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for Electricity Price Formation

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## Abstract

The wide range of models needed to support the various short-term operations for electricity generation demonstrates the importance of accurate specifications for the uncertainty in market prices. This is becoming increasingly challenging, since electricity hourly price densities exhibit a variety of shapes, with their characteristic features changing substantially within the day and over time, and the influx of renewable power, wind and solar in particular, has amplified these effects. A general-purpose, analytically tractable representation of the stochastic price formation process would have considerable value for operations control and trading, but existing empirical approaches or the application of standard density functions are unsatisfactory. We develop a general four parameter stochastic model for hourly prices, in which the four moments of the density function are dynamically estimated as latent state variables and furthermore modelled as functions of several plausible exogenous drivers. This provides a transparent and credible model that is sufficiently flexible to capture the shape-shifting effects, particularly with respect to the wind and solar output variations causing dynamic switches in the upside and downside risks. Extensive testing on German wholesale price data, benchmarked against quantile regression and other models in out-of-sample backtesting, validated the approach and its analytical appeal.

*JEL codes:* C01, C21, C22, C32, C53, Q41, Q47

*Keywords:* Electricity Prices, Density Estimation, Skewness, Quantiles, Risk

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## 1. Introduction

Price formation in wholesale electricity spot markets is known to be a complex function of many fundamental drivers, interactions, time-varying specifications and stochastic shocks. Various factors characterise the idiosyncratic dynamics, and the reasons why the stochastic models for price formation may be challenging to formulate have invited many explanations, see Lucia and Schwartz (2002), Knittel and Roberts (2005), Chen and Bunn (2010), Panagiotelis and Smith (2008), Benth et al. (2013), Aïd et al. (2013) and Weron (2014) among others.

In particular, power markets are local and resource-dependent. In some markets, the production of electricity may be a commodity spread between gas, oil or coal; in others it

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may be a result of infrastructure investments in nuclear facilities or large reservoirs, whilst elsewhere, and increasingly, it relates to the use of renewable resources such as wind, solar, hydro, biomass, geothermal or tidal currents. Furthermore electricity is produced to meet demand instantaneously; it is not easily storable, and in responding to inelastic consumers, the prices are prone to exhibit substantial volatility. And, with liberalised power markets being far from perfectly competitive, often composed of a small oligopoly of generators, at times of scarcity market power effects can result in price spikes substantially above fundamental levels. In contrast and increasingly, with some producers of electricity having inflexible production facilities, eg district heating facilities or nuclear plants, their aversion to shut down/start-up costs may incentivise them to make negative offers to the market during transient periods of oversupply (particularly from wind), resulting in “downspikes”.

Thus, to the extent that the underlying commodity properties dominate the price formation and these may be nonstationary, power prices will accordingly follow them as random walks; but if the natural renewables dominate, mean reversion will emerge from the dynamics of weather or transient supply outages. Unsurprisingly, therefore, specification tests on daily power series for mean reversion, unit roots, fractional cointegration or trend stationarity have varied in their indications over time, between locations and according to whether spikes have been trimmed out of the data (eg Haldrup and Nielsen (2006); Escribano et al. (2011); De Vany and Walls (1999); Koopman et al. (2007); Bunn and Gianfreda (2010); Nan et al. (2014)). Furthermore, with many markets being in structural transition, as governments seek to incentivise the replacement of fossil fuels with renewables, price formation will veer between different processes as wind, solar and hydro availabilities fluctuate. Finally, the granularity of power markets is fine and, because of the lack of storage, arbitrage between price formation at different times of the day or year is very restricted; thus we see quite different price distributions at offpeak hours during the night from those in the morning, midday or evening peaks. In the context of all of this, therefore, it is easily understandable why attempts to model the power price processes have led to different models for different times of the day and seasons, with regime switching and time varying specifications, nonlinear formulations as well as skewed and fat-tailed distributions all having been applied.

The complexity and evolutionary nature of these various influences on power price formation is well illustrated by the changing shapes of the German price densities since 2007. The German power market provides the main reference for European prices and, having also been at the forefront with its high penetration of renewable energy, is the most attentively observed in the region. In Figure 1 we display three selected daily time series for hours 3, 12 and 19 in 2007, and contrast these with the same series only four years later in 2011. The 2007 series exhibit the conventional patterns of a fossil fuel dominated power system, mostly coal and some gas at that time, with periods of volatility clustering and positive spikes. In contrast, 2011 shows the situation after some substantial penetration by wind and solar facilities. The price distributions are remarkably different. The predominantly positive skewness has transformed to negative skewness. In the supplementary Appendix 6.1, we display the series for all 24 hours and these fully demonstrate the diversity and rapid evolution in time series properties. The penetration of solar facilities in particular, by residential and commercial end-users, is continuing to diminish the midday need for conventional generation and eroding what used to be a daily peak, see Moody’s (2012). Thus, there is complexity in evolution, which materialises annually, as well as the time of day distinctiveness becom-

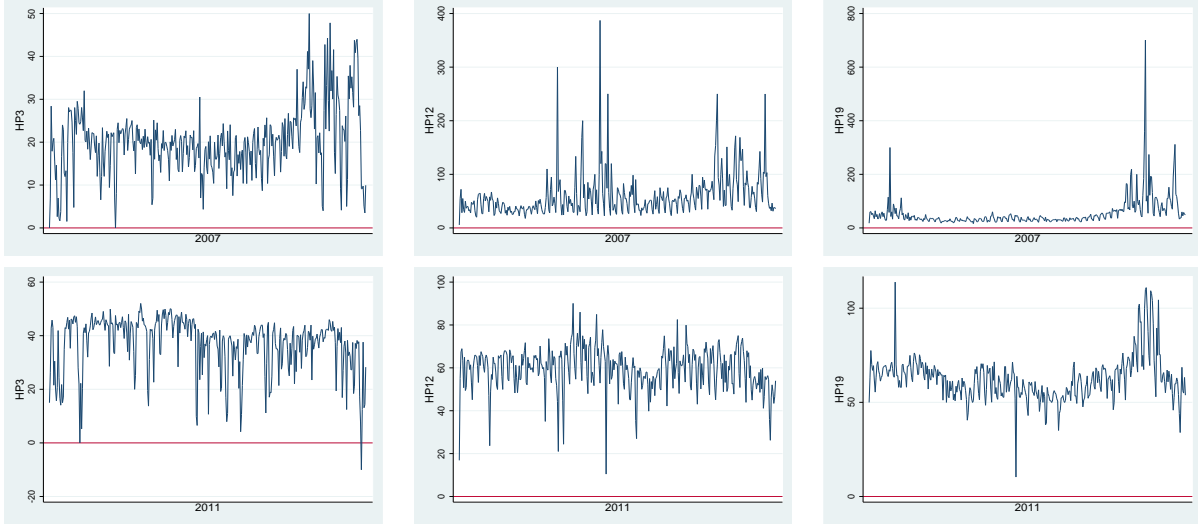


Figure 1: Daily time series for electricity prices (in €/MWh) for hours 3, 12 and 19 in 2007 (first row) and in 2011 (second row). Data source: EPEX ([www.epexspot.com](http://www.epexspot.com))

ing increasingly sensitive to changes in the weather. It is therefore a research question of considerable practical relevance to evaluate if the various hourly price densities can be determined, not apparently as empirical idiosyncracies, but as variations of a general parametric stochastic process, the parameters of which drive the appropriate changes in shape through their dependence upon the evolution of fundamental exogenous factors. That is the aim of this study.

To pursue this, we have searched for a general three or four parameter density specification with special stochastic and analytical features. From a large selection of distribution functions, we identified a few that are sufficiently flexible to provide adequate fit to the wide range of shapes displayed in the German hourly prices over 2006–2016. Furthermore, using generalized linear multivariate estimation for the four moments, sufficient to define the distributions, we were able to relate these moments to several key fundamental dynamic drivers in a plausible way, as well as to an autoregressive representation of the latent estimates to capture behavioural persistence. With respect to the latter feature, the formulation thereby represents to some extent the widely used conditional heteroscedasticity approach for volatility estimation and generalises it to the higher moments as well; the persistence of stochastic skewness being the particular new feature of power prices that we wish to capture. Overall, the latent four moment dynamic modelling within a Skewed  $t$  density representation was considered most appropriate on the balance of its empirical performance and closed form analytical properties. Day ahead predictive densities were then recursively estimated out-of-sample and benchmarked successfully against analogous quantile regression and other methods.

In the next section we consider the practical relevance of more accurate hourly price formation modelling for short-term electricity operations and then review some of the related background research on electricity price modelling, forecasting and density estimation. In Section 3, we discuss the case of Germany and present key results from the distributional analysis of power prices and fundamental drivers. The multifactor modelling approach is

described in Section 4, with the empirical support and the dependence of higher moments on fundamental drivers in Section 4.1, and the relative performance in forecasting density quantiles compared with other benchmarking techniques in Section 4.2. Finally, we summarise the research contributions in Section 5.

## 2. Operational Contexts of Power Price Density Modelling

Modelling the day-ahead electricity price formation process at hourly resolution has attracted extensive research (see Weron (2014), for a review) motivated by the practical considerations of a wide range of operational decisions for which these models provide support. Typically, in most wholesale power markets worldwide, the main-market, day-ahead prices emerge as a vector for all 24 hours of a particular day from auctions held around midday of the previous day. Whilst there are usually some demand and supply responses in the subsequent intra-day trading and real-time balancing, substantial operational commitments in practice need to be planned in advance of the day ahead auction, but with outcomes contingent upon those prices. Forecasts or simulations based upon the price formation models of the day-ahead auction prices are therefore essential to such short-term advance planning.

For example, unit commitment decisions for production facilities are often done days in advance, especially if single or two shift daily schedules are being considered (Hobbs et al., 2001; Tseng and Barz, 2002). From a risk management perspective, Stoft (2002) argues that day ahead hourly price forecasts are crucial for generators making offers to the day ahead auction in a way that recovers start-up costs over their expected dispatched periods. With the relative attractiveness of the intra-day and balancing markets, many generators face the decision problem of how much capacity to offer to the day ahead auction and how much to retain for the intra day and balancing opportunities (Soares et al., 2017; Ding et al., 2017). This will depend upon the relative price risks. The economic operation of gas-fired plants require positive spark spreads and the forecasts of power prices will therefore influence not only the offer strategy to the electricity auctions, but also planning the linked activities in the day ahead gas market, pipeline commitments and/or calls upon any swing option contracts for variable gas off-takes (Eydeland and Wolyniec, 2003; Harris, 2006, Jaillet et al., 2004). Similarly, when trading across interconnectors, transmission capacity may have to be acquired in advance of the day ahead auctions, the value of which will be a real option on the anticipated locational spreads across the day-ahead auction prices (Deng et al., 2001; Carmona and Durreleman, 2003; Bunn and Martoccia, 2010). All of these operational decisions are made in advance of the day-ahead auction prices, often with an element of optionality, and as a consequence, their valuations depend upon the probability densities of the hourly prices. The specifications of the stochastic hourly price-formation in the models that have been presented to support these decisions are often, however, quite simple mean-reverting (Ornstein-Uhlenbeck) or seasonal autoregressive processes, suitable for long-term analysis, but without any conditional dependence upon the exogenous factors such as weather and demand forecasts, as well as fuel prices, that are highly informative in the short term.

We expand on three specific illustrative contexts that are currently being actively researched and where short term conditional models of price formation would appear to be crucial for model adequacy:

- *Optimal Battery Storage and Electric Vehicle Charging Operations:* The operation of an electricity pumped storage facility on a daily cycle, based upon day ahead prices, is a well-established textbook example of optimised operational planning (Sioshansi and Conejo, 2017). Recently the linking of batteries to wind facilities has engaged various researchers in the application of stochastic optimisation techniques (Kim and Powell, 2011, Li et al., 2011, Jiang et al., 2013, Ding et al., 2016, Abdullah et al., 2017). The two stage models of Ding et al. (2016) and Abdullah et al. (2017) make use of the 24 day-ahead forecasts, but these forecasts are derived simply as historical densities. Many papers related to operating electric vehicle charging facilities have similar properties. Typical formulations have been as optimal stopping rules for charging and discharging as a function of mean reverting Gaussian spot prices (Jiang and Powell, 2016). In practice, decision making is likely to be more episodic, as with storage, based upon daily expectations for prices (Sioshansi, 2011). The day-ahead storage optimisation is particularly challenging because valuation depends upon a spread for charging and discharging and the agent has to decide upon both bid and offer hourly quantities for the auction. Accurate price risk modelling is clearly important to operate profitably across these short-term spreads.
- *Risk Management by Renewable Energy Producers:* In the absence of a link to storage, wind or solar producers can face considerable short term revenue risk from both prices and output volumes. This can be managed by means of a financial product that has payoffs to cover low volumes and low prices. Such an option contract is an example of an energy “quanto” option<sup>1</sup>, extensively discussed in Caporin et al. (2012), Benth et al. (2015) and Brik and Roncoroni (2016). These are offered as bespoke weather derivative products by the insurance industry (eg Munich Re<sup>2</sup> and Endurance Re<sup>3</sup>. To understand the payoffs from such options, their prices and the optimal design of strikes, a joint stochastic model for the power price and production (Wind or PV) is required. Typically solved by Monte-Carlo methods, not only are stochastic models for wind and prices needed, but also their correlation. In Benth and Ibrahim (2017) a simple AR(3) model with constant Gaussian noise is assumed for the hourly prices. Other derivative products, specific for particular hours, designed to help with the short term uncertainties for renewable energy producers include the “cap/floor futures” which have been introduced in Germany<sup>4</sup> and Australia<sup>5</sup>. These effectively manage the risk, on an hourly basis, of high and low prices, above and below specified thresholds. Pricing these derivatives evidently requires accurate estimation, particularly regarding the tails, of the density functions involved.

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<sup>1</sup>A long term version of this is sometimes called a *Proxy Revenue Swap*, eg for Capital Power’s Bloom Wind Farm (see <https://www.environmental-finance.com/content/sections/weather-risk-hub/weather-is-the-new-fuel-risk.html>).

<sup>2</sup><https://www.munichre.com/weatherandcommodity/en/group/index.html>

<sup>3</sup>[https://platts.com/IM.Platts.Content/ProductsServices/ConferenceAndEvents/emea/EU-Power/presentations/Ralph\\_Renner.pdf](https://platts.com/IM.Platts.Content/ProductsServices/ConferenceAndEvents/emea/EU-Power/presentations/Ralph_Renner.pdf)

<sup>4</sup><https://www.eex.com/en/products/energiewende-products/german-intraday-cap-futures>

<sup>5</sup>[https://www.asxenergy.com.au/products/overview\\_of\\_the\\_australian\\_el](https://www.asxenergy.com.au/products/overview_of_the_australian_el)

- *Demand-side Engagement:* Operational interest in day-ahead price extremes is also reflected on the demand-side and can be expected to increase with more consumer empowerment and distributed resources. Exposure to price extremes and their operational remedies can often be quite specific and detailed. For example, in Britain, drawing upon the theory of peak load pricing, the TSO recovers transmission charges from the demand side based upon their consumption in the highest three, non-consecutive trading periods in the winter (the so-called “triads”, National Grid, 2015). There is a large incentive to reduce demand in these periods but they are only known ex post, and commercial forecasting services have emerged to provide forecasts<sup>6</sup>. Generally, small distribution-connected turbines are started and run to reduce the net demand of retailers during these periods. But, it has been estimated (Frontier, 2017) that the search cost for these extremes involves targeting over 100 trading periods to ensure coverage of the maximum 3. This is one specific example but, evidently, improved modelling of the extreme price risks has benefits beyond this “triad chasing” to the extent that suppliers can influence demand-side engagement in a more timely and economic manner.

Regarding the need for more accurate price formation models in the above, and other, operational contexts, although there is a large amount of published work on modelling the fundamental drivers of power prices and a growing body of work on the disruptive effects of renewable generation, there is relatively little on the specification of electricity density forecasts. Most of the research on price formation has been in terms of relating their expected values to exogenous factors, such as fuel prices, demand, reserve margin as well as lagged effects, and the model formulations have often justified nonlinear, regime-switching and time-varying specifications (eg Huisman (2008); Karakatsani and Bunn (2008); Chen and Bunn (2010), amongst many). In parallel, stochastic models, often motivated by the need to support derivative pricing, have become increasingly elaborated to take account of non-normality, jumps and mixed processes (eg Benth et al. (2007); Panagiotelis and Smith (2008); Frestad et al. (2010)).

Regarding the particular changes in price formation induced by wind, Gelabert et al. (2011), looking at Spain, demonstrated the negative effects of renewables on price levels. In Texas, similar evidence of negative effects is presented by Baldick (2012), and also by Woo et al. (2011), the latter observing that an increase in wind generation reduces electricity prices but increases the variance and this happens to varying extents throughout the day. The price-wind-demand interrelationship is discussed in the Australian context by Cutler et al. (2011), in which they observe a general lack of correlation between wind and demand, emphasise that demand is the more important driver, but also note periods of low (high) market prices associated with high (low) wind generation at all hours of the day. The additional effects of regional imports and exports, induced by wind variations, have also been investigated by Mulder and Scholtens (2013) in The Netherlands, and by Mauritzen (2013) who identified Nordic hydropower as a natural complement to Danish intermittent wind generation.

From a forecasting perspective, Cruz et al. (2011) compare the predictive accuracy of

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<sup>6</sup>Npower Triad Warning Service <https://www.npower.com/business-solutions/buying-energy/demand-management/triadwarningservice/> and Flexitricity Triad Management <https://www.flexitricity.com/en-gb/solutions/triad/>

several univariate, multivariate, linear and nonlinear models for Spanish day-ahead prices, including hourly load and wind generation forecasts as explanatory variables, with results justifying the multivariate specifications. Similarly, Kristiansen (2012) developed a forecasting model for hourly day-ahead prices in Nord Pool based on an autoregressive model with exogenous variables, with the extended specification adding value to the previous work of Weron and Misiorek (2008).

