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Price Formation of Pledgeable
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Abstract

I derive the equilibrium price function of securities that can be pledged as collateral, in an economy where rational investors receive heterogeneous information. Although the distribution of borrowers' and lenders' payoffs is truncated, the linear equilibrium price is in closed form. That allows to investigate the impact of pledgeability on informational efficiency. The margin premium depends on the price of liquidity and yields a “pledgeability bias”, which increases the conditional volatility of the price function. The impact of pledgeability on prices contributes to explain excessive comovement and seemingly violations of the law of one price.

Keywords: REE, Collateral, Analytic solution, Price informativeness.

JEL: G12, G14

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1. Introduction

The price of securities that can be pledged as collateral is pivotal to the orderly functioning of financial markets. On the one hand, a drop in the price of pledgeable securities lowers the availability of funding in the economy and can be self-reinforcing when lenders react through higher haircuts, margin calls, or fire sales (see Gorton and Metrick (2012), Diamond and Rajan (2011)). On the other hand, to sustain economic recovery after the recent turmoil, policy makers have engaged in an effort to sustain the price of different classes of pledgeable assets (e.g. asset purchase programs in the Euro area, Japan, the UK, and the US).

How does the fact that a security can be pledged as collateral affect information aggregation in equilibrium? What are the implications for market participants and regulators?

To answer these questions, I provide a model where investors receive heterogeneous signals on the final value of securities they can either pledge, accept as collateral, or trade spot. I show that a Rational Expectations Equilibrium (REE) exists in the spot market, and it has an analytic solution notwithstanding the truncation of investors' payoff. The linear price function is unique and sheds light on the linkage between pledgeability and price informativeness.

The presence of a margin premium is well known in models of risk-sharing with symmetric information. Garleanu and Pedersen (2011) show that the Consumption Capital Asset Pricing Model (CCAPM) needs to be augmented by a margin premium to capture the pledgeability value of a security. Chabakauri (2013) expands such result to a dynamic setting. Rytchkov (2013) adds the feature that margin constraints are time-varying.

Extant literature does not investigate how pledgeability affects information aggregation. I find that spot prices in equilibrium are more volatile when margins are low, and this relationship is stronger when liquidity is cheaper. What was hitherto understood as a premium is a bias in terms of price efficiency. That has relevant implications for investors and policy makers alike.

Among collateralized loans, repurchase agreements (repos) constitute a primary source of short-term funding. The repo market capitalization – considering mortgage-backed securities alone – was nearly twice as big as that of US treasury debt in 2008. Gorton and Metrick (2012) report an outstanding repo volume of \$10tn in the US, whereas the International Capital Market Association (2014) estimates the European market in almost €6tn.

Bankruptcy remoteness is a key explanation to the popularity of repos (see Gorton and Metrick (2010)). Securities pledged in repos are exempted from the bankruptcy estate, so that litigation costs and counterparty risk play a negligible role. That simplifies the modeling of investment decisions, since the identity of counterparties does not affect collateralized loan agreements and the latter can be cleared in centralized markets.

To study price informativeness, I model investors who are fully rational and receive heterogeneous signals on the final value of their endowment of pledgeable securities. They exploit all available information to maximize utility through investment decisions in both the repo and the spot market.

Information aggregation in this setup has to deal with the truncated distribution of payoff functions: a borrower receives the liquidation value of collateral when securities perform and are repurchased at the final date, whereas his lender keeps the collateral and its final value in case of borrower's default.

Merton (1974) shows that option pricing theory can be used to price risky loans. Ideally one would apply the same approach to include the value of collateralized loans in the equilibrium price of a security that may be pledged to lever initial endowments. That is not straightforward in a REE, for two reasons: *i*) Risk-Neutral Valuation (RNV) for the pricing of options relies on assumptions over the statistical distribution of the value of underlying assets. Since the price of the pledgeable security in a REE is determined endogenously, such assumptions do not hold generally. *ii*) RNV formulas comprise transcendental functions whose argument is the value of underlying assets at inception. Since the equilibrium price of pledgeable securities at the initial date is the main object of this study, a novel approach is needed to derive its expression in analytic form.

