1+R_{t-1})N_{t-1} \\
& +\lambda_2\Big[\kappa_{t-1}\big[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\big]\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1})-Lev_{t-1}\Big]\Bigg].
\end{aligned}
$$

The first-order condition yields:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial \lambda_2}: \quad & \kappa_{t-1}\big[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\mu G^{e'}_{t-1}(\bar{\omega}_t)\big]\frac{(1+R_t^k)}{(1+R_{t-1})}(1+Lev_{t-1})-Lev_{t-1}=0 \\
\frac{\partial \mathcal{L}_t}{\partial Lev_{t-1}}: \quad & E_{t-1}\Big[[1-G_{t-1}(\bar{\omega}_t)]-\kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t)-G_{t-1}(\bar{\omega}_t)]\Big] \\
& \frac{1+R_t^k}{1+R_{t-1}}(1+R_{t-1})N_{t-1} \\
& +E_{t-1}\Big[\lambda_2\Big(\kappa_{t-1}\big[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)\big]\frac{(1+R_t^k)}{(1+R_{t-1})}-1\Big)\Big]=0 \\
\frac{\partial \mathcal{L}_t}{\partial \bar{\omega}_t}: \quad & -E_{t-1}\Big[G^{e'}_{t-1}(\bar{\omega}_t)+\kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)]\Big] \\
& \frac{1+R_t^k}{1+R_{t-1}}\Big[1+Lev_{t-1}\Big](1+R_{t-1})N_{t-1} \\
& +E_{t-1}\Big[\lambda_2\Big(\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t)\frac{1+R_t^k}{1+R_{t-1}}(1+Lev_{t-1})\Big)\Big] \\
& -E_{t-1}\Big[\lambda_2\Big(\kappa_{t-1}\mu G^{e'}_{t-1}(\bar{\omega}_t)\frac{1+R_t^k}{1+R_{t-1}}(1+Lev_{t-1})\Big)\Big]=0 \\
& -E_{t-1}\Big[G^{e'}_{t-1}+\kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t)-G^{e'}_{t-1}(\bar{\omega}_t)]\Big]\frac{1+R_t^k}{1+R_{t-1}}
\end{aligned}
$$

$$
\begin{aligned}
& \left[1 + Lev_{t-1}\right](1+R_{t-1})N_{t-1} \\
& + E_{t-1}\left[\lambda_2\left(\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t) - \kappa_{t-1}\mu G^{e'}_{t-1}(\bar{\omega}_t)\right)\frac{1+R^k_t}{1+R_{t-1}}(1+Lev_{t-1})\right] = 0 \\
\lambda_2 = \quad & \frac{\left[G^{e'}_{t-1}(\bar{\omega}_t) + \kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - G^{e'}_{t-1}(\bar{\omega}_t)]\right]\frac{1+R^k_t}{1+R_{t-1}}(1+Lev_{t-1})(1+R_{t-1})N_{t-1}}{\kappa_{t-1}\left[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - \mu G^{e'}_{t-1}(\bar{\omega}_t)\right]\frac{1+R^k_t}{1+R_{t-1}}(1+Lev_{t-1})} \\
\lambda_2 = \quad & \frac{\left[G^{e'}_{t-1}(\bar{\omega}_t) + \kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - G^{e'}_{t-1}(\bar{\omega}_t)]\right]}{\kappa_{t-1}\left[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - \mu G^{e'}_{t-1}(\bar{\omega}_t)\right]}(1+R_{t-1})N_{t-1}.
\end{aligned}
$$

Then, I substitute λ_2 into the first-order condition of leverage, and obtain:

$$
\begin{aligned}
0 = \quad & E_{t-1}\Bigg[\Big[[1 - G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\Big]\frac{1+R^k_t}{1+R_{t-1}}(1+R_{t-1})N_{t-1} \\
& + \frac{\left[G^{e'}_{t-1}(\bar{\omega}_t) + \kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - G^{e'}_{t-1}(\bar{\omega}_t)]\right]}{\kappa_{t-1}\left[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - \mu G^{e'}_{t-1}(\bar{\omega}_t)\right]}(1+R_{t-1})N_{t-1} \\
& \left(\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R^k_t)}{(1+R_{t-1})} - 1\right)\Bigg].
\end{aligned}
$$