With respect to the evolutionary nature of the model specifications, Paraschiv et al. (2014) considered the effects of both wind and solar generation on the day-ahead price formation in Germany, showing that there has been a continuous electricity price adaption process to market fundamentals, and that price drivers differ across hours with solar and wind generally reducing wholesale electricity prices. They argue that wind effects determine downspikes and even negative prices, whereas solar output balances the high demand during peak hours. Ketterer (2014) also studied the effect of wind in Germany looking at volatility dynamics as well as price levels. She showed that wind power reduces electricity price level but increases its volatility and, through rolling regressions over 3 years, found that the wind effect on mean prices was becoming less negative over time.

Finally, closer to the objectives of this paper, Serinaldi (2011) considered the short-term forecasting of Californian and Italian electricity price densities using the Johnson's  $S_U$  distribution, with time varying means and variances, but constant skewness and kurtosis, whilst Panagiotelis and Smith (2008) applied the skew- $t$  distribution in a daily vector autoregressive formulation. Otherwise, rather more researchers have approached the distributional specification through interval forecasts using semiparametric quantile regressions (eg Jónsson et al., 2014; Bunn et al., 2016), also demonstrating the time varying effects of wind, solar and other exogenous variables on particular quantile estimates. The formulation we describe below is an extension of these themes, with a time-varying specification of all four-moments from a parametric density representation of prices, with exogenous drivers and autoregressive latent variable persistence, benchmarked against forecasts from quantile regression and other models, including the simpler specification of Serinaldi (2011).

### 3. Evolution and Fundamentals of German Prices

In a relatively short period of time, since the turn of this century, the German electricity market has been characterized by a series of radical structural changes: liberalisation, emission trading, a nuclear power phase-out and, most recently, the growth in renewable generation. One consequence of the rapid penetration of wind in particular has been the need to allow negative prices to emerge in the German and coupled spot markets<sup>7</sup>. Negative prices may occur when demand falls and/or wind production is high and they signal an urgent need for generators to reduce output or for consumers to increase demand. However, producers of inflexible plants may prefer to pay for continuing to produce, as this may cost less, or be more practical, than stopping and restarting their plants over a short period of time. Less extreme than negative prices, is negative skewness, and for similar reasons, this is expected to be induced by low demand and/or high wind. Solar could have a similar effect,

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<sup>7</sup>Negative pricing has been introduced on the German/Austrian day-ahead market in 2008.



although being a midday producer, it does not generally coincide with low demand, and so its effect may be more manifest in reducing the otherwise positive skewness in those periods.

Germany is therefore an appropriate case study to develop and test a stochastic price formulation model in which there is explicit dependence of the shape of the densities on wind, solar and other short-term fundamental drivers. To this aim, we model the individual hourly electricity prices produced by the coupled German/Austrian day-ahead auction market, from January 2006 to December 2016. These 24 hour price vectors are recorded on a 7-day basis, thereby providing 24 hourly time series each containing 4018 observations<sup>8</sup>. In addition and on the same daily horizon, we have considered actual load, forecasted wind and solar PV generation, coal, gas,  $CO_2$  prices all quoted or converted in €/MWh. This data set has been carefully compiled from different sources, namely the EPEXSpot<sup>9</sup> (for hourly day-ahead prices), the four German TSOs websites (for actual and forecasted wind and solar generation aggregated on hourly level), ENTSO-e (for hourly actual loads), and Datastream<sup>10</sup> (for coal ICE API2 CIF ARA, TTF natural gas, and  $CO_2$  daily prices).

Figure 2 shows the evolution of average hourly curves for load, solar and wind production, computed yearly over 2006-2016 together with the ‘net load’ faced by conventional thermal generators after the feed-in of wind and solar production. The intra-daily load profiles show a slow decline in levels, whereas those for wind and solar show remarkable increases in output over this period. It should be mentioned that the full time series exhibit the usual annual time-varying patterns following calendar seasons, with higher solar PV generation during summer.

Descriptive statistics of price levels are reported in Table 2 in the supplementary Appendix 6.2 for all individual hours, showing the evolution of empirical sample moments across years.

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<sup>8</sup>Regarding the clock changes in spring and autumn, we have excluded from the analysis hour 25, observable at the autumn clock change, and for the spring clock change when the price for hour 3 was missing, we averaged the prices of the previous and following days. Indeed, when the clock is advanced to summer time the data reporting system automatically deletes the values/slots for hour 3 (i.e. from 2.00 am to 3.00 am). Furthermore, in our hour-by-hour approach, the leap year is not awkward as it is simply an extra day influencing the weekly seasonality that we already included in our modelling.

<sup>9</sup>The day-ahead auction DE/AT (Phelix) hourly prices are the reference prices for delivery of electricity on the following day in 24 hour intervals on the German/Austrian TSO zones. The physical deliveries are made within any of the 4 German TSOs zones as well as in the Austrian Power grid. However, EEX is planning to split it into two different zones, and, given our focus on the German market, we limit our analysis on the German TSOs hence considering only actual and forecasted variables (as wind and solar PV) registered in this market by Tennet, 50Herz, Transnet and Amprion. Thus, load and renewable data for Austria are not included.

<sup>10</sup>We have also interpolated Datastream quotations over missing weekends and holidays. The tickers of used series are, respectively: LMCYSPT for the settlement prices of coal Intercontinental Exchange API2 cost, insurance and freight Amsterdam, Rotterdam and Antwerp converted in €/MWh using the USEURSP rates from US\$ to Euro (WMR&DS); TRNLTTD for the 1st Future Day settlement prices for the natural gas TTF NL quoted in €/MWh; and finally, EEXEUAS for the EEX-EU  $CO_2$  Emissions E/EUA in €. The four German TSOs used to retrieve wind and solar data are: Tennet ([www.tennetso.de](http://www.tennetso.de)), Amprion ([www.amprion.net](http://www.amprion.net)), Transnet BW ([www.transnetbw.com](http://www.transnetbw.com)) and 50Hertz ([www.50hertz.com](http://www.50hertz.com)). Lagged actual load has been used as a proxy for the public unavailability of forecasted load, hence the load observed yesterday is considered the best forecast for today. Through the modelling, we have been attentive to the data that would be available to the market at the time when participants make their bids and offers to the day ahead auction.

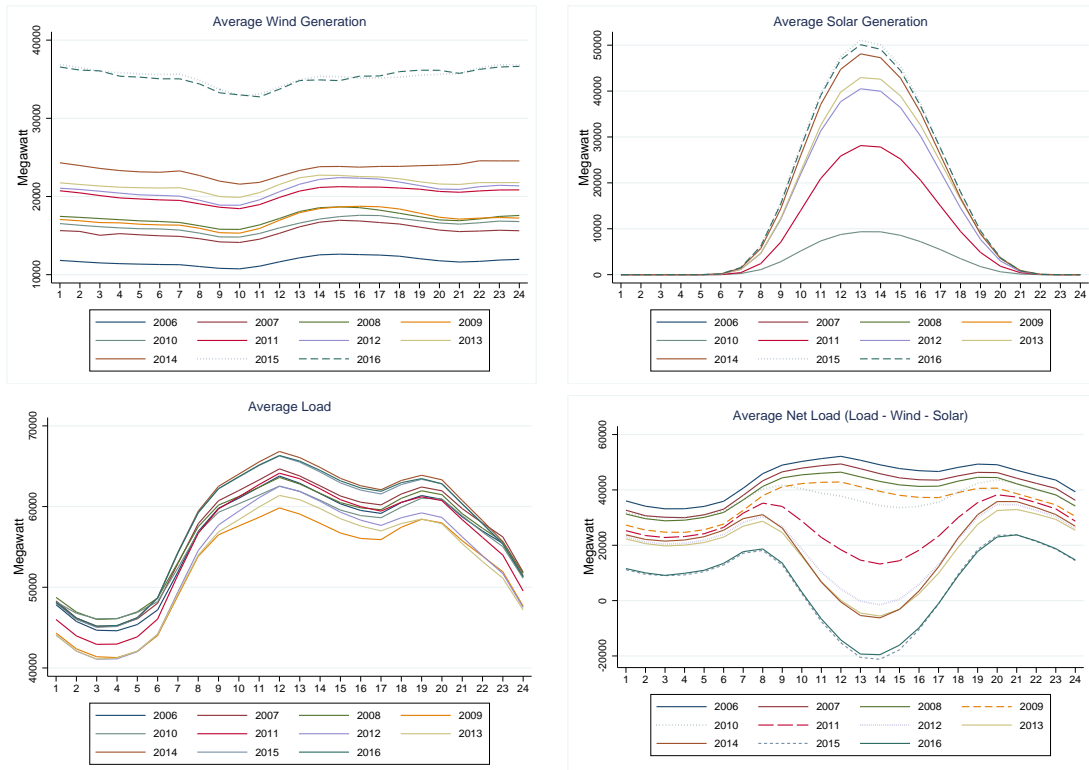


Figure 2: Average Intra-Daily Profiles for Load, Net Load, Wind and Solar Generation. Data sources: Tennet ([www.tennetso.de](http://www.tennetso.de)), Amprion ([www.amprion.net](http://www.amprion.net)), Transnet BW ([www.transnetbw.com](http://www.transnetbw.com)) and 50Hertz ([www.50hertz.com](http://www.50hertz.com)) for actual wind and solar generation; and ENTSO-E ([www.entsoe.eu](http://www.entsoe.eu)) for actual load.

Thus, it can be observed that negative skewness, on average, characterized prices densities at times of steep increase and decline in load, ie mid morning and late evening at hours 6, 7, 8, 23 and 24, at the beginning of our sample, but after 2010, negative skewness started to occur across the daytime hours as well, when solar generation created a new cycling requirement on the thermal generators. These observations motivated our modelling choice of selecting non-censored distributions with both positive and negative mean and skewness. Thus, to fit the deseasonalised densities of hourly prices, adjusted for holidays, we have considered 3 different classes of distributions with these features and also the capabilities to work within the multivariate formulations we require for estimation in the next section. The first one is a class of 4-parameter distributions, the *Johnson's  $S_U$*  (in its original parametrization as in Johnson, 1949 and in its alternative parametrization as in Johnson, 1954; see respectively *JSU<sub>0</sub>* and *JSU*), the *sinh-arcsinh* (as in Jones and Pewsey, 2009; see SHASHo and SHASHo2), the *skew exponential power* (as in Fernandez et al., 1995; see SEP1 and SEP2), the *skew- $t$*  (as in Azzalini, 1986, in Azzalini and Capitanio, 2003 and in Jones and Faddy, 2003; respectively ST1, ST2 and ST5). The second one is a 3-parameter family represented by the *skew-normal* distributions, specifically the skew normal ‘type 1’ which is a special case of the skew exponential power with  $\tau = 2$  (SN1). Thirdly, we selected as a baseline, the 2-parameter *normal* distribution (NO) as this is often used for simplicity in operational models (see previous section).

To identify the best fitting density functions, yearly values of three measures of the

goodness-of-fit (the Kolmogorov-Smirnov *KS*, the Cramer-von Mises *CVM*, and the Anderson-Darling *AD*) are reported in Tables 3-10 in Appendix 6.3. According to these measures of fit and accounting for the absolute maximum, the squared and the weighted squared differences (to give more attention on the tails), as well as the AIC criterion to discourage over-parameterisation, we observed the general superiority of the skew student-t distributions (specifically ST1 and ST2). Also recorded in these tables are details of the computational estimation times (for a PC with Intel Core Duo i7-3520M CPU 2.9 GHz and 8GB RAM) which reveal some of the computational difficulties, particularly with the two JSUs. It is also interesting to observe that SN1 and NO emerged to perform well during midday hours which is when, as we observed earlier, the historic positive skewness had been tempered to become more symmetrical through the impact of solar. When we further repeated the analysis on just the deseasonalized prices over the shorter sample 2010-2016 which we use in the multivariate modelling in the next section (shorter because forecast solar data was only available from 2010), we observed that ST2 and JSU were the two best fitting distributions (the former on hours 1-8 & 24, whereas the latter over the hours 9-23; see Table 11 in Appendix 6.3).

On balance, however, we considered the Skewed-t to be most appropriate on the basis of its general fitting and analytical properties. We note that the skewed-t had previously been used for hourly Australian prices in Panagiotelis and Smith (2008) whilst Serinaldi (2011) used the the Johnson's  $S_U$  distribution for Californian and Italian electricity price densities. The flexibility of the skew-t distribution to the range of shapes is shown in Figures 3-5, where two forms of the *skew-t* distribution were compared with the JSU, SN1 and NO, for the same motivating sample of hours and years that we displayed in Figure 1. Regarding the skew-t variants, comparing their performances, we decided to focus upon the second skew-t, in which the pdf of the skew-t type 2 distribution, denoted  $ST2(\mu, \sigma, \nu, \tau)$ , is defined by

$$f_Y(y|\mu, \sigma, \nu, \tau) = \frac{2}{\sigma} f_{Z_1}(z) F_{Z_2}(\omega) \quad (1)$$

for  $-\infty < y < +\infty$ , where  $-\infty < \mu < +\infty$ ,  $\sigma > 0$ ,  $-\infty < \nu < +\infty$  and  $\tau > 0$ , and where  $z = (y - \mu)/\sigma$ ,  $\omega = \nu\lambda^{1/2}z$ ,  $\lambda = (\tau + 1)/(\tau + z^2)$  and  $f_{Z_1}$  is the pdf of  $Z_1 \sim TF(0, 1, \tau)$  (a t-distribution with  $\tau > 0$  degrees of freedom treated as continuous parameter) and  $F_{Z_2}$  is the cdf of  $Z_2 \sim TF(0, 1, \tau + 1)$ . The mean and the variance of  $Y$  are given by  $E(Y) = \mu + \sigma E(Z)$  and  $Var(Y) = \sigma^2 Var(Z)$ , where  $E(Z) = \nu\tau^{1/2}\Gamma(\frac{\tau-1}{2}) / [\pi^{1/2}(1 + \nu^2)^{1/2}\Gamma(\frac{\tau}{2})]$  for  $\tau > 1$  and  $E(Z^2) = \tau/(\tau - 2)$  for  $\tau > 2$ . This distribution is the univariate case of the multivariate skew-t distribution introduced by Azzalini and Capitanio (2003).

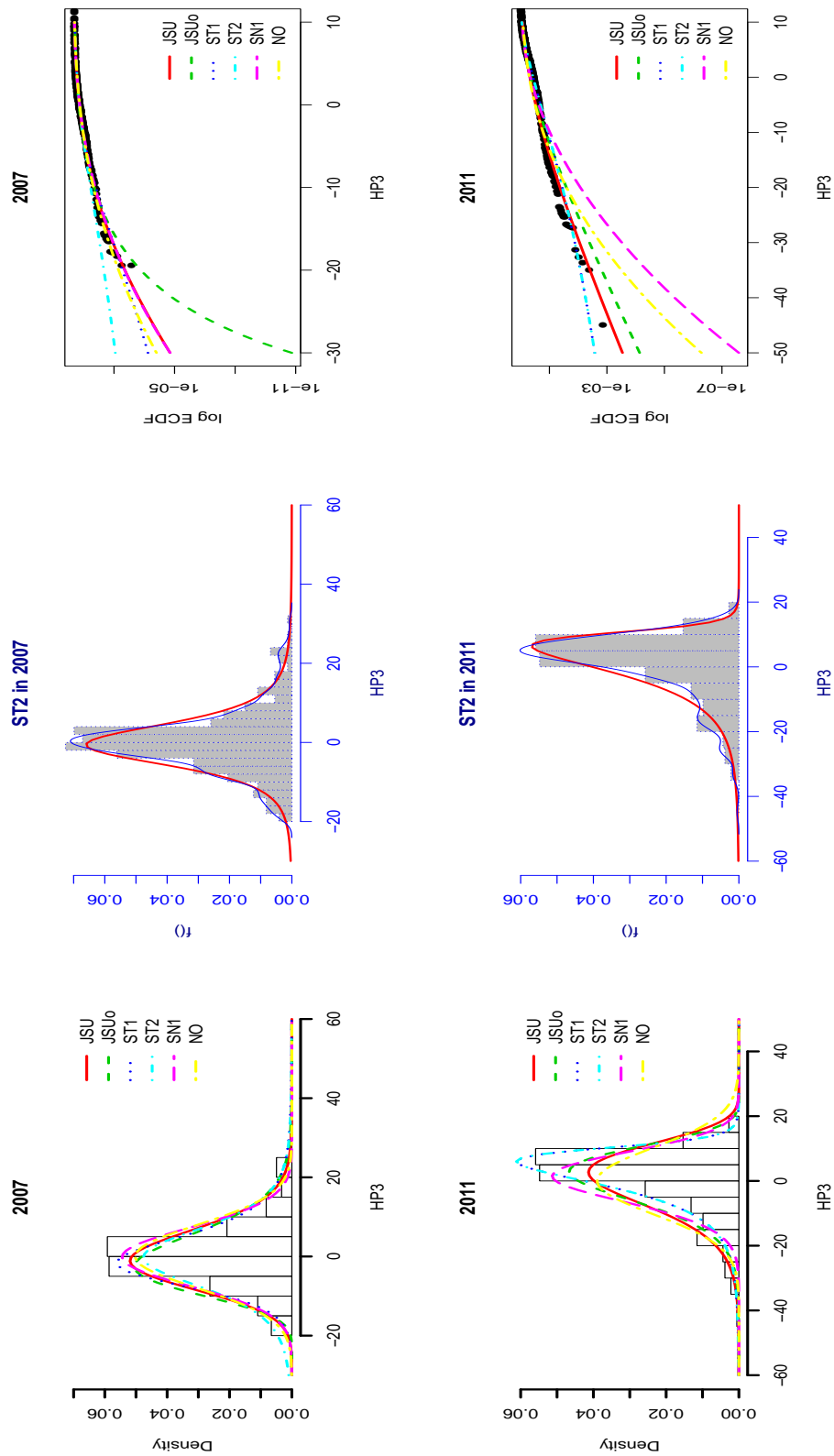


Figure 3: Comparisons of Density Fits (on the first column), the skewed-t fit vs a kernel density (in the middle) and the ECDF with Density in logarithmic scale for hour 3 in 2007 and 2011 (in rows). Descriptive statistics of deseasonalized prices:  $\mu = 0.000$ ,  $\sigma = 7.939$ ,  $\nu = 0.546$  and  $\tau = 4.487$  in 2007; and  $\mu = 0.000$ ,  $\sigma = 10.234$ ,  $\nu = -1.320$  and  $\tau = 4.610$  in 2011.