The first issue has been addressed and resolved in the literature on option pricing, as explained in the next section. I show that the second issue can be avoided in the case of pledgeable securities and thus an equilibrium price in closed-form exists. The value a security grants on top of its discounted expected cash flows when it is pledged as collateral can be represented as the product between an exogenous option value and the endogenous security price at inception. The option value in this representation does not depend on trading strategies. Investors price it in their optimal demand/supply of the security, but that has no feedback effect on the RNV of the option. That excludes transcendental functions and numerical approximation from the derivation of equilibrium prices. Such methodology allows deriving closed form solutions in a broad set of models dealing with collateralized loans.

The repo option value is influenced by risk management constraints of clearing houses. These are central clearing counterparties (CCPs) which – in the spirit of recommendations from the Financial Stability Board and in line with Dodd-Frank legislation and European Market Infrastructure Regulation – novate repos and impose margins (haircuts) to comply with risk constraints set by regulators.

The price function of pledgeable securities in the unique linear REE shows that, as investors internalize in spot transactions the option value that is implicit in repos:

- i*) the equilibrium spot price is less informative – in the sense that its conditional variance is higher for any fundamental value of the security;
- ii*) a “pledgeability bias” detaches the spot price of a pledgeable security from its fundamental value;
- iii*) clearing houses and regulators' policies affect pledgeability bias and price informativeness, and their impact on equilibrium prices is asymmetric depending on market conditions;
- iv*) securities with identical expected cash-flows may trade at different prices because of different pledgeability, and changes in repo terms may contribute to explain asset prices comovement.

Result (i) is due to the effect of leverage on noise trading. When uninformed traders can pledge securities, they affect equilibrium prices beyond the relative size of their endowment in the economy. That has relevant consequences for informational efficiency: any change in the pledgeability of a security affects the informativeness of its price in equilibrium.

Result (ii) has been addressed by an extensive literature, as I show in the next section. In models of risk-sharing, the common finding is that the price of a security accounts for its aptitude to provide liquidity when investors are cash-constrained. Since I want to address also the issue of price informativeness, I derive the same qualitative result in a REE where investors receive heterogeneous information. That shows how allowing a security to be used as collateral changes investors' incentives: since a buyer of pledgeable securities in the spot market is a seller of that collateral in the parallel market for loans, his incentive to trade may be blurred and the "buy cheap" rule does not need to apply. A similar result was discussed by Brunnermeier and Pedersen (2009), in the case of a spot price just below the expected liquidation value of the security. I show that the same may happen in equilibrium for any expected liquidation value – depending on lending conditions – simply because the spot price of pledgeable securities determines the amount of liquidity an investor can borrow to lever his endowment. On top of the standard strategic complementarity typical of a REE with heterogeneous signals, the linkage between spot price and repo borrowing makes price movements self-reinforcing and threatens the orderly functioning of financial markets.

Result (iii) is the consequence of regulators and clearing houses setting risk constraints and margin policies. By influencing the collateral value of a pledgeable security, risk constraints affect its appeal to investors and its spot price. However, traders can attain a higher leverage when liquidity is cheaper. The impact of any change in margin is thus stronger in a thriving economy, where there is little perceived risk and much availability of liquidity. Such result arises through the total cost of a repo transaction, which has two components. On the one hand, the margin clearing houses set on repo transactions – *i.e.* the difference between spot price of the security and its first-leg repo price – determines how much of a security spot price can be exploited in the repo market to attain leverage. On the other hand, the repo rate required by repo buyers – measured as the difference between the prices of second and first leg of the repo – is set by the interplay between demand and supply of loans. Margins have little effect when the cost of liquidity is relatively higher. That is likely to happen during a crisis, so the model implies that low margins are not an effective tool to sustain the value of collateral and support financial stability in a turmoil.