This simplifies to:

$$
\begin{aligned}
0 = \quad & E_{t-1}\Bigg[\Big[[1 - G_{t-1}(\bar{\omega}_t)] - \kappa_{t-1}[\Gamma_{t-1}(\bar{\omega}_t) - G_{t-1}(\bar{\omega}_t)]\Big]\frac{1+R^k_t}{1+R_{t-1}} \\
& + \frac{\left[G^{e'}_{t-1} + \kappa_{t-1}[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - G^{e'}_{t-1}(\bar{\omega}_t)]\right]}{\kappa_{t-1}\left[\Gamma^{e'}_{t-1}(\bar{\omega}_t) - \mu G^{e'}_{t-1}(\bar{\omega}_t)\right]} \\
& \left(\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R^k_t)}{(1+R_{t-1})} - 1\right)\Bigg]. \qquad \text{(D.15)}
\end{aligned}
$$

Given that $Q^k_{t-1}K_{t-1} = M_{t-1} + N_{t-1}$ and $Lev_t = \frac{M_t}{N_t}$, the bank zero-profit condition, defined above as $\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R^k_t)}{(1+R_{t-1})} = \frac{Lev_{t-1}}{(1+Lev_{t-1})}$, can also be rewritten as:

$$
\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R^k_t)}{(1+R_{t-1})} = \frac{M_{t-1}}{(N_{t-1}+M_{t-1})}
$$

$$\kappa_{t-1}\left[\Gamma_{t-1}(\bar{\omega}_t) - \mu G_{t-1}(\bar{\omega}_t)\right]\frac{(1+R_t^k)}{(1+R_{t-1})} = \frac{Q_{t-1}^k K_{t-1} - N_{t-1}}{(Q_{t-1}^k K_{t-1})} \tag{D.16}$$

Entrepreneur net worth evolves according to:

$$N_t = \gamma^e V_t + w,$$

where V is entrepreneur equity and γ^e is the percentage of entrepreneurs who survive, obtain the aggregate profit, and receive w, which is a transfer payment when new entrepreneurs enter the next period.

Entrepreneur equity is of the form:

$$\begin{aligned}
V_t = \quad & (1+R_{t-1}^k)Q_{t-1}^k K_{t-1} - (1+R_{t-1})(Q_{t-1}^k K_{t-1} - N_{t-1}) \\
& + \mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R_{t-1}^k)Q_{t-1}^k K_{t-1} \\
= \quad & (1+R_{t-1}^k)Q_{t-1}^k K_{t-1} - (1+R_{t-1})Q_{t-1}^k K_{t-1} + (1+R_{t-1})N_{t-1} \\
& + \mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R_{t-1}^k)Q_{t-1}^k K_{t-1} \\
= \quad & Q_{t-1}^k K_{t-1}[(1+R_{t-1}^k) - (1+R_{t-1}) + \mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R_{t-1}^k)] \\
& + (1+R_{t-1})N_{t-1}.
\end{aligned}$$

Then, I write the entrepreneur net worth as:

$$\begin{aligned}
N_t = & \gamma^e\Big(Q_{t-1}^k K_{t-1}[(1+R_{t-1}^k) - (1+R_{t-1}) + \mu\kappa_{t-1}G_{t-1}(\bar{\omega}_t)(1+R_{t-1}^k)] \\
& + (1+R_{t-1})N_{t-1}\Big) + w_t.
\end{aligned} \tag{D.17}$$

The aggregate firm value is given by:

$$V_t = Y_t - W_t L_t - \Upsilon I_t + \beta\frac{\lambda_{z,t+1}}{\lambda_{z,t}}Q_{t+1}^k K_{t+1},$$

which equals the current dividend and the present discounted value of future dividends. The relative price of capital is given by:

$$Q_t^k = \frac{V_t}{K_t} + s_t^*,$$

where the price of capital includes two components: average capital price and a speculative component. The latter component can be interpreted as a price bubble that can explain the fluctuations in the capital market and macroeconomic aggregates. The relative price of capital can be rewritten as:

$$Q_t^k = \frac{Y_t - W_t L_t - \Upsilon_t^{-1} I_t}{K_t} + \beta \frac{\lambda_{z,t+1}}{\lambda_{z,t}} Q_{t+1}^k \frac{K_{t+1}}{K_t} + s_t^*.$$

D.6 Households

The objective of a household is to maximize its utility subject to the budget constraints:

$$\begin{aligned} \text{maximize} \quad & E_0 \Sigma_{t=0}^{\infty} \beta^t \left\{ \zeta_{c,t} (\log(C_t - bC_{t-1}) - \psi_l \frac{(L_t)^{1+\sigma_l}}{1+\sigma_l} \right\} \\ \text{subject to} \quad & (1+\tau^c) P_t C_t + T_t \leq (1-\tau^l) W_t L_t + (1+R_t) T_{t-1}. \end{aligned}$$