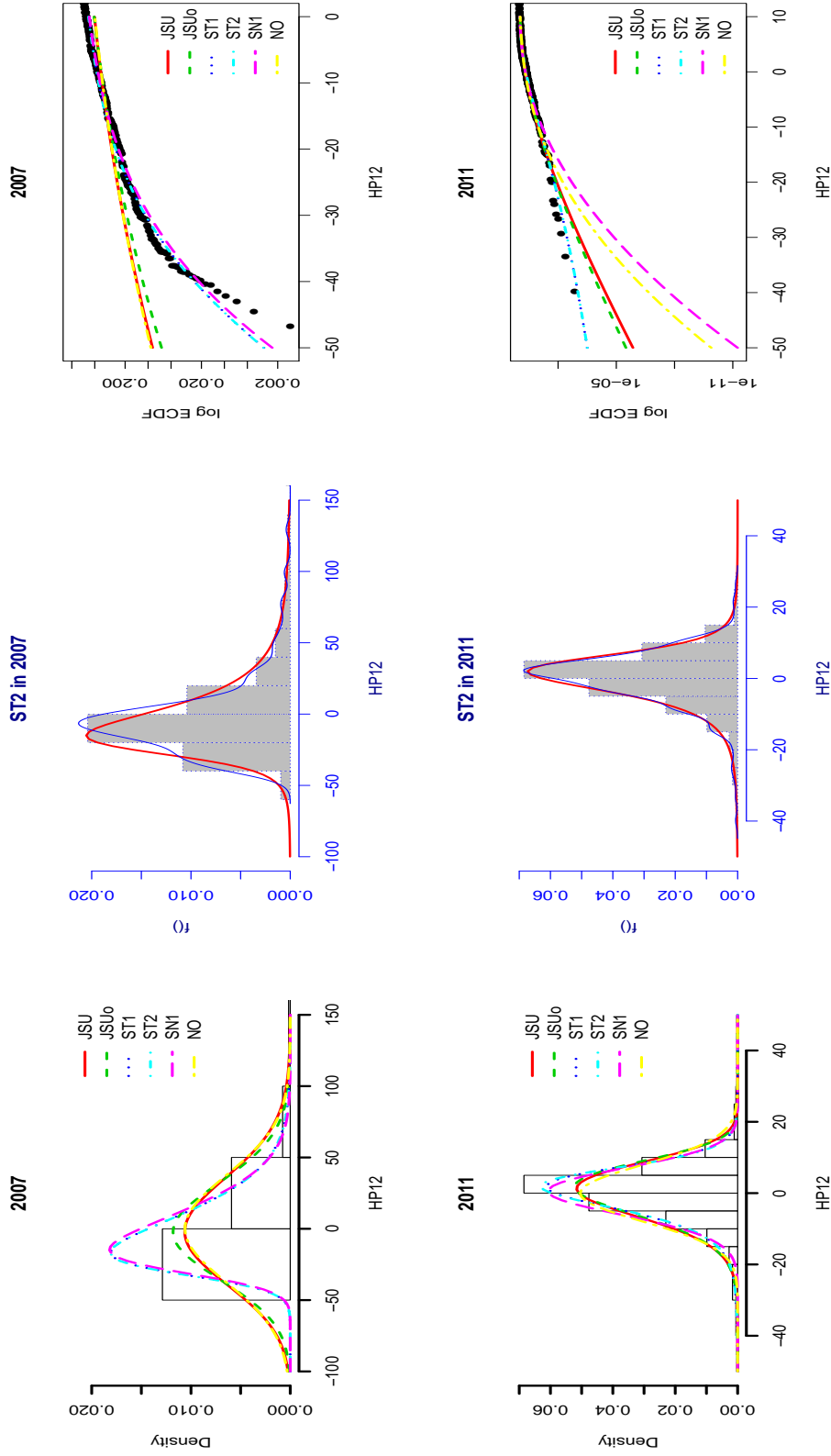


Figure 4: Comparisons of Density Fits (on the first column), the skewed-t fit vs a kernel density (in the middle) and the ECDF with Density in logarithmic scale for hour 12 in 2007 and 2011 (in rows). Descriptive statistics of deseasonalized prices:  $\mu = 0.000$ ,  $\sigma = 37.618$ ,  $\nu = 3.642$  and  $\tau = 23.575$  in 2007; and  $\mu = 0.000$ ,  $\sigma = 7.908$ ,  $\nu = -0.948$  and  $\tau = 6.362$  in 2011.

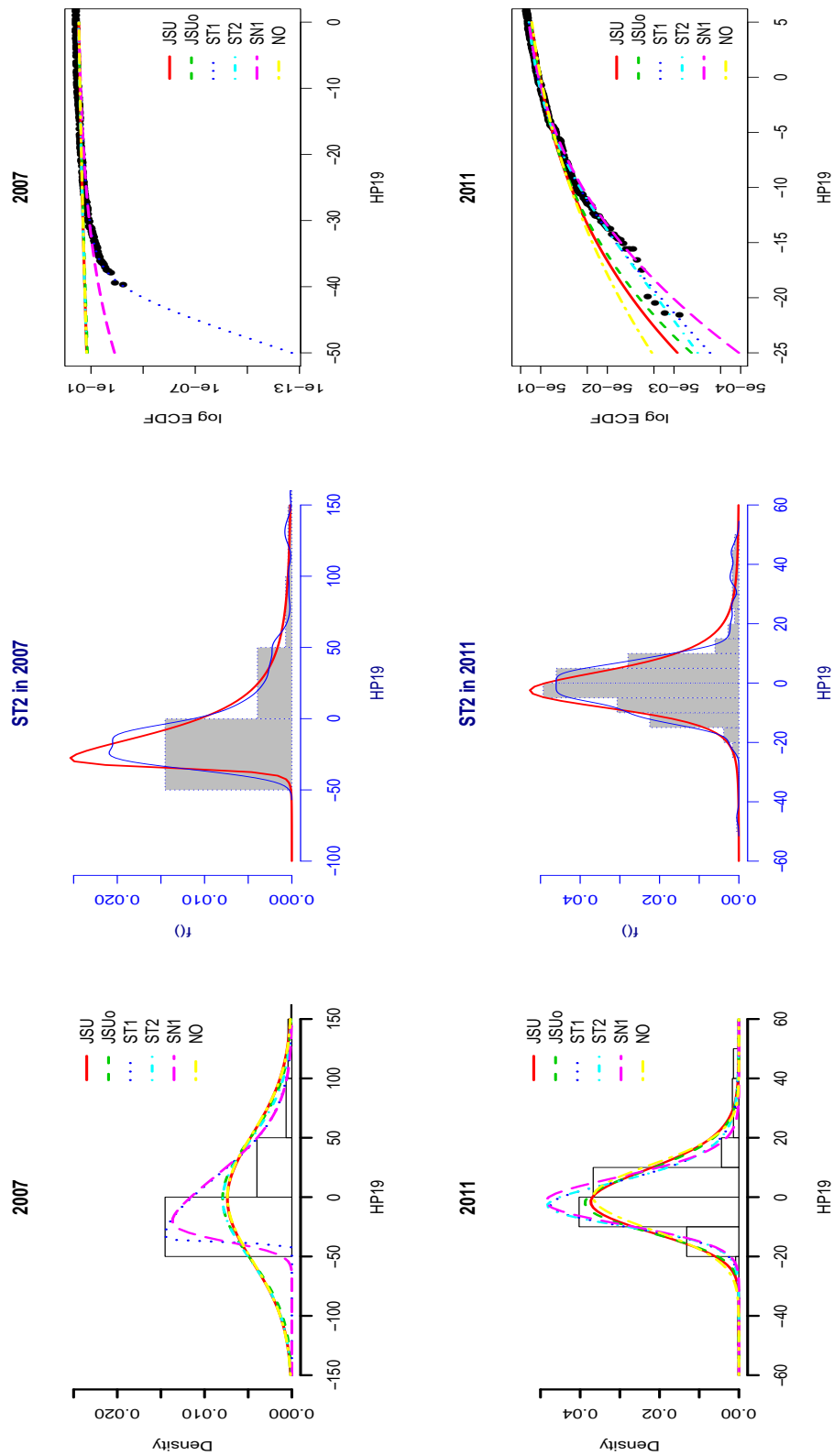


Figure 5: Comparisons of Density Fits (on the first column), the skewed-t fit vs a kernel density (in the middle) and the ECDF with Density in logarithmic scale for hour 19 in 2007 and 2011 (in rows). Descriptive statistics of deseasonalized prices:  $\mu = 0.000$ ,  $\sigma = 54.069$ ,  $\nu = 6.172$  and  $\tau = 60.714$  in 2007; and  $\mu = 0.000$ ,  $\sigma = 10.870$ ,  $\nu = 1.357$  and  $\tau = 7.921$  in 2011.

#### 4. Linear Multifactor Dynamic Estimation of the Density Moments

With the attractive flexibility of the four parameter skew-t densities for fitting the various hourly prices, the consequent research question is whether the specification of the first four moments of the skew-t can be well represented in terms of the dynamics of fundamental short term drivers of price formation. To this end, we considered whether these exogenous variables could be formulated and estimated within a generalized model for the first four moments (representing the level  $\mu$ , the volatility  $\sigma$ , skewness,  $\nu$ . and kurtosis,  $\tau$ ), resulting from an extension of the Generalized Linear Models in Nelder and Wedderburn (1972); Generalized Additive Models in Hastie and Tibshirani (1986) and Hastie and Tibshirani (1990); and the Generalized Additive Models for Location, Scale and Shape, `gamlss`, as in Rigby and Stasinopoulos (2005) and Stasinopoulos and Rigby (2007). In so doing, we seek to substantially extend the scope of applications undertaken with similar methodology by Serinaldi (2011), Matsumoto et al. (2012) and Scandroglio et al. (2013). Within this framework we formulate the hourly electricity price as a response variable whose distribution function varies according to multiple exogenous factors. From the previous section we choose to represent the response variable (the deseasonalised hourly electricity price) as a the skew-t density with the mean,  $\mu$ , standard deviation,  $\sigma$ , skewness,  $\nu$ , and kurtosis,  $\tau$  modelled as multifactor linear functions as follows.

Let  $Y$  be the response variable, it is assumed that independent observations  $y_i$  for  $i = 1, \dots, n$  have distribution function  $F_Y(y_i; \theta^i)$

with  $\theta^i = (\theta_1^i, \dots, \theta_p^i)$  a vector of  $p$  distribution parameters accounting for position, scale and shape. Generally,  $p$  is less than or equal to four, since the 4-parameter families provide enough flexibility.

Given an  $n$  length vector of the response variable  $\mathbf{y}^T = (y_1, \dots, y_n)$ , let  $g_k(\cdot)$  for  $k = 1, \dots, p$  be monotonic link functions relating the distribution parameters to explanatory variables and random stochastic effects to account for extra not explained variability through an additive model given by

$$g_k(\theta_k) = \eta_k = \mathbf{X}_k \beta_k + \sum_{j=1}^{J_k} \mathbf{Z}_{jk} \gamma_{jk} \quad (2)$$

where  $\theta_k$  and  $\eta_k$  are vectors of length  $n$ , e.g.  $\theta_k^T = \{\theta_k^1, \dots, \theta_k^n\}$ ,  $\beta_k^T = \{\beta_{1k}, \dots, \beta_{J_k k}\}$  is a parameter vector of length  $J_k$ ,  $\mathbf{X}_k$  is a known design matrix of order  $n \times J_k$ ,  $\mathbf{Z}_{jk}$  is a fixed known  $n \times q_{jk}$  design matrix and  $\gamma_{jk}$  is a  $q_{jk}$ -dimensional random variable.

The linear predictors  $\eta_k$ , for  $k = 1, \dots, p$  comprise a parametric component  $\mathbf{X}_k \beta_k$  and additive components  $\mathbf{Z}_{jk} \gamma_{jk}$ ; the first term represents a linear function of explanatory variables and the second one represents random effects. For sake of parsimony, we used only linear components in link functions. We assumed identity link functions,  $g_1(\cdot)$  and  $g_3(\cdot)$ , for the expected hourly prices and their skewness; whereas logarithmic link functions were used for  $g_2(\cdot)$  and  $g_4(\cdot)$  to ensure positivity for standard deviation and kurtosis, obtained by taking the exponential of the filtered log series.

Based upon the conventional considerations of market fundamentals and with regard to the information that would generally be available to market participants by the time they make offers and bids into the day ahead auction, we have expressed the expected hourly

price in reduced form as a function of its value observed on the day before ( $y_{t-1}$ ), as well as on electricity load observed on the day before ( $load_{t-1}$ , in thousands of MW), forecasts for wind and solar PV generation ( $fwind_t$ , and  $fsolar_t$ , also in thousands of MW) available to the market prior to the auction, lagged prices of coal, gas and  $CO_2$  allowances (respectively  $coal_{t-1}$ ,  $gas_{t-1}$ ,  $co2_{t-1}$ ), together with calendar holidays and weekends in a dummy variable,  $hol_t$ , which takes value 1 for weekends and German holidays<sup>11</sup>.

Formally, in its basic formulation, this dynamic multi-factor skew t “MFST” model has  $g_1(\theta_1) = \eta_1 = E(y_t) = \mu_t$ , with a time-varying latent mean

$$\mu_t = \alpha_1 + \gamma_1 y_{t-1} + \beta_{11} hol_t + \beta_{12} load_{t-1} + \beta_{13} fwind_t + \beta_{14} fsolar_t \quad (3)$$

$$+ \beta_{15} coal_{t-1} + \beta_{16} gas_{t-1} + \beta_{17} co2_{t-1}. \quad (4)$$

We assumed that the dispersion, estimated dynamically as a time varying latent volatility (standard deviation) state variable, is a function of fundamental drivers through  $g_2(\theta_2) = \eta_2 = \log(\sqrt{Var(y_t)}) = \log(\sigma_t)$ , with

$$\log(\sigma_t) = \alpha_2 + \beta_{21} hol_t + \beta_{22} load_{t-1} + \beta_{23} fwind_t + \beta_{24} fsolar_t + \beta_{25} coal_{t-1} + \beta_{26} gas_{t-1} + \beta_{27} co2_{t-1}. \quad (5)$$

Distinct from previous formulations (eg Serinaldi, 2011), we extend the scope of dynamic multifactors to the third and fourth moments through the time varying latent parameters,  $g_3(\theta_3) = \eta_3 = \nu_t$  and  $g_4(\theta_4) = \eta_4 = \log(\tau_t)$  as follows

$$\nu_t = \alpha_3 + \gamma_3 hol_t + \beta_{31} load_{t-1} + \beta_{32} fwind_t + \beta_{33} fsolar_t + \beta_{34} coal_{t-1} + \beta_{35} co2_{t-1} + \beta_{36} gas_{t-1} \quad (6)$$

$$\log(\tau_t) = \alpha_4 + \gamma_4 hol_t + \beta_{41} load_{t-1} + \beta_{42} fwind_t + \beta_{43} fsolar_t + \beta_{44} coal_{t-1} + \beta_{45} co2_{t-1} + \beta_{46} gas_{t-1}. \quad (7)$$

In order to investigate possible persistence in the latent moments, we also considered a new expanded formulation to include the autoregressive dynamics of the latent moments into the above mean, variance, skewness and kurtosis equations, thus defining an autoregressive multifactor skew-t model, “AR-MFST”. In the “AR-MFST” models, the 1-lag autoregressive variables  $\mu_{t-1}$ ,  $\sigma_{t-1}$ ,  $\nu_{t-1}$  and  $\tau_{t-1}$  are included in each of the corresponding equations 4-7 above. Furthermore, extending this concept to include possible implicit effects between these latent moments, we also considered a vector autoregression version in which the first lags of all four of the latent moments are included in each latent moment equation, 4-7, to give the “VAR-MFST” formulation<sup>12</sup>.

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<sup>11</sup>To avoid over-parametrization, we have included weekly seasonality together with holidays in one single dummy variable, equal to one over weekends and holidays such as: New Year, Good Friday, Easter Monday, Labour Day, Ascension Day, Whit Monday, German unit day, Christmas Eve, Christmas Day, Boxing day and New Years Eve. Given our focus on the German TSOs market, we have not considered Austrian public holidays nor regional holidays, such as Pentecost, Corpus Christi Assumption of Mary, Reformation day, All Saints day and Repentance Day.

<sup>12</sup>In the former case,  $\mu_{t-1}$  is replacing lagged prices. And autoregressive terms have been extracted after the estimation of the basic MFST model presented in eqs. 4-7.



All parameters are estimated by maximizing the likelihood function<sup>13</sup>, through an adaptation of Cole and Green (1992) algorithm, which uses the first, second and cross derivatives of the likelihood function with respect to the distribution parameters,  $\theta(\mu, \sigma, \nu, \tau)$ . As the computation of cross derivatives proved to be intractable, we adopted a generalization of the algorithm developed in Rigby and Stasinopoulos (1996a), and Rigby and Stasinopoulos (1996b) for fitting the mean and dispersion additive models which does not require the cross derivatives. Initial values for autoregressive sample parameters have been obtained by fitting the model to the entire sample, then lagged series have been included (and later in the forecasting performance, updated through the rolling iterations).

Optimisation in ML estimations such as this are often sensitive to starting values, leading to local optima, or even singularities with small samples, but with the length of time series in our study, neither of these were problems<sup>14</sup>. Overfitting, on the other hand, was a serious consideration and for that reason we restricted our factor specifications to plausible drivers on the basis of what is known about electricity price formation and we undertook extensive out-of-sample testing in a forecasting context with some challenging benchmark testing.

Whilst it should be recalled that the main objective of this modelling is to elucidate a general-purpose price formation model, flexible enough to accommodate the range of shapes that hourly power prices may take, in a way that relates these shape changes parametrically to fundamental exogenous drivers, the estimation process also derives the time-varying latent estimates of the moments. For several financial engineering applications in particular, such latent estimates of the instantaneous volatility and skewness states are attractive alternatives to historic estimates based upon lagged finite rolling windows. For example, Figure 6 shows the time series of the latent moments for hour 12, whereas similar series are shown for hour 19 in Figure 10 in Appendix 6.1.