Result (iv) is the logical consequence of result (ii) in a setting with multiple pledgeable securities: rational investors price securities with the same expected final value differently, when repo margins allowed by regulators and imposed by clearing houses are different. Failures of the law of one price have been explained by Vayanos and Weill (2008) in the presence of search costs. Their solution is numerical, whereas I show this result analytically. Moreover, when market-wide variables such as perceived systemic risk or regulatory provisions change, margins on different securities react all

in the same direction. The new margins affect the pledgeability bias of all pledgeable securities simultaneously and that produces excess comovement of their prices.

The model considers two classes of traders. One class consists of investors who trade their endowment of securities and liquidity in the *spot* market. Spot traders are homogeneous ex ante and decide to play the role of either buyer or seller only after receiving heterogeneous private signals on the future liquidation value of the securities they are endowed with. The second class of traders are institutions who can also access the *repo* market – namely leveraged banks or institutional traders. Repo traders receive private signals of the same quality as those available to spot investors and face a similar investment allocation decision. The only difference is that they can choose to either lever (shorten) their endowment of securities through repos (reverse repos).

The informational framework of the model borrows from Easley and O’Hara (2004), with the extension of a repo market for collateral. If the repo margin is too high, the repo market shuts down and their qualitative results are confirmed. However, in order to price repo contracts in a REE, traders in the present paper exhibit Constant Relative Risk Aversion (CRRA) and the value of securities follows independent log-Normal distributions.

The fact that pledgeable securities are traded in both the repo and the spot market channels all results of the paper. There are of course limitations to the analysis in the model. It neglects roll-over risk and lacks an empirical test of the price function. The main testable implication of the model is that risk constraints bias the price of collateral. Such effect is stronger when liquidity is cheap, whereas there is little scope for inflating the value of collateral and avoid an illiquidity spiral in a turmoil.

2. Related literature

This paper is related to several works from the literature on market microstructure, the law of one price, and option pricing in general equilibrium.

The effect of margins on asset prices has been a key explanation to the liquidity spiral during the crisis of 2007-2009. Brunnermeier and Pedersen (2009) and Gorton and Metrick (2012) point out that the tightening of constraints on collateralized borrowing played an important role in contagion. Institutions relying on collateralized loans were hit by a tightening of lending conditions, and prices were hit by both fire sales and higher perceived risk. Adrian and Shin (2010) show that margin constraints matter for the pricing of risk, as variations in VaR are followed by a lagged balance sheet shrinkage. Such a finding is consistent with Gromb and Vayanos (2002), who elaborated the seminal result by Shleifer and Vishny (1997) on the limits of arbitrage in the context of financial intermediaries. The price function derived in the present paper supports the linkage between VaR and asset values. However, that does not happen through deleveraging strategies. The channel this paper points out is the impact pledgeability constraints have on the ability for financial institutions to borrow against collateral. Margins depend on the VaR of collateral, and restrictions in the

pledgeability of a security determine “fire-sale” prices even before investors deleverage through fire-sales.

Brunnermeier and Pedersen (2009) find that margins based on past volatility are destabilizing for securities whose fundamental value is unobservable. That creates margin spirals that exacerbate both an improvement and a worsening of credit conditions. Such result is supported by the findings of the present model, since higher margins lower the spot price of pledgeable assets and further reduce their ability to raise liquidity. The backward-looking nature of haircuts is the main linkage between margin rules and market stability also in Chowdhry and Nanda (1998), who consider the impact of margin calls on a 100% VaR constraint with no shock on fundamentals. Margin adjustments over roll-over dates are destabilizing and make investors unable to repay their loans. The consequent portfolio reallocation determines a downward liquidity spiral. Gromb and Vayanos (2002) investigate the equilibrium consequences of limits to arbitrage when investors face risk constraints. The authors develop a model of general equilibrium that, likewise in the present paper, is populated by agents who differ in both funding and investment opportunities. They find that borrowing constraints – once more, a security-specific 100% VaR=0 constraint – may limit arbitrageurs’ ability to move prices towards fundamentals. Adrian and Shin (2014) study leverage procyclicality in a model of moral hazard with asset substitution. The requirement of a constant VaR per dollar of asset value makes leverage procyclical, whereas the backward looking nature of its estimate induces a lag between changes in volatility and leverage. The finding that market liquidity declines as fundamental volatility increases is consistent with the present paper, in so far as clearing houses set repo haircuts on the basis of past volatility of collateral.