I solve the problem by the Lagrangian method. The household problem in the Lagrangian form is given by:

$$\begin{aligned} \mathcal{L} = \quad & E_0 \Sigma_{t=0}^{\infty} \beta \Bigg[\zeta_{c,t} (\log(C_t - bC_{t-1}) - \psi_l \frac{(L_t)^{1+\sigma_l}}{1+\sigma_l} \\ & + \lambda_{z,t} \Big((1+\tau^c) P_t C_t + T_t - W_t L_t - (1+R_t) T_{t-1} \Big) \Bigg]. \end{aligned}$$

The first-order condition with respect to consumption and deposit is of the form:

$$\frac{\partial \mathcal{L}_t}{\partial C_t} : \qquad \lambda_{z,t} (1+\tau_c) P_t - \frac{\zeta_{c,t}}{C_t - bC_{t-1}} + b\beta E_t \frac{\zeta_{c,t+1}}{C_{t+1} - bC_t} = 0 \qquad \text{(D.18)}$$

$$\frac{\partial \mathcal{L}_t}{\partial T_t}: \qquad \lambda_{z,t} - \beta E_t \lambda_{z,t+1}(1 + R_{t+1}) = 0 \tag{D.19}$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t}: \qquad -\psi_l (L_t)^{\sigma_l} - \lambda_{z,t} W_t = 0. \tag{D.20}$$

D.7 Resource Constraint

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon_t \mu_{\Upsilon,t}} + \frac{a(u_t)K_t}{\Upsilon_t} + \frac{\Theta(1-\gamma^e)V_t}{P_t},$$

where D_t is the entrepreneur monitoring cost,

$$D_t = \kappa_{t-1} \mu G(\bar{\omega})(1 + R_t^k) \frac{Q_{t-1}^k K_t}{P_t},$$

and G_t denotes government consumption,

$$G_t = z_t^* g_t.$$

The resource constraint can be rewritten as:

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon_t \mu_{\Upsilon,t}} + \frac{a(u_t)K_t}{\Upsilon_t} + \frac{\Theta(1-\gamma^e)}{\gamma^e}(N_t - w). \tag{D.21}$$

D.8 Monetary Policy

The monetary authority sets the policy rate according to:

$$R_t - R = \rho_p(R_{t-1} - R) + (1 - \rho_p)[a_\pi(E_t \pi_{t+1} - \pi_t) + a_{\Delta y}(Y_t - Y).] \tag{D.22}$$

D.9 Exogenous Shocks

Price markup shock:

$$\nu_{p,t} = \rho_{\nu_p} \nu_{p,t-1} + \epsilon_t^{\nu_p}. \tag{D.23}$$

Investment-specific technology shock:

$$\mu_{\Upsilon,t} = \rho_{\mu_\Upsilon} \mu_{\Upsilon,t-1} + \epsilon_t^{\mu_\Upsilon}. \tag{D.24}$$

Government spending shock:

$$g_t = \rho_g g_{t-1} + \epsilon_t^g. \tag{D.25}$$

Technology trend shock:

$$\mu_{z,t}^* = \rho_{\mu_z^*} \mu_{z,t-1}^* + \epsilon_t^{\mu_z^*}. \tag{D.26}$$

Stationary technology shock:

$$\gamma_t = \rho_\gamma \gamma_{t-1} + \epsilon_t^\gamma. \tag{D.27}$$

Entrepreneur risk shock:

$$\sigma_{e,t} = \rho_{\sigma_e} \sigma_{e,t-1} + \epsilon_t^{\sigma_e}. \tag{D.28}$$

Consumption preference shock:

$$\zeta_{c,t} = \rho_{\zeta_c} \zeta_{c,t-1} + \epsilon_t^{\zeta_c}. \tag{D.29}$$

Marginal efficiency of investment shock:

$$\zeta_{i,t} = \rho_{\zeta_i} \zeta_{i,t-1} + \epsilon_t^{\zeta_i}. \tag{D.30}$$

Collateral requirement shock:

$$\kappa_t = \rho_\kappa \kappa_{t-1} + \epsilon_t^\kappa. \tag{D.31}$$

Appendix E Detrended Model

$$y_t = \gamma_t (\frac{u_t k_{t-1}}{\mu_z^* \Upsilon})^\alpha (l_t)^{1-\alpha} - \Phi \tag{E.1}$$

$$r_t^k = \alpha \gamma_t \left(\frac{\Upsilon \mu_{z,t}^* l_t}{u_t k_{t-1}} \right) \lambda_{1,t} \tag{E.2}$$

$$\lambda_1 = \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \left(\frac{1}{\alpha} \right)^\alpha \frac{(r_t^k)^\alpha w^{(1-\alpha)}}{\gamma_t} \tag{E.3}$$