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<sup>13</sup>The estimation has been carried out using the statistical software R and some of the "gamlss" library models (<http://www.gamlss.org/>).

<sup>14</sup>There are two algorithms to maximize the likelihood function: the first is the CG algorithm, a generalization of the Cole and Green (2002) algorithm which uses first, (expected or approximated) second and cross derivatives of the likelihood function with respect to the distribution parameters; the second one is the RS algorithm, a generalization of the algorithm used by Rigby and Stasinopoulos (1996a) and Rigby and Stasinopoulos (1996b) for fitting the mean and the dispersion of additive models and it does not use cross derivatives. Singularities in the likelihood function similar can potentially occur, especially when the sample size is small as in our yearly analysis. The RS algorithm has an outer cycle which maximizes the penalized likelihood with respect to  $\beta_k$  and  $\gamma_{jk}$  for  $j = 1, \dots, J_k$  in the model successively for each  $\theta_k$  in turn, for  $k = 1, \dots, p$ ; and at each calculation in the algorithm, the current updated values of all quantities are used. Roughly, the RS algorithm starts initializing the fitted values  $\theta_k$  and random effects  $\gamma_{jk}$  in the first outer and inner cycle iterations by regressing partial residuals against a 'weighted' design matrix to obtain updated parameter estimates. Then, the linear predictors  $\eta_k$  are evaluated and updated, and the cycle is repeated until the change in the penalized likelihood is sufficiently small. At this point, convergence is obtained and results produced. On the contrary, in the CG algorithm all weight matrices are evaluated and updated after fitting of all  $\theta_k$ . Further details on the functioning of algorithms and the maximization of the likelihood can be found in Rigby and Stasinopoulos (2005) and in Stasinopoulos et al. (2008). In all our estimation, we used the RS algorithm and control parameters set to: 0.5 as for the the convergence criterion for the algorithm (*c.crit* and *cc*), and the tolerance level for the backfitting algorithm (*bf.tol*); 1000 for *n.cyc*, for *cyc*, as the number of cycles of the algorithm, and for the number of cycles of the backfitting algorithm (*bf.cyc*). Finally, *Inf* was used for the global deviance tolerance level (*gd.tol*), to allow the algorithm to converge even if the global deviance changes dramatically during the iterations.

Residual analysis for enlarged models is presented in Appendix B.5, Figures 11 and 12, which contain the ACF and PACF of residuals from the main models under consideration; in addition, ACF and PACF for level, squared and cubic residuals are shown in Figures 13 and 14. The PACFs indicated some residual correlations which was expected. We did not include any nonlinear specification in the models, particular with respect to load. Supply functions are known to be nonlinear at low and high levels and whilst we chose a simple linear response as a robust assumption, it will presumably be underspecified. There are also interaction effects. In some of the subsequent modelling we have therefore included AR(7) to capture serial effects of up to a week. For additional insight, Table 17 in Appendix 6.4.3 presents the descriptive statistics of normalized (quantile) residuals for a sample of hours. The *normalized quantile* residuals are given by  $\hat{r}_i = \Phi^{-1}(u_i)$  where  $\Phi^{-1}$  is the inverse cumulative distribution function of a standard normal variate,  $u_i = F(y_i|\hat{\theta}^i)$  and  $F(y|\theta)$  is the cumulative distribution function with  $\theta = (\mu, \sigma, \nu, \tau)$ . If the “MFST” models are correctly specified, then the normalized quantile residuals should behave as standard normal ones, see Dunn and Smyth (1996). In our results, these residuals generally exhibit zero mean, unit variance, zero asymmetry and kurtosis equal to three (except for the simpler SN1 and NO distributions) thus indicating model adequacy (the results for other hours are similar and available on request).

#### 4.1. Dependence of Higher Moments on Exogenous Fundamental Drivers

In order to avoid over-elaboration and to justify the multifactor, time-varying representation of the moments, we developed a series of progressively more complex specifications. According to the multifactor formulations of eqs. 4-7, four models were successively estimated:

- M1: time-varying mean, but all other moments constant;
- M2: time-varying mean and standard deviation, with constant skewness and kurtosis;
- M3: time-varying mean, variance and skewness, constant kurtosis;
- M4: all four moments time varying.

In all cases, we also specified the models with and without each of the renewable energy drivers (wind and solar) and the results confirmed their incremental impacts (these have not been reported for lack of space, but they are available on request).

Expectations for these factor effects follow from previous work, but as such only inform price levels and variance (eg from Karakatsani and Bunn (2010), Ketterer (2014), Paraschiv et al. (2014), Cló et al. (2015) and Bunn et al, 2016, among others) as this research is the first to consider the higher skewness and kurtosis drivers. Thus, previous research would imply:

1. positive autoregression at lag one in price levels and the higher moments, reflecting some *adaptive behavior and persistence*;
2. positive effect of load (i.e. demand) on the mean power price because of the increasing fundamental marginal cost supply function, and on volatility, reflecting the conventional “inverse leverage” observation that at times of high demand and prices, volatility also increases (in contrast to the usual leverage in equity markets with higher volatility at lower prices);

	Hour 3				Hour 12				Hour 19			
	$\mu$	$\log(\sigma)$	$\nu$	$\log(\tau)$	$\mu$	$\log(\sigma)$	$\nu$	$\log(\tau)$	$\mu$	$\log(\sigma)$	$\nu$	$\log(\tau)$
$hol_t$	-	+	-	+	-	+	-	-	-	$\pm$	-	+
$y_{t-1}$	+				+				+			
$\mu_{t-1}$	+	+	-	+	-	-		-	$\pm$	-	-	-
$\log(\sigma_{t-1})$	+	+			-	+		-	+	+	-	-
$\nu_{t-1}$		-	+		+	+	-		-	-	-	+
$\log(\tau_{t-1})$				-		+			+	+		+
$load_{t-1}$	+	-	+	-	+	-	-	+	+	$\pm$	$\pm$	-
$fwind_t$	-	+	-	+	-	+	-	-	-	-	-	+
$fsolar_t$					-	-	-		-	-	-	+
$gas_{t-1}$	+	+	-	+	+	+	+	-	+	+	+	-
$coal_{t-1}$	+	$\pm$	-	-	+	$\pm$	-	$\pm$	+	-	-	-
$co2_{t-1}$	+	+	-	-	+	+			+	+	+	-

Table 1: High level summary of significant signs for the full model M4 and its AR and VAR variations.

3. fuel prices and  $CO_2$  generally increasing the mean power price because of their input cost;
4. wind (and solar) reducing electricity prices (especially for peak hours as consequence of balancing excess demand), but increasing the volatility.

A high level summary of the significant signs of the full model specifications for the MFST, AR-MFST and VARM-MFST models is shown in Table 1, whereas the full results for estimated coefficients and t-values are in Tables 12-14 in Appendix 6.4.1 for a sample of hours. The entries in Table 1 are for the significant (5%) coefficients and their signs. Where plus and minus signs are overlaid, the indications from all three MFST variations differ. The R-squares (the generalised R-squared in Nagelkerke (1991)), the Global Deviance (defined as a function of the log-likelihood, formally  $GD = -2\log\mathcal{L}$ ), and information criteria about the progressive modelling are reported in Tables 15 and 16 in Appendix 6.4.2. These results present a generally coherent interpretation, mostly consistent with expectations but with some new insights. The most important observations are the shape-shifts induced by wind and solar. Both wind and solar production do indeed reduce the skewness of hourly electricity prices and this is more evident at hour 12 when solar is at its maximum level. In addition, both increase the kurtosis of these prices at peak hour 19. The consistency of the results across all of the modelling progressions, with the various inclusions of wind and solar, adds considerably to the robustness of the fundamental specification. The key factor influences are:

- 1'** Lagged price is positive for all four MFST models on price level, indicating adaptive behaviour consistent with expectations. However, for the latent moment autoregressions the results are mixed. Holidays, as expected, reduce business activities and therefore price levels. Latent volatility was positively persistent on itself, as expected and had a positive effect on price levels in periods 3 and 19, but negative in period 12. For the higher moments, some mixed results might suggest over-fitting with the AR MFST and particularly VAR MFST.
- 2'** Load (i.e. demand) has a positive coefficient on the price levels, consistent with conventional expectations and previous research, but negative on price variance for hours 3 and 12. Hour 3 is when more negative price spikes are observed compared to hours 12 and 19 and low load levels over night will therefore manifest higher volatility.

- 3' Among fuel prices, we find that *natural gas* increases mean and volatility. During the day it increases skewness, whereas it reduces the kurtosis. At night it reduces skewness and increases kurtosis. These effects seem plausible as peaking generation during the day may be associated with positive spikes, whereas its use at night might be counteracting negative price tendencies. *coal* increases mean but it reduces skewness. We also confirm that  $CO_2$  affects price levels and volatility.
- 4' Wind lowers prices and lowers skewness. Hence the appearance of negative skewness, as noted earlier, can be confirmed as a wind-induced factor. Solar has effects during the day, with negative effects on price levels, volatility and skewness. The wind and solar effects are consistent across all modelling specifications.

It is questionable whether the gains of expanding the dynamic specifications in the model from 3 to 4 parameters, and in extending the specifications to AR and VAR latent moments is worthwhile. In one respect, whilst the interpretations of the kurtosis factor coefficients are generally significant, intuitions regarding their sign are equivocal, and the gains in fit from  $M3$  to  $M4$  are small. The dynamic three parameter improvement over a two parameter model is substantial, and the skewness story is persuasive; but less so for the kurtosis in going to four parameters. Likewise, the AR and VAR inclusions of the latent estimates produced equivocal estimations and this suggests specification or overfitting problems, despite their conceptual appeal. Thus, in the next section we report results on out-of-sample forecast testing performed on rolling windows of 365 days<sup>15</sup>. The motivation is not primarily to suggest a forecasting method, but to test the robustness of the specifications.

#### 4.2. Forecasting Performance

For robustness and as a check against over-fitting, we test the performance of the MFST, AR-MFST and VAR-MFST models through one period ahead, out-of-sample forecasting, using rolling window estimations and also considering all one-, two-, three- and four-moment specifications. We compare against two benchmarking techniques, one being the well-established semi parametric quantile regression method to derive empirical interval limits and the other being a conventional approach for estimating the conditional means and variances of electricity prices using ARMA-GARCH type models.

Benchmarking against quantile regressions is a challenging test insofar as these estimates of particular points on the density function (eg 5%, 95% intervals) make no distributional assumptions and are purely empirical. The MFST approach will only be as accurate as quantile regression at specific quantiles if its parametric specification is appropriate. More precisely, we compare M1- AR1- and VARM1-MFST versus Quantile Regressions, including an AR(7) structure together with drivers, holidays and seasonality. When the first two moments are considered, an ARX-EGARCHX(1,1), specified with a student-t and a skewed-student-t for the innovations, will estimate volatility persistence and its formulation clearly recalls the  $\log(\sigma_t)$  in the M2-MFST versions, which will only capture this to the extent that the drivers of heteroscedasticity are in the exogenous multifactor dynamics. But, in the

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<sup>15</sup>Not reported here were results based on rolling windows of 730 days and an expanding window of 365 days. These confirmed those found in the rolling approach presented here.

ARMA-GARCH framework, the innovations cannot change density shape as effectively as in the MFST, and then the AR- and VAR-MFST should capture some of the persistence effect through the inclusion of lagged latent moments. In addition, we compare the (AR)M2-MFST (that is specified with two moment equations) with the model formulated in Serinaldi (2011) based on the Johnsons'  $S_U$  with daily and hourly dummies, quadratic and cubic loads, and price functionals.

Quantile methods, following Koenker and Bassett (1978), are extensively applied as regression models for expressing specific percentiles of a response variable as a function of exogenous factors. Their main attractive features are firstly their semiparametric formulation of interval estimates of the predictive distribution; and secondly, the fact that they distinguish the impact of explanatory variables to different intervals of the distribution. Thus, in the conventional way, we let  $q \in [0, 1]$  be a quantile of interest,  $Y_t$  be the dependent variable (that is the hourly electricity price levels) and  $X_t$  a  $d$ -dimensional vector of explanatory variables (e.g. load, forecasted wind and solar generation; gas, coal, and  $CO_2$ ) with a constant included. The linear conditional quantile function is given by  $Q_q(Y_t|X_t) = X_t\beta_q$  and we have used the fundamental drivers as in eq.(4) to forecast and compare selected quantiles:

$$Q_q(y_t) = \alpha^q + \sum_{i=1}^7 \gamma_i^q y_{t-i} + \beta_1^q hol_t + \beta_2^q load_{t-1} + \beta_3^q fwind_t + \beta_4^q fsolar_t + \beta_5^q coal_{t-1} + \beta_6^q gas_{t-1} + \beta_7^q co2_{t-1} \quad (8)$$

As observed by Chernozhukov and Umantsev (2001), data scarcity may be a problem in estimating the extreme tails of the distribution.

Benchmarking with the ARX-EGARCHX(1,1) model we test both student-t and skew student-t error distributions. Again, we have specified an AR(7) structure with the same factors as eq.8, and in this case with the conditional variance following an EGARCH(1,1) process also driven by the load, wind, solar and lagged fuel drivers. Formally, the conditional mean being as follows

$$y_t = c + \sum_{i=1}^7 \phi_i y_{t-i} + \lambda_1 hol_t + \lambda_2 load_{t-1} + \lambda_3 fwind_t + \lambda_4 fsolar_t + \lambda_5 coal_{t-1} + \lambda_6 gas_{t-1} + \lambda_7 co2_{t-1} + \theta \varepsilon_t \quad (9)$$

and the conditional variance as follows

$$\begin{aligned} \log(\sigma_t^2) &= \omega + \alpha \log(\sigma_{t-1}^2) + \beta g(\varepsilon_{t-1}) + \varphi_1 hol_t + \varphi_2 load_{t-1} + \varphi_3 fwind_{t-1} + \varphi_4 fsolar_{t-1} \\ &\quad + \varphi_5 coal_{t-1} + \varphi_6 gas_{t-1} + \varphi_7 co2_{t-1} \end{aligned} \quad (10)$$

with  $g(\varepsilon_t) = \theta \varepsilon_t + \varrho[|\varepsilon_t| - E(|\varepsilon_t|)]$  with the conditional density set to be a Student-t,  $\varepsilon_t|I_{t-1} \sim t(0, \sigma_t^2)$ , or a skew Student-t,  $\varepsilon_t|I_{t-1} \sim st(0, \sigma_t^2)$ .

We expect that the four parameter, dynamic, multifactor skew-t models ("MFST" and "AR-MFST") to perform better than the ARMAX-GARCHX method, to the extent that the latter is only able to capture mean and variance dynamics. Even if the GARCH element would add the extra responses to conditional volatility and the persistence of shocks, the AR-MFST model is expected to exhibit competitive performance due to the extra flexibility in the density shape and the lagged latent moment persistence.

Having a full dataset of 2557 observations from the beginning of 2010 (because of the solar data not being available before), we perform an in-sample estimation using a rolling

window of 365 days. We then forecast the next observation as an out-of-sample test, and recursively advance the process through the time-series<sup>16</sup>.

We assess the calibration of the forecast densities by simply using conventional tests, such as Kupiec (1995) unconditional (UC), Christoffersen (1998) conditional coverage (CC) and the Engle and Manganelli (2004) dynamic quantile (DC) tests on yearly basis. In the coverage tests, the null hypothesis is that the calibration frequencies at each quantile are correct. Thus, in the results, a p-value greater than 0.01 indicates that we cannot reject the null hypothesis under a significance test at 1% (hence we are looking for high p values for well calibrated coverage). Tables 18-20, 21-23 and 24-26 in Appendix 6.5 summarise the results for hour 3, 12 and 19, for lower, middle and upper quantiles respectively. Results for a sample of other hours are available in Tables 27, 28, and 29. On this basis, we observe that the this forecasting process on a rolling window of 365 observations provides poor results for *quantile regression*, most likely for the sample size reasons indicated in Chernozhukov and Umantsev (2001).

For hour 3, most practical interest would be in the lower quantiles, for the occurrence of downspikes, and here the MFST and the AR-MFST outperform the quantile regression and the GARCH methodology. For hour 12, practical interest would be in both the high and low quantiles, and here again the MFST and AR-MSFT performances in terms of coverage are better than the two benchmarks especially at lower quantiles (even with the shortest rolling window). For hour 19, most practical interest would be in the high quantiles for the risk of high prices, and we see again that the MFST models outperform the benchmarks. In addition, with the lower quantiles, the flexibility and forecasting ability of MFST models continues to be beneficial with the benchmarks being outperformed. Furthermore, we observe that the ARM3-MFST outperforms the ARM4-MFST in the out-of-sample forecasting testing, and this provides empirical evidence, as suspected, that there is little to be gained through modelling with the fourth moment being dynamic, and potential overfitting can be avoided. Likewise, the VAR representation proved to be less robust than the AR for out-of-sample, and together with the mixed interpretation of the coefficients in Table 1, appears to reflect overspecification.

Finally, to summarise the main comparisons, we show in in Figure 7 the theoretical number of hits (exceedances at particular quantiles) versus the empirical frequencies across the quantiles, for three key models. For illustration we plot the graphs for hour 12 in 2011 and 2015 (similar results for other hours are available on request, showing the same indications). This clearly displays the superiority of the proposed ARM3-MFST, being closest to the theoretical curve, compared to the JSU density model used by Serinaldi (2011) and the EGARCH with skew student t which would perhaps be the most appropriate of the conditional volatility models typically used for Value-at-Risk analysis.

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<sup>16</sup>Thus, we start by estimating the models using the first 365 observations, and from this forecast the 1%, 2%, 5%, 25%, 50%, 75%, 95%, 98% and 99% quantiles of the 366<sup>th</sup> observation. Thereafter, we estimate the models using observation 2 to 366 to forecast quantiles of observation 367, and so on. The recursive (V)AR-MFST models have been initialized by using the lagged filtered moments obtained from the MSFT estimation on the whole sample; then, the autoregressive terms have been recursively updated by rolling estimations and used into the forecasting procedures.