This work is also related to the literature on the failure of the law of one price under funding constraints. Garleanu and Pedersen (2011) in particular point out the failure of the law of one price during the 2007-2009 crisis. They show that rational investors react to the tightening of margin requirements in a way that increases the equilibrium return on collateral. The CCAPM must be augmented with the margin on the security – multiplied by the difference between collateralized and uncollateralized borrowing cost – in order to capture the consensus return on pledgeable assets. I confirm such result in a framework with heterogeneous information: the fact that haircuts affect the equilibrium spot price of pledgeable securities implies that assets with the same expected future cash flows trade at different prices. Securities paying the same expected cash flow are priced differently if their pledgeability is dissimilar.

To find a REE in an economy where investors trade repos is not a trivial exercise. This paper deals with nonlinear payoffs by treating repos as portfolios of options. The problem is that the price of pledged securities in REE depends on investors’ optimal strategies rather than on an exogenous diffusion process. In fact, the Black and Scholes (1973) option pricing model builds upon a partial equilibrium analysis where no-arbitrage arguments allow the pricing formula to be independent of preferences and information. The Black and Scholes formulas are not generally applicable to a REE model, where investors use the spot price as informative signal and at the same time affect it

through their optimal decisions. The equilibrium price of the underlying asset changes depending on how market forces aggregate investors' preferences and information into a unique value that is regret-free for all market participants.

Such issue has been resolved in a range of specific frameworks. DeMarzo and Skiadas (1998) use the notion of Equivalent Martingale Measure to show that, in a set of quasi-complete economies where agents have the same HARA, a REE exists wherein the regret-free price has either a Normal or log-Normal distribution – conditional on private signals – depending on assumptions over the risk aversion coefficients. Kreps (1982) shows that, under a limited set of trading strategies, an equilibrium exists that corresponds to the Black and Scholes (1973) one. Among these conditions, the fact that investors exhibit CRRA plays a role recognized by Brennan (1979), Bick (1987), and Kim (1992). Brennan (1979) shows that, within CARA/Normal and CRRA/log-Normal frameworks, risk neutral valuation is compatible with optimal strategies in general equilibrium. Bick (1987) studies an economy where the endogenously determined spot price of a risky asset follows a geometric Brownian motion, so that the price dynamic assumed by Black and Scholes (1973) is self-fulfilling. Kim (1992) confirms that CRRA agents' optimal strategies generate an equilibrium that is compatible with the Black and Scholes formula when the value of the underlying is lognormally distributed. Borrowing from the aforementioned results, the model outlined in the next section is compatible with Black and Scholes (1973) option pricing in REE. This allows to deal with non-linear payoffs, ensuring that an analytic solution for the pledgeable security price function can be derived.

3. The Model

Consider an economy that lasts two dates $t = 1, 2$, with no discounting. All decisions are taken at the initial date, when investors receive heterogeneous signals on the future value of securities and choose the investment strategies which maximize their final expected utilities.

Investors have a power utility function. The fact that random variables distribute lognormally simplifies the derivation of conditional distributions. Moreover, as mentioned above, these assumptions allow to apply the well known Black and Scholes formulas and to derive a closed-form expression for the price function in the linear REE.