$$K_{p,t} = \lambda_{1,t} \nu_{t,p} y_t + \beta \zeta^p E_t \left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}} \right)^{(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}})} K_{p,t+1} \right] \tag{E.4}$$

$$F_{p,t} = \nu_{t,p} y_t + \beta \zeta^p E_t \left[\left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}} \right)^{(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}})} F_{p,t+1} \right] \tag{E.5}$$

$$p_t = \left[(1-\zeta_p) \left(\frac{1 - \zeta_p \left(\frac{\tilde{\pi_{t+1}}}{\pi_{t+1}} \right)^{(\frac{\nu_{t+1,p}}{1-\nu_{t+1,p}})}}{1-\zeta_p} \right)^{\frac{1}{1+\nu_p}} + \zeta_p (\tilde{\pi} p_{t-1})^{\frac{1}{1+\nu_p}} \right]^{1+\nu_p} \tag{E.6}$$

$$K_{w,t} = (l_t)^{1+\sigma_L} + \beta\zeta^l E_t\left[\left(\frac{\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}(1+\sigma_L)} K_{w,t+1}\right] \tag{E.7}$$

$$F_{w,t} = \frac{l_t}{\nu_l} p_t \lambda_z + \beta\zeta^l E_t\big(\tilde{\pi}_{w,t+1}(\mu^*_{z,t+1})^{\iota_\mu}(\mu^*_z)^{1-\iota_\mu}\big)^{\frac{1}{1-\nu_l}}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\nu_l}{1-\nu_l}}\frac{1}{\pi_{t+1}} F_{w,t+1}\Big] \tag{E.8}$$

$$w_t = \big[(1-\zeta_l)(\tilde{w}_t)^{\frac{1}{1+\nu_l}} + \zeta_l(\tilde{\pi}(\mu^*_{z,t})^{\iota_w}(\mu^*_z)^{1-\iota_w} w_{t-1})^{\frac{1}{1+\nu_l}}\big]^{1+\nu_l} \tag{E.9}$$

$$k_t = \frac{(1-\delta)k_{t-1}}{\mu^*_{z,t}} + \left(1 - S\left(\zeta_{I,t}, \frac{i_t \mu^*_{z,t}\Upsilon}{i_{t-1}}\right)\right) i_t \tag{E.10}$$

$$\begin{aligned} &\lambda_{z,t} q^k_t\left[1 - S\left(\frac{\zeta_{I,t}\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\right) - \frac{\zeta_{I,t} i_t \mu^*_{z,t}\Upsilon}{i_{t-1}} S'\left(\frac{\zeta_{I,t} i_t \mu^*_{z,t}\Upsilon}{i_{t-1}}\right)\right] \\ -&\frac{\lambda_{z,t}}{\mu_{\Upsilon,t}} + \frac{\beta\lambda_{z,t+1} q_{t+1}}{\mu^*_{z,t+1}\Upsilon}\left(\frac{\zeta_{I,t+1}\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\right)^2 S''\left(\frac{\zeta_{I,t+1} i_{t+1}}{i_t}\right) = 0 \end{aligned} \tag{E.11}$$

$$m_{t-1} = \frac{q^k_{t-1}k_{t-1} - n_{t-1}}{\pi\mu^*_{z,t}} \tag{E.12}$$

$$a'(u_t) = r^k_t \tag{E.13}$$

$$(1 + R^k_{t+1}) = \frac{(1-\tau^k)(u_{t+1}r^k_{t+1} - a(u_{t+1})) + (1-\delta)q^k_{t+1}}{\Upsilon q^k_t}\pi_t + \tau^k\delta \tag{E.14}$$

$$
\begin{aligned}
0 =& E_{t-1}\Bigg[\Bigg(1-\kappa_{t-1}\Gamma_{t-1}(\bar{\omega}_t)\Bigg)\frac{1+R^k_t}{1+R_{t-1}} \\
&+ \frac{\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t)}{\kappa_{t-1}\Gamma^{e'}_{t-1}(\bar{\omega}_t)-\kappa_{t-1}\mu G^{e'}_{t-1}(\bar{\omega}_t)} \\
&\Bigg(\kappa_{t-1}\big[\Gamma_{t-1}(\bar{\omega}_t)-(1-\mu)G_{t-1}(\bar{\omega}_t)\big]\frac{(1+R^k_t)}{(1+R_{t-1})}-1\Bigg)\Bigg]
\end{aligned}
\tag{E.15}
$$