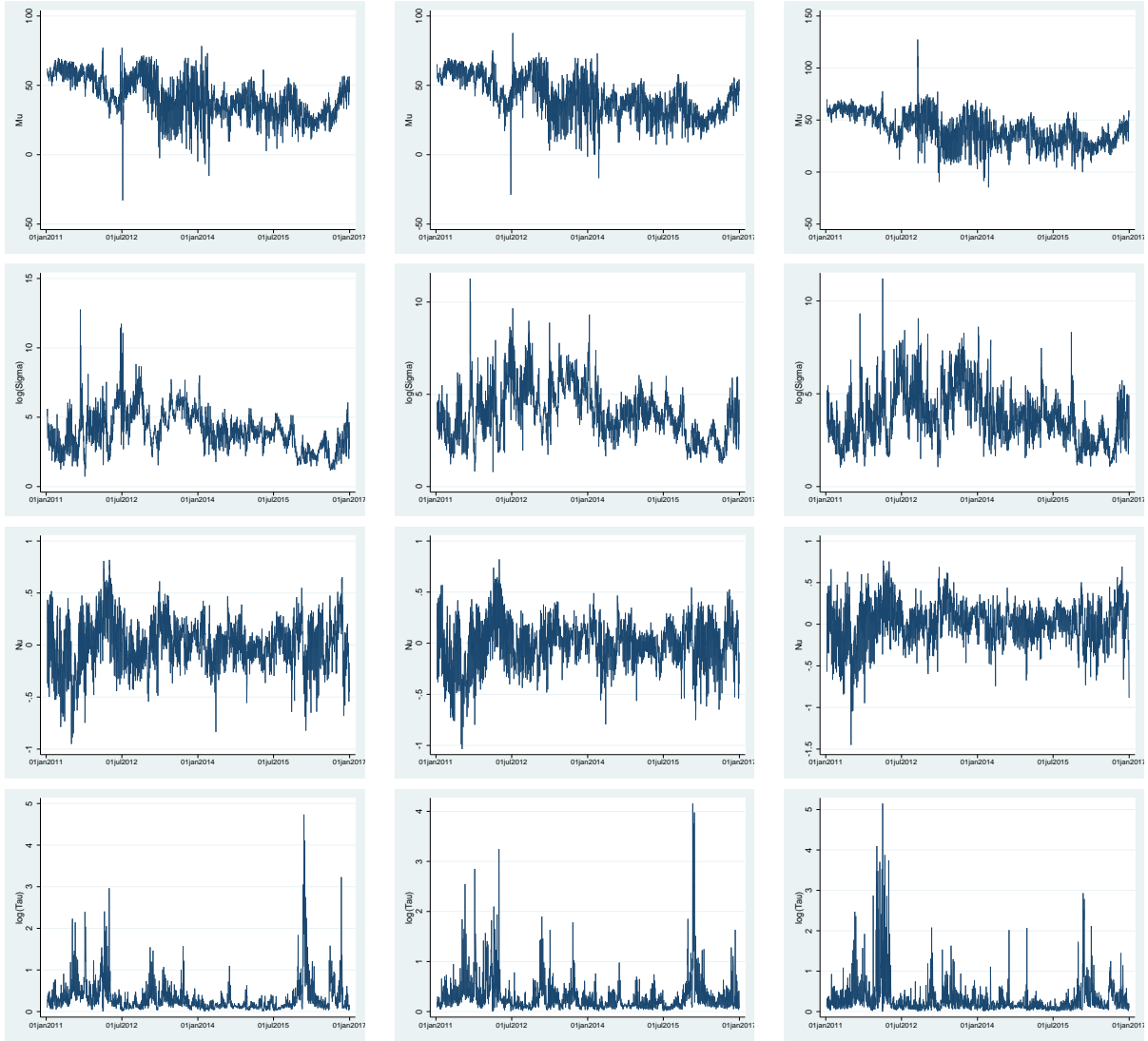


Figure 6: Filtered time series of mean ( $\hat{\mu}_t$ ) on the first rows, volatility ( $\log(\hat{\sigma}_t)$ ) on the second rows, skewness ( $\hat{\nu}_t$ ) on the third rows and kurtosis ( $\log(\hat{\tau}_t)$ ) on the last rows, from a skew-t representation of prices at hour 12 based on the MFST (first column), AR-MFST (second) and VAR-MFST models (third column) estimated on a rolling window of 365 days and continuously updated from 2010 towards the end of our sample. Static filtered moments can also be extracted once the model is estimated on the full sample.

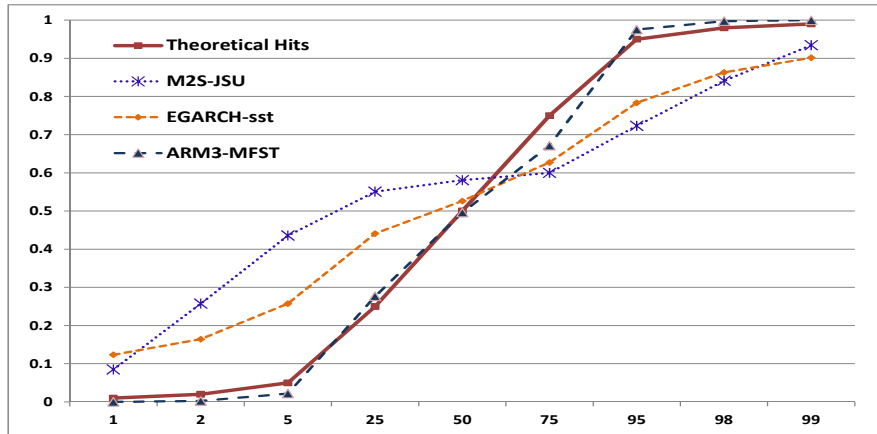
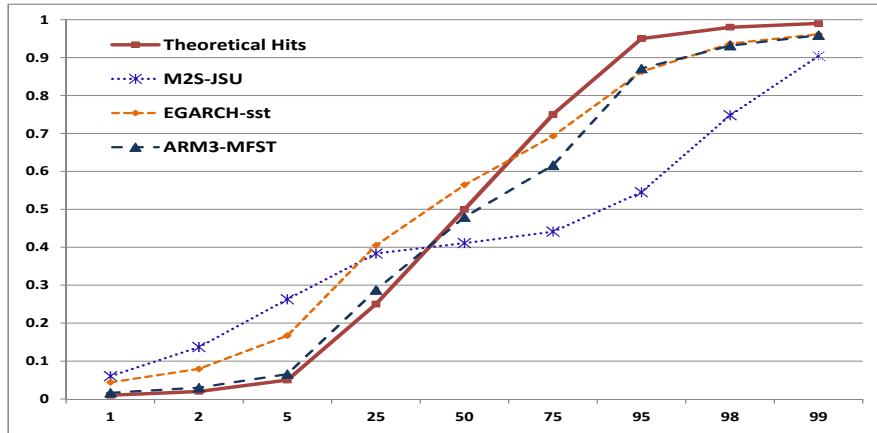


Figure 7: Theoretical versus Empirical Hits of Selected Models across Quantiles for hour 12 in 2011 (first row) and in 2015 (second row). Theoretical Hits are simply 1%, 2%, 5%, 25%, 50%, 75%, 95%, 98%, 99%; M2S-JSU is the model proposed by Serinaldi (2011); EGARCH-sst is the AR(7)X-EGARCH(1,1)-X with a skew student-t distribution as formulated in eqs. 9-10; ARM3-MFST is the ‘MFST’ model with three moment equations and autoregressive terms included (as formulated in Section 4).



## 5. Final Comments and Conclusion

The development of an accurate, flexible and analytically tractable representation for electricity price processes has considerable practical value for operations control, risk management and financial products, and the increasing penetration of renewable energy, through solar and wind, is not only adding to the complexity of price formation, but also the greater need for short-term hedging products. Closed form solutions are desirable for hedging, but the choice of an appropriate density function is awkward in power markets because of the rapid shape changes that may occur over time due to intermittent wind and solar output as well as the usual demand and fuel price shocks. The increasing interest in battery storage operations and EV charging logistics is also motivating the use of analytics that require accurate short-term models of the hourly prices. Furthermore, whilst generators have always needed to plan unit commitment several days in advance, retailers are now looking to anticipate demand-side engagement at the day ahead stage, and both generators and retailers are carefully considering their offer quantities to the day ahead auctions in the light of reserving capacities for the profitable intra day and balancing markets.

With this context and methodological need in mind, we have examined the applicability of a stochastic price formation model based upon dynamic latent moments estimated within a skewed- $t$  density to accommodate the range of shapes that hourly power prices exhibit, and furthermore related these shape changes dynamically to a set of exogenous drivers. This is conveniently and transparently achieved through linear multifactor representations of the first four moments of this density, estimated as dynamic latent variables. Benchmarked against quantile regression and ARMAX-GARCHX, the method performed well. In particular, the ability to capture the swings between positive and negative skewness by time of day and according to the amount of renewable energy generation is an appealing feature and could provide a useful ingredient into the daily optimisation of trading positions and operational scheduling. Furthermore, since the evolution of market structure is represented through the exogenous variables, the frequent re-specification that a more empirical approach would require is avoided. The MFST approach therefore offers shape flexibility and stability of specification. Furthermore, introducing lagged latent moments into the model, as AR-MFST, is attractive for interpreting persistence in skewness and volatility, as well as adaptive evolution in the mean, and this was robust to out-of-sample testing.

The approach was not outperformed by the nonparametric quantile regression benchmark, which does not offer analytical solutions, but would have been expected to estimate the percentiles more precisely. In fact our proposed approach was clearly better in terms of out-of-sample performance. This is reassuring for the general functional form of the skew- $t$  and its multifactor drivers. Regarding its comparison to an ARX-EGARCHX, also in this case, the (V)AR-MFST models outperformed, showing that extreme quantiles were better calibrated by the multifactor skew- $t$  models compared to the GARCH model. We observe that the inclusion of conditional volatility and lagged higher moments made the specification more dynamic and superior to the GARCH approach.

It is an open question if the dynamic specification of the fourth moment adds value, with the three moment version performing better out of sample, but for situations where this may be useful, eg portfolio optimisation models that explicitly use the first four moments (Giesecke et al. (2014)), the formulation appears sufficiently robust. As regards, the choice

of further exogenous variables, the reserve margin and the flow of imports/exports in the German context would be potentially valuable and may improve accuracy. There is certainly scope for a more comprehensive multifactor modelling of the price formation process, but the main methodological objectives of this research have nevertheless been well supported by the specification, as applied.

Beyond the electricity case which motivated this study, this general approach should have quite wide appeal in capturing the shape-shifting properties in the market price densities not only of a wide range of commodities but also for prices in any market where the key ingredient is a role for significant and time-varying exogenous drivers in determining dynamic shifts in the density shapes.

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## 6. APPENDICES

### 6.1. Reference Figures



Figure 8: 24 daily time series in 2007, one for each of the separately determined hourly prices.





Figure 9: 24 daily time series in 2011, one for each of the separately determined hourly prices.



Figure 10: Filtered time series of mean ( $\hat{\mu}_t$ ) on the first rows, volatility ( $\log(\hat{\sigma}_t)$ ) on the second rows, skewness ( $\hat{\nu}_t$ ) on the third rows and kurtosis ( $\log(\hat{\tau}_t)$ ) on the last rows, from a skew-t representation of prices at hour 19 based on the M4-MFST (first column), ARM4-MFST (second) and VARM4-MFST models (third column) estimated on a rolling window of 365 days and continuously updated from 2010 towards the end of our sample. Static filtered moments can also be extracted once the model is estimated on the full sample.

## 6.2. Yearly Descriptive Statistics across 24 Hours





Table with 15 columns (SE1 to RG) and multiple rows grouped by hour (HP 4, HP 5, HP 6). Each group contains 12 rows of statistics (sec, KS, CVM, AD, AIC) for each of the 10 models (SEP1 to RG). Values are numerical, with some cells containing '>' or '>>' to indicate values exceeding the column's maximum. Bolded values represent the best-performing model in each category for that hour.

Table 4: Goodness-of-fit statistics and Computational Time for hours 4-5-6. KS = Kolmogorov-Smirnov, CVM = Cramer-von Mises, and AD = Anderson-Darling statistics; AIC = Bayesian Information Criterion. “>” and “>>” mean respectively ‘greater’ and ‘grater than thousands’. “er” means that an error occurred in the process of fitting the density, then it has been reported in the computational times (in elapsed seconds) of estimated models. The general model formulation (without indication of coefficients and adapted accordingly to the number of moments in used distribution) is:  $\mu_t = E(y_t) = y_{t-1} + hol_t + fwind_t + fsolar_t + load_{t-1} + coal_{t-1} + gas_{t-1} + co2_{t-1}$ ,  $log(\sigma_t) = \nu_t = log(\tau_t) = hol_t + fwind_t + fsolar_t + load_{t-1} + coal_{t-1} + gas_{t-1} + co2_{t-1}$ . SN1 used the first three equations, whereas NO, LO, GU and RG used only the first two.















hours	KS					CVM					AD				
	JSU	JSU <sub>0</sub>	ST1	ST2	ST5	JSU	JSU <sub>0</sub>	ST1	ST2	ST5	JSU	JSU <sub>0</sub>	ST1	ST2	ST5
1	0.0603	0.0822	0.0294	<b>0.0291</b>	0.4210	1.9100	4.3528	0.5453	<b>0.5277</b>	148.5070	12.8945	3.9139	<b>3.8630</b>	727.8232	
2	0.0794	0.0688	0.0260	<b>0.0259</b>	0.4584	4.2045	2.9519	0.3020	<b>0.2985</b>	176.7370	27.9584	3.1511	<b>3.1483</b>	841.7994	
3	0.0885	0.0620	<b>0.0255</b>	<b>0.0255</b>	0.4610	5.7829	2.1785	0.2036	<b>0.2023</b>	179.0372	38.1926	<b>2.8697</b>	<b>2.8709</b>	851.0370	
4	0.0808	0.0735	0.0184	<b>0.0182</b>	0.4619	5.5244	4.2863	0.1258	<b>0.1224</b>	180.2509	37.4213	1.7491	<b>1.7386</b>	855.9105	
5	0.0733	0.0639	0.0194	<b>0.0190</b>	0.4567	3.8933	2.6782	0.1071	<b>0.1021</b>	176.3345	26.8176	1.2072	<b>1.1912</b>	840.1847	
6	0.0645	0.0769	0.0170	<b>0.0167</b>	0.4600	2.7978	4.5500	0.1375	<b>0.1369</b>	178.3401	21.0471	<b>1.3620</b>	<b>1.3644</b>	848.2432	
7	0.0636	0.0775	0.0205	<b>0.0203</b>	0.4549	3.0892	5.1891	0.1415	<b>0.1403</b>	175.8737	24.0319	1.5832	<b>1.5664</b>	838.3356	
8	0.0328	0.0464	0.0264	<b>0.0227</b>	0.3877	0.5175	2.1116	0.1587	<b>0.1478</b>	128.0669	inf	inf	<b>2.0238</b>	644.3292	
9	<b>0.0261</b>	0.0832	0.0293	<b>0.0407</b>	0.2952	<b>0.2976</b>	6.5921	0.6850	0.7924	72.8623	<b>3.0777</b>	inf	<b>2.0238</b>	644.3292	
10	<b>0.0343</b>	0.0648	0.0520	0.0558	0.2121	<b>0.9334</b>	2.7193	1.6108	1.1460	33.5401	<b>6.2472</b>	inf	10.6042	409.4537	
11	<b>0.0467</b>	0.0621	0.0695	0.0641	0.1910	1.7901	2.7249	<b>1.7070</b>	1.8615	25.7141	<b>11.3990</b>	inf	17.9298	223.8301	
12	<b>0.0566</b>	0.0632	0.0742	0.0688	0.1897	2.4602	2.9940	<b>2.2254</b>	2.4308	24.1867	<b>15.5861</b>	inf	21.8025	175.5929	
13	<b>0.0513</b>	0.0642	0.0647	0.0650	0.2146	2.4087	2.8771	2.5482	<b>1.9983</b>	32.0021	<b>15.4343</b>	19.2628	24.5901	217.4465	
14	<b>0.0466</b>	0.1034	0.0537	0.0557	0.2738	<b>1.4318</b>	7.5778	2.1409	2.2748	58.8315	10.4153	15.3257	16.2581	346.9655	
15	<b>0.0426</b>	0.0726	0.0529	0.0540	0.3414	<b>0.8479</b>	3.6727	1.7213	1.8447	96.3967	<b>8.0750</b>	11.0307	11.7799	512.1744	
16	<b>0.0336</b>	0.0756	0.0454	0.0480	0.3075	<b>0.7256</b>	5.0136	1.3505	1.5281	78.6884	<b>5.9686</b>	8.8552	9.8153	435.6142	
17	<b>0.0327</b>	0.0593	0.0478	0.0526	0.2545	<b>0.6182</b>	2.0496	1.4480	1.3419	51.1149	<b>4.4347</b>	inf	15.5496	309.8492	
18	0.0307	<b>0.0245</b>	0.0334	0.0388	0.2623	0.4800	<b>0.4532</b>	0.8896	1.1539	53.7730	3.7851	3.1608	6.7847	321.4886	
19	0.0352	<b>0.0313</b>	0.0376	0.0386	0.3264	0.8963	<b>0.5020</b>	0.9327	1.1456	85.7751	8.0516	4.4464	7.1876	466.0824	
20	<b>0.0170</b>	0.0217	0.0355	0.0425	0.2643	<b>0.1570</b>	0.1950	0.6848	1.0053	55.6637	1.5136	<b>1.5098</b>	4.7497	331.0547	
21	<b>0.0239</b>	0.0337	0.0362	0.0338	0.2122	<b>0.1983</b>	0.6995	0.3370	0.4910	37.0782	<b>1.6723</b>	5.7331	5.6475	240.7256	
22	<b>0.0209</b>	0.0560	0.0509	0.0466	0.1806	<b>0.2626</b>	3.2594	0.8717	0.7708	27.8992	<b>1.4116</b>	22.7194	9.6025	192.3555	
23	<b>0.0472</b>	0.0612	0.0667	0.0606	0.1679	<b>1.4743</b>	4.0055	1.6974	1.6342	22.7213	<b>7.6127</b>	29.9739	16.5928	165.2484	
24	0.0489	0.1069	0.0384	<b>0.0376</b>	0.3323	1.3256	7.6553	1.2046	<b>1.1320</b>	92.5652	8.2081	53.8549	<b>6.5296</b>	495.1395	

Table 11: Comparing JSU, JSU<sub>0</sub>, ST1, ST2 and ST5 over the full sample of deseasonalized electricity prices 2010-2016. KS = Kolmogorov-Smirnov, CVM = Cramer-von Mises, and AD = Anderson-Darling statistics; AIC = Bayesian Information Criterion. “Inf” means ‘infinite number’.

## *6.4. Progressive Modelling*

### *6.4.1. Estimated Coefficients*

Eq.	Hour 3													
	MI	M2	M3	M4	AR-M1	AR-M2	AR-M3	AR-M4	VAR-M2	VAR-M3	VAR-M4	VAR-M4		
$\mu_t$	const 5.351 19.663 2.266 2.366 0.182 0.506 -1.271 -0.200	$t_{-1}$ -3.811 4.283 33.302 33.422 14.422 20.469 -2.924 -23.545	$t_{-2}$ -10.293 1.090 2.790 24.160 18.238 15.258 0.654 -18.077	$t_{-3}$ -0.340 0.190 2.790 25.551 19.583 15.072 0.369 -18.531	$t_{-4}$ -5.929 0.388 2.533 2.400 9.312 17.809 -6.227 -30.460	$t_{-5}$ -3.436 -0.234 2.400 25.193 13.575 19.259 0.701 -2.091	$t_{-6}$ -9.749 12.504 3.067 21.418 15.744 13.665 0.701 -17.219	$t_{-7}$ -11.694 0.227 3.067 21.193 15.840 13.665 0.739 -0.152	$t_{-8}$ -10.041 0.191 2.969 15.418 13.665 13.665 0.472 -16.776	$t_{-9}$ -4.393 0.247 2.081 15.119 7.219 7.988 -0.448 -0.147	$t_{-10}$ -0.927 0.221 2.739 10.195 3.353 5.749 0.599 0.107	$t_{-11}$ -8.590 0.116 10.195 3.353 7.665 7.665 0.415 -0.156	$t_{-12}$ -5.445 -2.342 8.590 0.116 10.195 3.353 7.665 0.415	
$\log(\sigma_t)$	1.772	2.510 -0.054 -0.008 0.029 0.074	1.873 -0.063 0.043 0.044 0.088	1.810 0.069 0.057 0.061 0.066	1.798 0.057 -0.011 0.033 0.076	1.429 -12.871 -13.589 2.109 6.807	7.933 -0.058 -0.053 0.046 0.063	6.426 -11.380 2.520 11.453 6.170	2.309 -0.052 -0.062 0.023 0.052	10.827 -12.610 -9.587 -2.193 3.591	2.859 -0.046 -0.154 -0.006 0.029	9.259 -9.587 -6.797 -0.519 2.167	0.945 -0.058 -0.223 0.010 0.010	0.105 0.097 0.116 0.077 0.077
$\nu_t$	-2.966	-11.181 -2.714	5.064 3.727 -0.628 -0.188 -3.927 -4.655 -4.590	4.535 0.096 -0.603 -0.214 -0.156 -1.399 -0.030	-2.805 -10.837	-12.292 4.229 4.727 -4.233 -7.972 -3.415 -5.400 -5.614	4.739 2.313 -0.609 -3.772 -5.003 -3.176 -4.871 -4.135	3.948 3.431 -4.045 -6.162 -2.807 -4.971 -5.034	-2.640 2.988 0.034 -0.221 -0.050 -0.004 -3.225 -0.026	1.772 1.346 -0.927 -0.260 -0.233 -1.318 -2.330 -5.380	0.161 0.153 0.173 -0.260 -0.233 -1.318 -2.330 -5.380	0.042 0.072 -0.001 -0.179 -5.275	0.042 0.072 -0.001 -0.179 -5.275	0.105 0.125 0.097 0.160 -0.670
$\log(\tau_t)$	1.051	14.531 2.251	13.881	1.531 -0.045 -0.017 0.173 -0.118 0.555 0.022	1.089 14.767	12.134 2.263	13.562	0.985 -1.392 -0.141 4.373 -3.615 1.697 2.431	2.283	11.740	2.170	13.391	0.009 0.848 -0.034 -0.718 0.084 -0.292 0.516 0.025	0.155 0.186 0.013 -0.203 -0.750 0.009

Table 12: MFST Progressive Modelling for HP3.

Eq.	MI	M2	M3	M4	AR-M1	AR-M2	AR-M3	AR-M4	VAR-M2	VAR-M3	VAR-M4
$\mu_t$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$	$\frac{t-Var}{t-1}$
drivers	16.316	14.638	5.768	0.572	15.556	14.017	4.345	8.248	13.536	18.705	18.506
const	0.188	0.197	0.294	0.258	0.204	0.215	0.322	0.272	0.222	0.183	0.227
$load_t-1$	2.306	2.368	2.766	2.691	2.665	2.736	3.175	3.072	2.825	3.266	2.827
$load_t-2$	1.844	1.780	16.608	2.691	18.779	19.652	19.652	17.310	2.825	17.876	15.888
$gas_t-1$	0.483	0.460	0.383	0.307	0.505	0.471	0.393	0.334	0.398	0.602	0.513
$gas_t-2$	0.570	0.572	11.799	0.734	12.384	0.698	0.779	11.478	0.696	12.902	7.651
$coal_t-1$	0.570	0.572	11.799	0.734	12.384	0.698	0.779	11.478	0.696	12.902	7.651
$coal_t-2$	0.570	0.572	11.799	0.734	12.384	0.698	0.779	11.478	0.696	12.902	7.651
hol	-13.118	-12.243	-7.139	-8.197	-13.328	-12.419	-7.059	-8.000	-12.294	-6.868	-7.513
$fwind_t$	-0.266	-0.249	-0.183	-0.188	-0.272	-0.253	-0.193	-0.191	-0.253	-0.178	-0.187
$fsolar_t$	-0.242	-0.233	-0.211	-0.205	-0.242	-0.234	-0.213	-0.207	-0.232	-0.234	-0.234
$\mu_t-1$	0.115	0.114	8.087	8.277	-0.018	-0.023	-0.037	-1.430	-0.031	-1.240	-0.021
$\mu_t-2$											
$\mu_t-3$											
$\mu_t-4$											
log( $\sigma_t$ )	1.750	1.851	1.760	1.559	1.768	1.597	1.544	4.733	1.601	5.694	1.676
$load_t-1$		-0.009	-0.032	0.000	0.000	-0.008	-0.006	-1.820	-0.008	-0.012	-0.004
$load_t-2$		-0.041	-0.041	-0.065	-0.074	-0.021	-0.023	-1.292	-0.021	-0.024	-0.008
$gas_t-1$		0.004	0.004	0.047	8.235	0.025	0.035	4.429	0.025	0.025	0.005
$gas_t-2$		0.004	0.004	0.047	8.235	0.025	0.035	4.429	0.025	0.025	0.005
hol		-0.044	0.004	-0.044	-0.079	-0.054	0.203	0.416	-0.058	0.000	0.191
$fwind_t$		0.000	0.002	2.443	-0.002	-1.414	0.001	1.348	-0.001	0.002	-0.001
$fsolar_t$		-0.002	-0.002	-2.240	-0.003	-3.101	-1.622	-1.848	-0.002	-0.002	-0.003
$\mu_t-1$											
$\mu_t-2$											
$\mu_t-3$											
$\mu_t-4$											
$\nu_t$	0.085	0.490	0.455	0.455	0.090	0.102	0.512	3.640	1.895	1.415	0.953
const			3.513	2.169	0.090	0.102	0.512	-4.233	-2.195	-1.611	1.163
$load_t-1$			-0.046	-0.032	-2.465	-0.041	-0.041	-0.172	-0.041	-0.157	-0.049
$load_t-2$			-0.141	-0.106	-1.888	-0.172	-0.172	-3.171	-0.145	-0.157	-0.049
$gas_t-1$			0.072	0.084	5.080	0.086	0.086	5.799	0.097	0.095	0.091
$gas_t-2$			-0.017	-0.029	-1.422	-0.008	-0.008	-0.416	-0.022	0.012	0.024
hol			-2.300	-1.720	-5.719	-2.461	-2.461	-9.456	-2.731	-2.731	-2.170
$fwind_t$			-0.028	-0.026	-5.617	-0.027	-0.027	-6.644	-0.028	-0.028	-0.028
$fsolar_t$			-0.011	-0.012	-4.275	-0.011	-0.011	-4.173	-0.012	-0.016	-0.029
$\mu_t-1$											
$\mu_t-2$											
$\mu_t-3$											
$\mu_t-4$											
log( $\tau_t$ )	1.877	2.097	2.353	14.525	1.892	2.137	14.743	2.381	2.135	14.684	1.877
const				-1.014							
$load_t-1$			0.088	4.837	16.103	2.137	14.743	2.381	14.168	14.502	0.004
$load_t-2$			-0.201	-2.020					0.443	0.296	0.004
$gas_t-1$			0.038	1.225					0.055	2.366	0.094
$gas_t-2$			-0.013	-0.297					-0.225	-2.151	0.801
hol			-1.092	-3.862					0.036	1.135	0.360
$fwind_t$			-0.020	-4.231					0.003	0.063	0.119
$fsolar_t$			-0.005	-0.954					-1.022	-3.752	0.057
$\mu_t-1$									-0.015	-2.856	-0.018
$\mu_t-2$									-0.006	-0.885	-0.018
$\mu_t-3$											-0.134
$\mu_t-4$											-0.698
$\tau_t-1$											1.218
$\tau_t-2$											1.185
$\tau_t-3$											0.273
$\tau_t-4$											0.029

Table 13: MFST Progressive Modelling for HP12.



Eq.	Hour 19												
	M1	M2	M3	M4	AR-M1	AR-M2	AR-M3	AR-M4	VAR-M2	VAR-M3	VAR-M4	VAR-M5	
$\mu_t$													
drivers	-7.260	-7.773	-2.359	-5.486	-12.536	-10.374	7.010	6.715	-13.674	-18.040	-13.144	-18.040	
const	18.763	3.713	0.731	0.476	0.508	0.475	13.568	0.182	0.447	6.262	6.262	6.262	
load $_t$ -1	2.521	2.735	3.209	2.744	3.298	3.483	20.354	3.627	3.779	3.930	3.930	3.930	
coal $_t$ -1	0.666	0.442	0.271	0.478	0.817	0.512	10.888	0.668	0.271	0.278	0.278	0.278	
gas $_t$ -1	0.303	0.369	0.425	-0.408	0.492	0.535	8.281	0.792	0.534	1.636	0.270	0.270	
coal	-8.853	-8.129	-5.886	-5.310	-9.014	-8.266	-20.924	-8.747	-7.992	-21.080	-7.432	-7.432	
wind	-0.245	-0.213	-0.183	-0.167	-0.271	-0.229	-26.635	-0.213	-0.218	-26.680	-0.236	-0.236	
fsolar $_t$	-0.477	-0.307	-0.267	-0.285	-0.520	-0.336	-15.385	-0.117	-0.181	-5.211	-0.635	-0.635	
$\mu_t$ -1	0.140	0.140	0.126	0.131	-0.160	-0.137	-4.795	-0.109	-0.146	-4.541	-0.262	-0.262	
$\mu_t$ -1									0.826	1.774	6.723	6.723	
$\mu_t$ -1										-2.208	-4.367	-4.367	
$\mu_t$ -1											0.000	0.000	
$\mu_t$ -1											0.249	0.249	
coal $_t$ -1	2.040	2.297	1.818	2.225	2.025	2.005	7.511	1.305	1.201	1.263	1.263	1.263	
coal $_t$ -1		-0.005	-0.006	-0.014		-0.004	-1.194	0.009	0.001	0.018	0.018	0.018	
gas $_t$ -1		-0.039	-0.044	-0.022		-0.033	-1.753	-0.071	0.027	1.093	0.007	0.007	
gas $_t$ -1		0.037	0.047	0.031		0.032	3.748	0.035	0.042	4.622	0.040	0.040	
coal		-0.004	0.037	0.031		0.001	0.214	0.009	0.020	2.730	0.067	0.067	
wind		-0.110	-0.184	0.003		-0.106	-2.246	0.118	-0.087	-1.850	-0.173	-0.173	
fsolar $_t$		-0.003	-0.003	0.003		-0.003	-5.335	-0.005	-0.005	-5.326	-0.008	-0.008	
$\mu_t$ -1		-0.031	-0.034	-0.017		-0.025	-5.335	-0.030	-0.026	-4.864	-0.014	-0.014	
$\mu_t$ -1						0.019	0.976	-0.022	0.028	1.302	0.084	0.084	
$\mu_t$ -1										-0.238	-4.875	-4.875	
$\mu_t$ -1											0.000	0.000	
coal $_t$ -1	1.391	1.514	0.314	0.744	1.240	1.389	9.285	-5.759	1.324	9.021	-1.058	-1.058	
coal $_t$ -1			-0.039	-0.043		-0.039	-3.768	0.078	0.078	8.760	0.063	0.063	
gas $_t$ -1		-0.166	-2.456	-0.068		-0.254	-3.604	-0.095	-0.254	-2.593	-1.849	-1.849	
gas $_t$ -1		0.109	4.952	0.076		0.160	5.895	0.088	0.160	7.326	0.491	0.491	
coal		0.421	10.447	0.349		0.197	3.494	0.111	0.349	10.920	0.306	0.306	
wind		-1.281	-8.721	-1.268		-0.302	-2.267	-0.320	-0.302	-7.761	-1.045	-1.045	
fsolar $_t$		-0.011	-3.355	-0.016		-0.017	-8.706	-0.017	-0.017	-12.830	-0.005	-0.005	
$\mu_t$ -1		-0.059	-6.659	-0.055		-0.160	-10.942	-0.154	-0.154	-27.569	-0.026	-0.026	
$\mu_t$ -1										-0.084	-0.084	-0.084	
$\mu_t$ -1										-0.037	-4.639	-4.639	
$\mu_t$ -1										-0.684	-3.575	-3.575	
const	1.339	1.701	1.723	4.624	1.349	1.725	15.372	1.726	1.696	14.542	2.002	2.002	
load $_t$ -1				-0.046			-3.408	3.414	3.414	3.921	6.961	6.961	
coal $_t$ -1				0.032			0.356	-0.033	-0.033	-2.962	-0.027	-0.027	
gas $_t$ -1				-0.074			-2.889	-0.143	-0.143	1.615	0.138	0.138	
coal				-0.037			-0.926	0.049	0.049	-1.318	-0.101	-0.101	
wind				1.700			3.666	1.598	1.598	9.186	1.904	1.904	
fsolar $_t$				0.061			6.253	0.046	0.046	14.450	0.050	0.050	
$\mu_t$ -1				0.357			4.015	0.308	0.308	2.845	-0.074	-0.074	
$\mu_t$ -1										-2.845	-2.845	-2.845	
$\mu_t$ -1										-0.268	-1.900	-1.900	
$\tau_t$ -1								0.000	0.000	0.418	3.107	3.107	
$\tau_t$ -1										-3.332	0.000	0.000	

Table 14: MFST Progressive Modeling for HP19.