$$
\kappa_{t-1}\big[\Gamma_{t-1}(\bar{\omega}_t)-(1-\mu)G_{t-1}(\bar{\omega}_t)\big]\frac{(1+R^k_t)}{(1+R_{t-1})} = \frac{q^k_{t-1}k_{t-1}-n_{t-1}}{(q^k_{t-1}k_{t-1})}
\tag{E.16}
$$

$$
\begin{aligned}
n_t = \frac{\gamma^e}{\pi\mu^*_z}\Bigg(& q^k_{t-1}k_{t-1}[(1+R^k_{t-1})-(1+R_{t-1})+\mu G_{t-1}(\bar{\omega}_t)(1+R^k_{t-1})] \\
&+(1+R_{t-1})n_{t-1})\Bigg)+w
\end{aligned}
\tag{E.17}
$$

$$
\lambda_{z,t}(1+\tau_c)P_t - \frac{\zeta_{c,t}\mu^*_{z,t}}{\mu^*_{z,t}c_t-bc_{t-1}} + b\beta E_t\frac{\zeta_{c,t+1}\mu^*_{z,t+1}}{c_{t+1}-bc_t} = 0
\tag{E.18}
$$

$$
\lambda_{z,t} + \beta E_t\frac{1}{\mu^*_{z,t+1}\pi_{t+1}}\lambda_{z,t+1}(1+R_{t+1}) = 0
\tag{E.19}
$$

$$
y_t = d_t + g_t + c_t + \frac{i_t}{\mu_{\Upsilon,t}} + \frac{a(u_t)k_t}{\Upsilon\mu^*_{z,t}} + \frac{\Theta(1-\gamma^e)}{\gamma^e}(n_t-w)
\tag{E.20}
$$

Appendix F Steady State

In the steady state, $\pi = \tilde{\pi}$ and $\pi^* = \pi$. I can use the steady-state forms from equations (E.6), (E.4), and (E.5) to derive:

$$\lambda_1 = \frac{1}{\nu_p}. \tag{F.1}$$

Given (E.11) and assuming $S = S'' = 0$, I obtain:

$$q = \frac{1}{\mu_\Upsilon}. \tag{F.2}$$

The steady-state form of R^k is:

$$R^k = ((1 - \tau^k) r^k + 1 - \delta) \frac{\pi}{\Upsilon} + \tau^k \delta - 1. \tag{F.3}$$

From (E.19), I find the steady-state form of R:

$$R = \frac{\pi \mu_z^*}{\beta} - 1. \tag{F.4}$$

Using (E.15) I can solve for $\bar{\omega}$.

Then, I use the detrended law of motion for net worth (E.17) and the detrended zero-profit condition (E.16) to solve numerically for k and n.

I then compute investment i:

$$i = \left[1 - \left(\frac{\frac{(1-\delta)}{\Upsilon}}{\mu_z^*}\right)\right] k. \tag{F.5}$$

Then, I can solve for w using:

$$\lambda_{1,t} = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{(r^k)^\alpha w^{(1-\alpha)}}{\gamma}. \tag{F.6}$$

I solve for l using:

$$r^k = \alpha \gamma \left(\frac{\Upsilon \mu_z^* l}{uk}\right) \lambda_{1,t}. \tag{F.7}$$

I solve for y using:

$$y = \gamma(\frac{uk}{\mu_z^* \Upsilon})^{\alpha}(l)^{1-\alpha} - \Phi. \tag{F.8}$$

I set a value for parameter g to obtain the government spending in the steady state,

$$G = g * y. \tag{F.9}$$

I use the resource constraint to solve for consumption c:

$$y = d + G + c + \frac{i}{\mu_\Upsilon} + \frac{a(u)k}{\Upsilon \mu_z^*} + \frac{\Theta(1-\gamma^e)}{\gamma^e}(n - w). \tag{F.10}$$

I solve for λ_z by using the first-order condition of consumption:

$$\lambda_z = \left(\frac{\frac{1}{(1+\tau_c)}}{c}\right)\left(\frac{\mu_z^* - b\beta}{\mu_z^* - b}\right) \tag{F.11}$$

$$F_p = \frac{(y\lambda_z)}{(1-\beta\zeta_p)} \tag{F.12}$$

$$F_w = \frac{(\lambda_z l(1-\tau^l))}{((1-\beta\zeta^l)(\nu_l))} \tag{F.13}$$

$$\psi^l = \frac{w\lambda_z}{L^{\sigma^l}} \tag{F.14}$$

$$m = \frac{(qk - n)}{\pi \mu_z^*}. \tag{F.15}$$

I can also solve for D:

$$D = \kappa\mu G(\bar{\omega})(1 + R^k)\frac{QK}{\mu_z^* \pi}.$$