	R-sqr		Global Deviance			AIC			SBC			
	Base	AR	VAR	Base	AR	VAR	Base	AR	VAR	Base	AR	VAR
Hour 1												
M1	0.7996	0.7818		15151	15362		15173	15384		15237	15448	
M2	0.8377	0.8217	0.8223	14613	14848	14838	14647	14884	14878	14746	14989	14995
M3	0.8466	0.8326	0.8360	14469	14686	14634	14515	14736	14696	14649	14882	14878
M4	0.8476	0.8337	0.8378	14451	14670	14606	14509	14734	14694	14679	14921	14951
Hour 2												
M1	0.7684	0.7591		15580	15673		15602	15695		15666	15760	
M2	0.8250	0.8174	0.8185	14866	14967	14952	14900	15003	14992	14999	15109	15109
M3	0.8360	0.8283	0.8330	14700	14810	14739	14746	14860	14801	14880	15006	14982
M4	0.8386	0.8320	0.8398	14659	14755	14633	14717	14819	14721	14887	15006	14978
Hour 3												
M1	0.7405	0.7284		15919	16029		15941	16051		16005	16115	
M2	0.8077	0.7988	0.7995	15153	15264	15255	15187	15300	15295	15287	15405	15411
M3	0.8193	0.8115	0.8152	14996	15098	15047	15042	15148	15109	15176	15294	15290
M4	0.8227	0.8153	0.8186	14946	15046	15000	15004	15110	15088	15173	15297	15345
Hour 4												
M1	0.6922	0.6817		16335	16413		16357	16435		16421	16500	
M2	0.7714	0.7625	0.7632	15576	15667	15659	15610	15703	15699	15709	15808	15816
M3	0.7855	0.7769	0.7811	15414	15508	15460	15460	15558	15522	15594	15704	15703
M4	0.7888	0.7801	0.7864	15374	15470	15397	15432	15534	15485	15601	15721	15742
Hour 5												
M1	0.6987	0.6888		16194	16271		16216	16293		16280	16357	
M2	0.7721	0.7646	0.7651	15483	15558	15554	15517	15594	15594	15616	15700	15711
M3	0.7855	0.7797	0.7867	15328	15390	15307	15374	15440	15369	15508	15586	15551
M4	0.7889	0.7734	0.7898	15287	15462	15270	15345	15526	15358	15514	15713	15615
Hour 6												
M1	0.7668	0.7609		15634	15688		15658	15712		15728	15782	
M2	0.8188	0.8165	0.8165	14990	15014	15013	15028	15054	15057	15139	15171	15186
M3	0.8334	0.8312	0.8325	14776	14801	14781	14828	14857	14849	14980	15021	15047
M4	0.8371	0.8341	0.8368	14719	14757	14716	14785	14829	14812	14978	15039	15092
Hour 7												
M1	0.7729	0.7673		16701	16753		16725	16777		16795	16847	
M2	0.8056	0.8082	0.8092	16304	16259	16246	16342	16299	16290	16453	16416	16419
M3	0.8248	0.8318	0.8344	16039	15925	15886	16091	15981	15954	16243	16144	16152
M4	0.8288	0.8339	0.8374	15981	15893	15838	16047	15965	15934	16240	16176	16215
Hour 8												
M1	0.8045	0.8056		17540	17516		17564	17540		17635	17610	
M2	0.8197	0.8224	0.8228	17334	17285	17278	17372	17325	17322	17483	17442	17451
M3	0.8382	0.8460	0.8525	17058	16922	16812	17110	16978	16880	17262	17142	17078
M4	0.8420	0.8487	0.8577	16997	16877	16720	17063	16949	16816	17256	17159	17097
Hour 9												
M1	0.7957	0.7998		17610	17545		17634	17569		17704	17640	
M2	0.8077	0.8092	0.8106	17455	17423	17404	17493	17463	17448	17604	17580	17577
M3	0.8241	0.8333	0.8427	17227	17079	16930	17279	17135	16998	17431	17299	17197
M4	0.8286	0.8404	0.8485	17162	16969	16835	17228	17041	16931	17421	17251	17212
Hour 10												
M1	0.7901	0.7949		17346	17275		17370	17299		17440	17369	
M2	0.8006	0.8013	0.8048	17215	17195	17150	17253	17235	17194	17364	17351	17322
M3	0.8126	0.8139	0.8349	17057	17028	16722	17109	17084	16790	17261	17248	16989
M4	0.8195	0.8222	0.8401	16961	16911	16640	17027	16983	16736	17220	17193	17017
Hour 11												
M1	0.8039	0.8034		17143	17140		17167	17164		17237	17234	
M2	0.8150	0.8122	0.8176	16995	17023	16948	17033	17063	16992	17144	17179	17121
M3	0.8328	0.8261	0.8450	16737	16827	16533	16789	16883	16601	16941	17047	16800
M4	0.8379	0.8322	0.8443	16658	16736	16545	16724	16808	16641	16917	17018	16921
Hour 12												
M1	0.8132	0.8106		17079	17107		17103	17131		17174	17201	
M2	0.8215	0.8192	0.8241	16965	16988	16918	17003	17028	16962	17114	17145	17091
M3	0.8341	0.8367	0.8432	16778	16729	16626	16830	16785	16694	16981	16949	16892
M4	0.8367	0.8380	0.8428	16736	16708	16633	16802	16780	16729	16995	16990	17009

Table 15: Informative Statistics of Enlarged Models. R-sqr is the ‘‘Cox-Snell’’ or Nagelkerke’s R squared, Global Deviance is  $GD = -2\mathcal{LL}$ , AIC and SBC are the Akaike and Schwarz Bayesian Information Criteria for estimated models, as formulated in eqs 2-7 and described therein.

	R-sqr		Global Deviance			AIC			SBC			
	Base	AR	VAR	Base	AR	VAR	Base	AR	VAR	Base	AR	VAR
Hour 13												
M1	0.8262	0.8222		16825	16876		16849	16900		16919	16970	
M2	0.8345	0.8311	0.8345	16700	16745	16693	16738	16785	16737	16849	16902	16865
M3	0.8510	0.8421	0.8549	16433	16573	16358	16485	16629	16426	16637	16793	16625
M4	0.8526	0.8515	0.8559	16405	16417	16340	16471	16489	16436	16663	16699	16717
Hour 14												
M1	0.8321	0.8282		16845	16896		16869	16920		16939	16990	
M2	0.8381	0.8357	0.8382	16752	16781	16742	16790	16821	16786	16901	16938	16915
M3	0.8590	0.8579	0.8644	16400	16411	16293	16452	16467	16361	16604	16631	16559
M4	0.8639	0.8631	0.8705	16309	16316	16176	16375	16388	16272	16568	16599	16552
Hour 15												
M1	0.8267	0.8227		16870	16920		16894	16944		16964	17014	
M2	0.8328	0.8297	0.8316	16779	16817	16789	16817	16857	16833	16928	16974	16962
M3	0.8541	0.8540	0.8605	16431	16425	16309	16483	16481	16377	16635	16644	16576
M4	0.8608	0.8612	0.8705	16311	16296	16120	16377	16368	16216	16570	16579	16497
Hour 16												
M1	0.8129	0.8111		16826	16843		16850	16867		16920	16937	
M2	0.8200	0.8186	0.8205	16728	16739	16713	16766	16779	16757	16877	16895	16886
M3	0.8365	0.8359	0.8429	16482	16483	16372	16534	16539	16440	16686	16703	16639
M4	0.8463	0.8478	0.8589	16324	16292	16099	16390	16364	16195	16582	16574	16476
Hour 17												
M1	0.7924	0.7919		16972	16971		16996	16995		17066	17066	
M2	0.8017	0.8001	0.8009	16855	16869	16859	16893	16909	16903	17004	17026	17032
M3	0.8176	0.8187	0.8227	16642	16620	16563	16694	16676	16631	16846	16839	16830
M4	0.8214	0.8217	0.8258	16589	16577	16518	16655	16649	16614	16848	16860	16895
Hour 18												
M1	0.7465	0.7442		17881	17898		17905	17922		17975	17992	
M2	0.7683	0.7657	0.7666	17652	17674	17664	17690	17714	17708	17801	17831	17836
M3	0.7858	0.7831	0.8006	17451	17477	17262	17503	17533	17330	17655	17696	17529
M4	0.7990	0.7965	0.8021	17289	17314	17243	17355	17386	17339	17548	17596	17620
Hour 19												
M1	0.7489	0.7417		17829	17895		17853	17919		17923	17989	
M2	0.7714	0.7654	0.7726	17589	17650	17569	17627	17690	17613	17739	17806	17742
M3	0.7915	0.7866	0.7895	17354	17408	17373	17406	17464	17441	17558	17627	17640
M4	0.8040	0.8001	0.8052	17196	17241	17176	17262	17313	17272	17455	17523	17553
Hour 20												
M1	0.7223	0.7156		17585	17640		17609	17664		17679	17734	
M2	0.7457	0.7413	0.7494	17361	17399	17317	17399	17439	17361	17510	17556	17490
M3	0.7695	0.7641	0.7658	17111	17163	17145	17163	17219	17213	17315	17383	17411
M4	0.7784	0.7727	0.7766	17010	17068	17024	17076	17140	17120	17269	17351	17401
Hour 21												
M1	0.7553	0.7494		16424	16478		16448	16502		16518	16572	
M2	0.7712	0.7673	0.7716	16252	16290	16242	16290	16330	16286	16401	16447	16415
M3	0.7805	0.7776	0.7882	16146	16174	16050	16198	16230	16118	16350	16393	16317
M4	0.7878	0.7854	0.7945	16060	16083	15973	16126	16155	16069	16319	16366	16350
Hour 22												
M1	0.7862	0.7732		15464	15608		15486	15630		15550	15695	
M2	0.7996	0.7894	0.7935	15299	15419	15369	15333	15455	15409	15432	15560	15526
M3	0.8138	0.8032	0.8155	15111	15246	15082	15157	15296	15144	15291	15442	15325
M4	0.8177	0.8070	0.8154	15058	15196	15082	15116	15260	15170	15285	15447	15427
Hour 23												
M1	0.8082	0.8013		15032	15118		15054	15140		15119	15204	
M2	0.8182	0.8123	0.8138	14897	14972	14952	14931	15008	14992	15030	15113	15109
M3	0.8335	0.8292	0.8298	14671	14732	14722	14717	14782	14784	14852	14928	14965
M4	0.8330	0.8304	0.8330	14679	14714	14674	14737	14778	14762	14906	14965	15019
Hour 24												
M1	0.8286	0.8191		14656	14788		14678	14810		14743	14874	
M2	0.8438	0.8365	0.8382	14419	14531	14504	14453	14567	14544	14552	14672	14661
M3	0.8556	0.8500	0.8528	14219	14312	14264	14265	14362	14326	14400	14508	14507
M4	0.8548	0.8500	0.8564	14233	14310	14200	14291	14374	14288	14460	14561	14545

Table 16: Informative Statistics of Enlarged Models. R-sqr is the ‘‘Cox-Snell’’ or Nagelkerke’s R squared, Global Deviance is  $GD = -2\mathcal{LL}$ , AIC and SBC are the Akaike and Schwarz Bayesian Information Criteria for estimated models, as formulated in eqs 2-7 and described therein.

### 6.4.3. Residuals Analysis

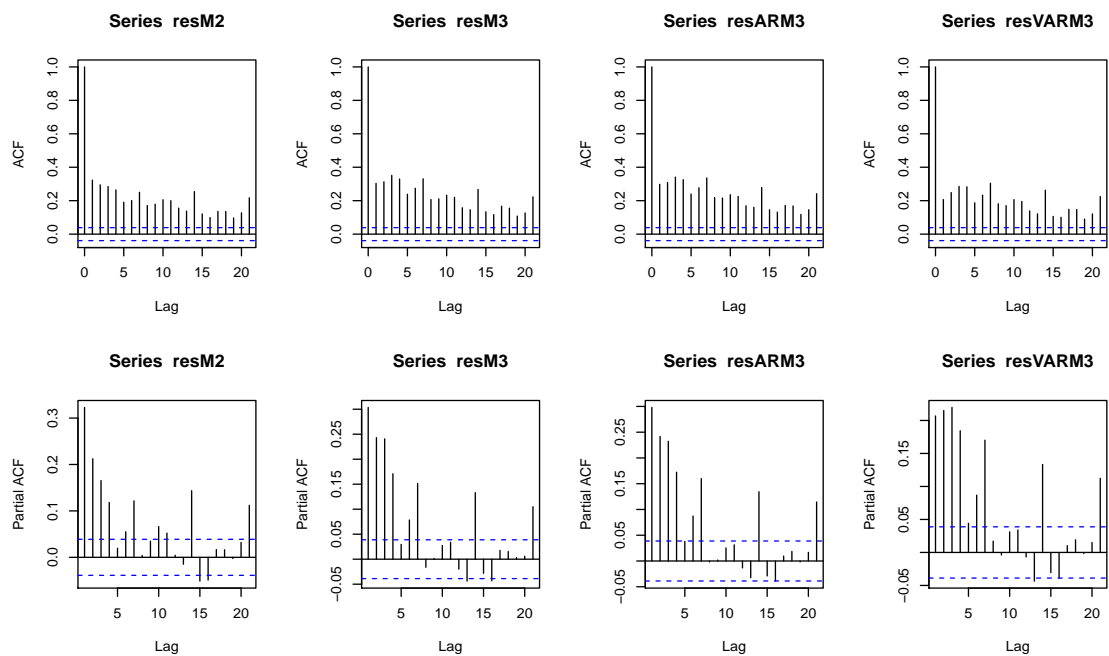


Figure 11: Autocorrelation and Partial autocorrelation Functions for Models M2, M3, ARM3 and VARM3 for hour 3.

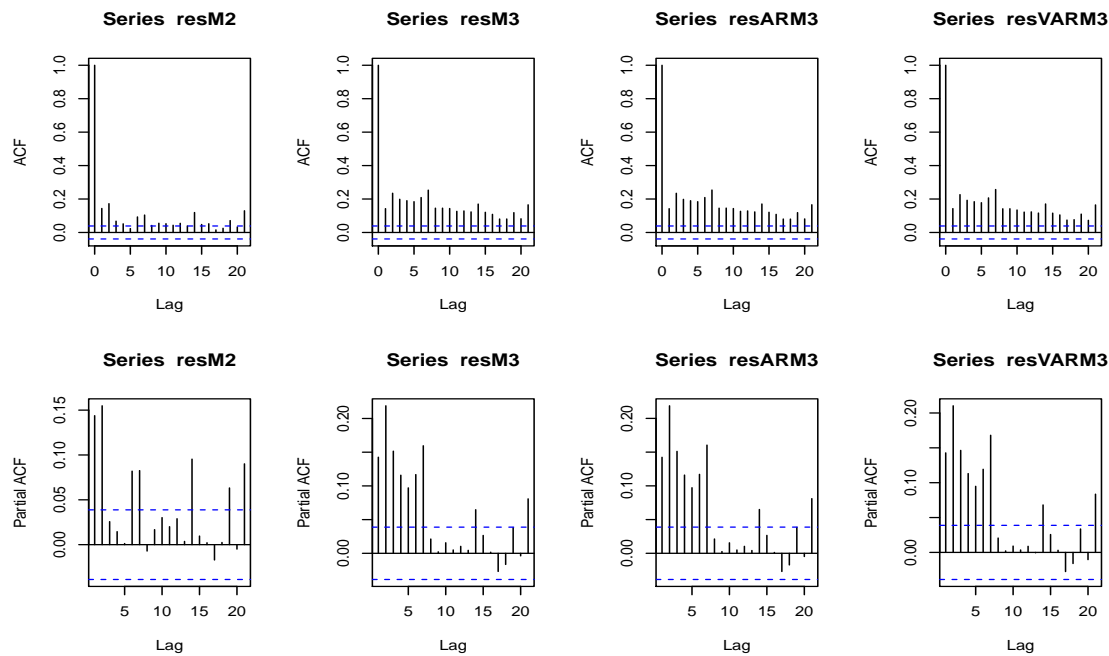


Figure 12: Autocorrelation and Partial autocorrelation Functions for Models M2, M3, ARM3 and VARM3 for hour 12.

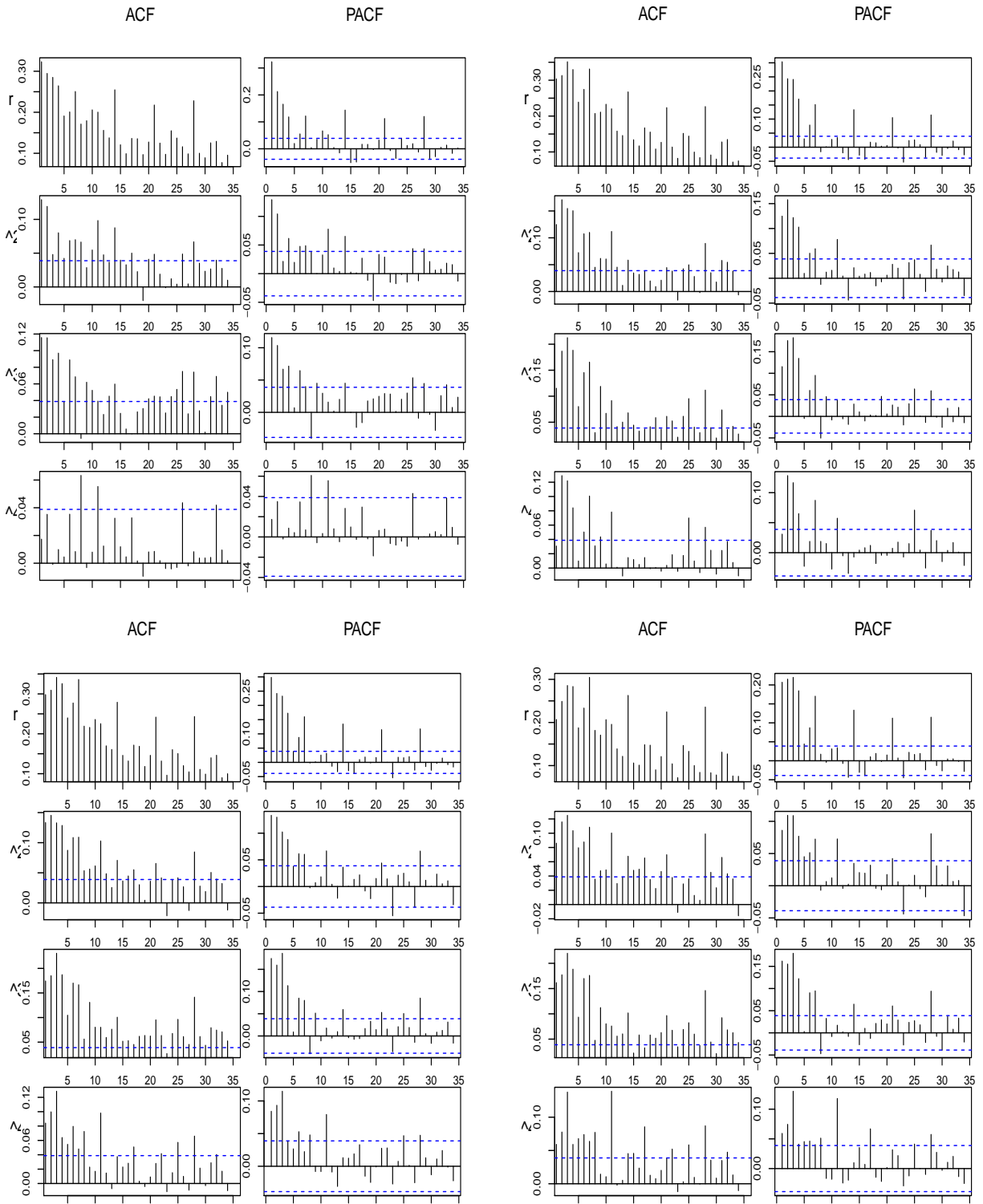


Figure 13: ACF and PACF for Residuals of Models M2 (top-left), M3 (top-right), ARM3 (bottom-left) and VARM3 (bottom-right) for hour 3. Levels of residuals on first rows,  $r_t^2$  on second rows,  $r_t^3$  on third rows and finally  $r_t^4$  on last rows.

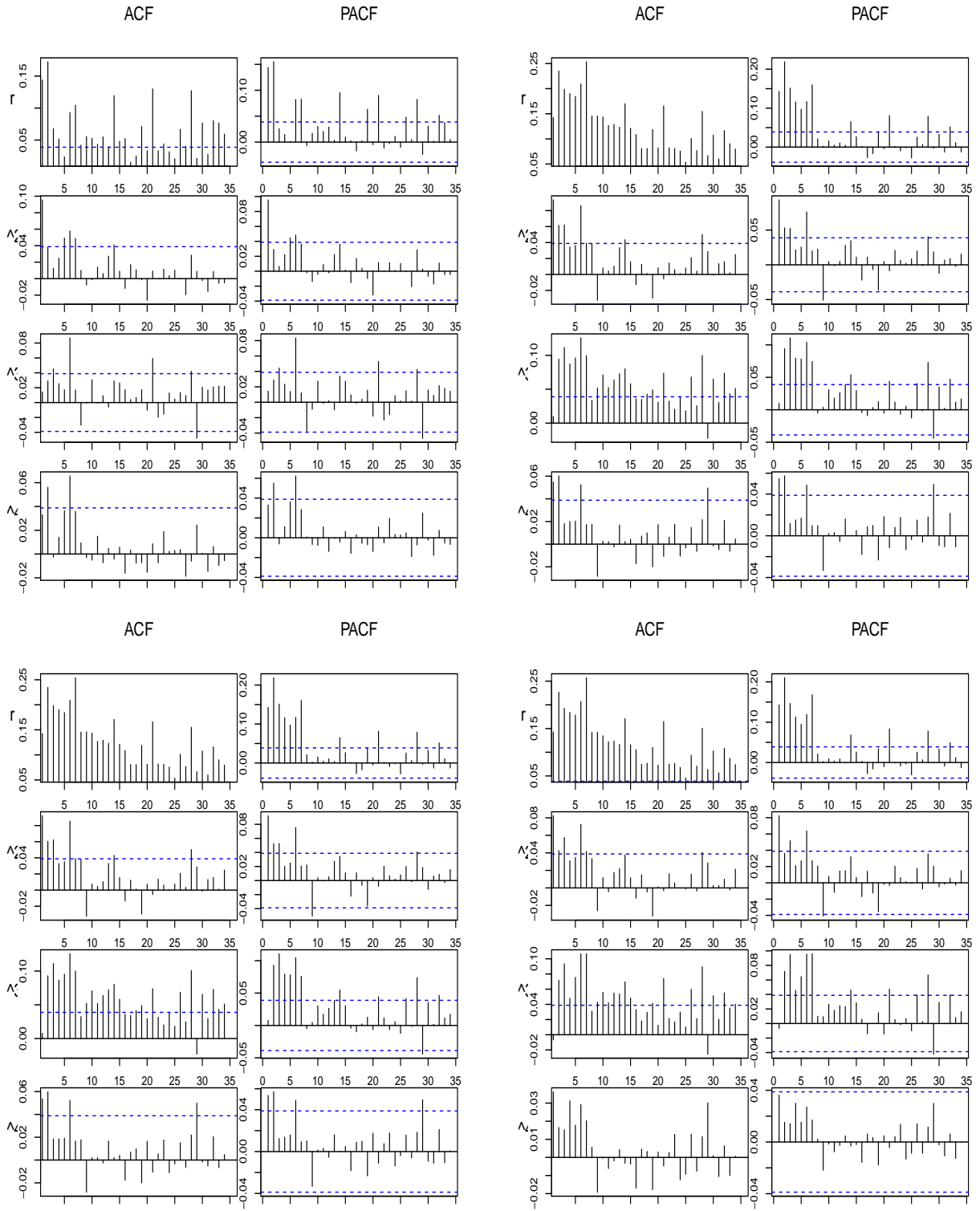


Figure 14: ACF and PACF for Residuals of Models M2 (top-left), M3 (top-right), ARM3 (bottom-left) and VARM3 (bottom-right) for hour 12. Levels of residuals on first rows,  $r_t^2$  on second rows,  $r_t^3$  on third rows and finally  $r_t^4$  on last rows.

		Base				AR				VAR			
		Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt	Mean	Var	Skew	Kurt
Hour 3													
JSU	M1	-0.004	1.001	-0.061	3.055	-0.002	0.999	-0.075	3.085				
	M2	-0.029	1.007	0.027	2.955	-0.029	1.007	0.025	2.965	-0.028	1.006	0.024	2.963
	M3	-0.029	1.011	0.026	2.941	-0.030	1.010	0.024	2.973	-0.029	1.007	0.025	2.981
	M4	-0.036	0.994	-0.017	3.210	-0.033	0.997	0.007	3.084	-0.029	0.989	-0.034	3.139
JSU <sub>o</sub>	M1	-0.002	0.998	-0.068	2.994	-0.002	0.998	-0.084	3.020				
	M2	-0.002	0.996	0.033	3.272	-0.002	0.996	0.032	3.325	-0.002	0.995	0.005	3.334
	M3	-0.001	0.997	0.036	2.908	-0.001	0.998	0.050	2.868	-0.001	0.998	0.005	2.889
	M4	-0.002	0.993	0.139	3.505	-0.001	0.996	-0.013	3.411	-0.001	0.994	0.053	3.272
ST1	M1	-0.006	1.001	-0.055	2.835	-0.008	1.000	-0.068	2.830				
	M2	-0.008	0.988	0.113	3.232	-0.009	0.987	0.117	3.255	-0.010	0.986	0.108	3.241
	M3	-0.009	0.974	-0.019	3.226	-0.012	0.971	-0.032	3.279	-0.014	0.958	-0.019	3.378
	M4	-0.009	0.978	-0.033	3.143	-0.016	0.972	-0.026	3.224	-0.027	0.974	-0.055	3.058
ST2	M1	-0.004	0.995	-0.058	2.876	-0.005	0.993	-0.068	2.886				
	M2	-0.009	0.992	0.091	3.189	-0.009	0.990	0.094	3.210	-0.011	0.988	0.088	3.205
	M3	-0.011	0.977	-0.027	3.165	-0.014	0.973	-0.041	3.222	-0.016	0.960	-0.028	3.337
	M4	-0.013	0.976	-0.045	3.125	-0.020	0.967	-0.019	3.223	-0.028	0.964	-0.031	3.092
SN1	M1	0.002	1.068	-10.506	> 240	0.001	1.067	-10.271	> 230				
	M2	-0.001	0.991	-0.068	4.254	-0.003	0.990	-0.052	4.222	-0.004	0.987	-0.054	4.113
	M3	0.007	0.982	-0.258	4.141	0.004	0.977	-0.251	4.035	0.000	0.962	-0.237	4.087
NO	M1	0.000	1.000	-10.527	> 240	0.000	1.000	-10.296	> 230				
	M2	-0.044	0.998	-0.855	5.024	-0.044	0.998	-0.810	4.861	-0.041	0.999	-0.815	4.874
Hour 12													
JSU	M1	0.000	0.999	0.054	3.189	0.001	0.999	0.065	3.199				
	M2	0.001	0.999	0.040	3.144	-0.003	0.999	0.043	3.136	0.004	0.999	0.049	3.131
	M3	-0.001	1.002	0.017	3.104	-0.004	1.002	0.016	3.085	0.000	1.004	0.027	3.094
	M4	0.006	0.973	0.014	3.287	-0.005	0.973	0.018	3.188	-0.003	1.007	0.003	3.050
JSU <sub>o</sub>	M1	0.000	1.000	0.054	3.093	0.000	1.000	0.067	3.100				
	M2	0.000	1.000	0.037	3.032	0.000	1.000	0.067	3.031	0.000	1.000	0.022	3.028
	M3	0.000	0.997	0.005	3.103	0.001	0.997	-0.002	3.148	0.001	0.996	-0.015	3.205
	M4	0.000	0.997	0.095	3.309	0.000	0.997	0.057	3.356	0.001	0.995	-0.053	3.553
ST1	M1	0.000	1.001	0.008	3.079	0.000	1.001	0.021	3.080				
	M2	0.000	1.002	0.010	3.059	-0.001	1.002	0.028	3.051	0.000	1.002	0.009	3.052
	M3	0.012	0.984	0.127	3.220	0.016	0.980	0.377	3.536	0.024	0.980	0.350	3.527
	M4	0.012	0.973	0.483	3.889	0.004	0.968	0.503	3.860	0.011	0.970	0.395	3.512
ST2	M1	0.001	1.001	0.039	3.085	0.001	1.001	0.056	3.090				
	M2	0.001	1.002	0.035	3.064	0.001	1.002	0.053	3.059	0.001	1.002	0.038	3.063
	M3	0.010	0.985	0.146	3.202	0.016	0.980	0.357	3.492	0.023	0.981	0.337	3.473
	M4	0.000	0.974	0.355	3.619	0.007	0.973	0.482	3.889	0.012	0.970	0.389	3.441
SN1	M1	-0.002	0.968	0.309	5.880			0.250	5.334				
	M2			0.134	4.705			0.100	4.104	0.000	1.003	0.092	5.079
	M3			0.114	3.784	-0.008	0.980	0.328	4.824	0.002	0.991	0.077	4.275
NO	M1	inf				inf							
	M2	inf				inf				inf			
Hour 19													
JSU	M1	0.000	0.997	0.096	3.203	0.000	0.998	0.110	3.227				
	M2	0.012	1.002	0.027	3.063	0.011	1.002	0.030	3.074	0.010	1.002	0.021	3.031
	M3	0.007	1.006	0.016	3.110	0.005	1.005	0.003	3.167	0.004	1.004	0.000	3.134
	M4	0.013	1.004	0.040	2.909	-0.001	0.991	0.027	2.871	0.005	0.996	0.040	2.907
JSU <sub>o</sub>	M1	0.001	0.999	0.115	3.136	0.001	0.999	0.130	3.155				
	M2	0.003	0.988	0.097	4.500	0.003	0.989	0.081	4.151	0.002	0.997	-0.087	3.489
	M3	0.002	0.992	0.407	3.831	0.002	0.990	0.380	3.780	0.002	0.993	0.260	3.499
	M4	-0.001	0.992	0.252	3.621	-0.001	0.991	0.256	3.554	-0.001	0.989	0.246	3.714
ST1	M1	0.001	1.003	0.063	2.971	0.001	1.003	0.074	2.976				
	M2	0.005	0.995	-0.086	3.209	0.008	0.991	-0.112	3.282	0.003	0.998	-0.071	3.142
	M3	-0.001	0.969	-0.077	3.325	0.003	0.955	-0.136	3.377	-0.001	0.965	-0.100	3.280
	M4	0.030	0.947	-0.007	3.447	0.014	0.920	0.307	3.647	0.017	0.960	-0.078	3.139
ST2	M1	0.005	0.994	0.086	2.983	0.005	0.993	0.100	2.983				
	M2	0.007	0.995	-0.062	3.152	0.010	0.992	-0.085	3.209	0.005	0.998	-0.050	3.094
	M3	0.033	0.957	0.504	3.624	0.026	0.959	0.484	3.629	0.023	0.959	0.497	3.660
	M4	0.016	0.924	0.352	3.850	0.012	0.922	0.295	3.630	0.013	0.921	0.269	3.728
SN1	M1			1.593	10.244	-0.004	0.940	1.615	10.361				
	M2	0.006	0.987	0.042	4.423	0.010	0.982	-0.004	4.128	0.001	0.996	0.030	4.025
	M3	-0.013	0.973	0.329	4.274	-0.011	0.977	0.351	3.976	-0.006	0.989	0.091	4.143
NO	M1	inf				inf							
	M2	0.023	1.000	0.643	4.870	0.028	1.000	0.533	4.491	0.013	1.000	0.553	4.306

Table 17: Descriptive Statistics of (Randomized Quantile) Residuals for Enlarged Models and selected hours.



### 6.5. Coverage Tests

The formulated models in Tables 18-26, without parameter indications and under the skew Student-t distribution unless diversely specified, are

- with just the first moment:
  - M1:  $\mu_t = c + \sum_{i=1}^7 y_{t-i} + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ ;
  - ARM1:  $\mu_t = c + \mu_{t-1} + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$  with  $\mu_{t-1}$  filtered from M1;
  - QReg:  $Q_q(y_t) = \alpha^q + \sum_{i=1}^7 \gamma_i^q y_{t-i} + \beta_1^q hol_t + \beta_2^q load_{t-1} + \beta_3^q fwind_t + \beta_4^q fsolar_t + \beta_5^q coal_{t-1} + \beta_6^q gas_{t-1} + \beta_7^q co_{2t-1}$ ;
- with the first two moments:
  - M2st:  $\mu_t$  as in M1 and  $\log(\sigma_t) = hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ ;
  - M2jsu: as M2 but with the Johnson's  $S_U$  distribution;
  - M2Ser:  $\mu_t = c + y_{t-1} + y_{t-2} + y_{t-7} + miny_{t-1} + load_t + load_t^2 + load_t^3 + \sum_{i=1}^6 d_i$  and  $\log(\sigma_t) = c + y_{t-1} + abs(y_{t-1} - y_{t-2}) + \sum_{i=1}^6 d_i$  under the Johnson's  $S_U$ ;
  - ARM2:  $\mu_t$  as in ARM1, and  $\log(\sigma_t) = c + \log(\sigma_{t-1}) + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$  with  $\mu_{t-1}$  and  $\log(\sigma_{t-1})$  filtered from M2;
  - VARM2:  $\mu_t = \log(\sigma_t) = c + \mu_{t-1} + \log(\sigma_{t-1}) + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ , with  $\mu_{t-1}$  and  $\log(\sigma_{t-1})$  filtered from M2;
  - AR-EGARCH-st and -sst: as in eqs. (9) and (10) under the Student-t and the skew Student-t distributions;
- with the three moments:
  - M3:  $\mu_t = c + y_{t-1} + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ , and  $\log(\sigma_t) = \nu_t = c + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ ;
  - ARM3:  $\mu_t$  and  $\log(\sigma_t)$  as in ARM1 and ARM2,  $\nu_t = c + \nu_{t-1} + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$  with  $\mu_{t-1}$ ,  $\log(\sigma_{t-1})$  and  $\nu_{t-1}$  filtered from M3;
  - VARM3:  $\mu_t = \log(\sigma_t) = \nu_t = c + \mu_{t-1} + \log(\sigma_{t-1}) + \nu_{t-1} + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ , with  $\mu_{t-1}$ ,  $\log(\sigma_{t-1})$  and  $\nu_{t-1}$  filtered from M3;
- with four moments:
  - M4:  $\mu_t$ ,  $\log(\sigma_t)$ , and  $\nu_t$  as in M3, and with  $\log(\tau_t) = c + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ ;
  - ARM4:  $\mu_t$ ,  $\log(\sigma_t)$  and  $\nu_t$  as in ARM3,  $\log(\tau_t) = c + \tau_{t-1} + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$  with  $\mu_{t-1}$ ,  $\log(\sigma_{t-1})$ ,  $\nu_{t-1}$  and  $\log(\tau_{t-1})$  filtered from M4;
  - VARM4:  $\mu_t = \log(\sigma_t) = \nu_t = \log(\tau_t) = c + \mu_{t-1} + \log(\sigma_{t-1}) + \nu_{t-1} + \log(\tau_{t-1}) + hol_t + load_{t-1} + fwind_t + fsolar_t + coal_{t-1} + gas_{t-1} + co_{2t-1}$ , with  $\mu_{t-1}$ ,  $\log(\sigma_{t-1})$ ,  $\nu_{t-1}$  and  $\log(\tau_{t-1})$  filtered from M4.























Year	Model	Q1	Q2	Q5	Q25	Q50	Q75	Q95	Q98	Q99
2011	M1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	QRreg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.36	0.65	1.00	0.00	0.10	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.63	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM2	0.11	0.27	0.99	0.00	0.08	0.00	0.00	0.00	0.00
	VARM2	0.11	0.27	0.99	0.00	0.08	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M3	0.11	0.27	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	ARM3	0.11	0.27	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	VARM3	0.11	0.27	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	ARM4	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
	VARM4	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
2012	M1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	QRreg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.86	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.68	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM2	0.26	0.00	0.00	0.00	0.71	0.00	0.00	0.00	0.00
	VARM2	0.12	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.83	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M3	0.11	0.27	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	ARM3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	VARM3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM4	0.00	0.00	1.00	0.00	0.25	0.00	0.00	0.00	0.00
	VARM4	0.00	0.00	1.00	0.00	0.04	0.00	0.00	0.00	0.00
2013	M1	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00
	ARM1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	QRreg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.72	0.92	1.00	0.00	0.68	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.37	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00	0.00
	M2set	0.50	0.74	0.90	0.00	0.57	0.00	0.00	0.00	0.00
	VARM2	0.12	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M3	0.12	0.25	0.99	0.00	0.43	0.00	0.00	0.00	0.00
	ARM3	0.00	0.00	0.00	0.00	0.19	0.00	0.00	0.00	0.00
	VARM3	0.12	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	M4	0.00	0.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
	ARM4	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00
	VARM4	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00
2014	M1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	QRreg	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
	M2set	0.34	0.63	0.91	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.23	0.01	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	VARM2	0.34	0.63	0.91	0.00	0.08	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M3	0.12	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	ARM3	0.34	0.63	1.00	0.00	0.07	0.00	0.00	0.00	0.00
	VARM3	0.12	0.25	0.99	0.00	0.43	0.00	0.00	0.00	0.00
	M4	0.00	0.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
	ARM4	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00
	VARM4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2015	M1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	QRreg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.10	0.25	0.95	0.00	0.23	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	VARM2	0.34	0.63	0.91	0.00	0.88	0.14	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.56	0.25	0.01	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M3	0.12	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	ARM3	0.34	0.63	1.00	0.00	0.08	0.00	0.00	0.00	0.00
	VARM3	0.12	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	M4	0.00	0.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
	ARM4	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.00
	VARM4	0.00	0.00	0.00	0.00	0.37	0.00	0.00	0.00	0.00
2016	M1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	ARM1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	QRreg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.10	0.25	0.95	0.00	0.45	0.02	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M2set	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	VARM2	0.34	0.63	0.91	0.00	0.00	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	EGARCHset	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	M3	0.10	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	ARM3	0.10	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	VARM3	0.10	0.25	0.99	0.00	0.00	0.00	0.00	0.00	0.00
	M4	0.00	0.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
	ARM4	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00
	VARM4	0.00	0.00	0.00	0.00	0.16	0.03	0.00	0.00	0.00

Table 28: Coverage tests for HP16. P-values of Coverage Tests: UC, CC and DC are respectively Kupiec's unconditional coverage, Christoffesen's conditional coverage and Engle and Manganelli's Dynamic Quantile tests.