Since I focus on the impact of margins on information aggregation, I follow Easley and O'Hara (2004) and assume that the final values of securities are independently distributed. That implies that the price of a pledgeable security in equilibrium does not depend on either information or net demand for other securities. In the first part of the model, I simplify notation and look at an economy with only one pledgeable security. That is sufficient to provide relevant results on pledgeability bias and on the impact of risk constraints. I introduce multiple pledgeable securities in Section 4.5, to address the issues of asset prices comovements and failures of the law of one price.

3.1. Players

The economy is populated by noise traders and a continuum of rational investors in the interval $[0, 1]$, with CRRA preferences on final consumption and a degree of relative risk aversion $\gamma > 1$. Their utility function is

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}. \quad (1)$$

Liquidity is the consumption good and serves as numeraire for the economy. Investors trade their endowment, which consists in liquidity and a non-dividend paying risky security. The security pays an amount v of the consumption good at the final date and can be pledged in repo transactions. Repos are agreements to sell an asset and repurchase it one period later at a price that is set at the initial date.

All investors know the specifications of the model. The security liquidation value is known to distribute as $\tilde{v} \sim \ln N(\ln \bar{v} - (2\tau_{\ln v})^{-1}, \tau_{\ln v}^{-1})$. Noise traders demand, independently of v , a random amount $\tilde{u} \sim \ln N(\ln \bar{u} - (2\tau_{\ln u})^{-1}, \tau_{\ln u}^{-1})$ of the risky asset in the spot market and they repo it. Noise traders act as providers of assets or liquidity, depending on the realization of \tilde{u} .

Before trading, investor $i \in [0, 1]$ receives a private signal that conveys the final risky payoff perturbed by noise. Signals are independent and amount to $s_i = e^{\ln v + \ln \frac{\epsilon_i}{\bar{\epsilon}}}$. The investor-specific error distributes as $\tilde{\epsilon}_i \sim \ln N(\ln \bar{\epsilon} - (2\tau_{\ln \epsilon})^{-1}, \tau_{\ln \epsilon}^{-1})$, ϵ_i is independent from v , the signal informativeness is $\tau_{\ln s} = \tau_{\ln v} + \tau_{\ln \epsilon}$, and the conditional precision of $\tilde{s}_i|v \sim \ln N(\ln v - (2\tau_{\ln \epsilon})^{-1}, \tau_{\ln \epsilon}^{-1})$ is exogenous and common to all informed investors.

Each investor has information set $\Omega_i = \{s_i, p\}$. Among them, spot investor $i \in [0, \mu]$ can only trade the pledgeable security in the spot market and is endowed with liquidity and securities at the initial date. To simplify notation and with no loss of generality, I assume such endowment w is the same among all traders and is equally split between liquidity m and a units of the pledgeable security. Differently from spot investors, repo traders $i \in (\mu, 1]$ can use repos to temporarily convert securities in liquidity and *vice versa*.

Spot traders can mix three different actions. They can Hold a security; Sell it on the spot market; and Purchase it. Repo traders have access to the repo market and can also Repo (or reverse Repo) their securities. In what follows, the initials of such actions label them as $\{H, S, P, R\}$, respectively.

The first step in solving investors' decision problem is to find what combinations of the four actions an investor undertakes to maximize his expected terminal utility, conditional on whether his information set induces to either increase or cut his exposure to the pledgeable security.

Label y_i^j identifies the share of endowment trader i allocates to action $j \in \{H, S, P, R\}$. A capital letter $Y_i^j = (a + \frac{m}{p})y_i^j$ stands for the corresponding amount of wealth, measured in securities, allocated to action j . The informed investor who wants to increase his exposure to the risky asset has the two following alternatives:

- 1) If he trades in the spot market, he purchases Y_i^P – or equivalently sells a negative amount Y_i^S – paying liquidity.
- 2) If he trades in the repo market, the decision over Y_i^R implies $Y_i^H = a - Y_i^R$ and the amount of liquidity an investor uses to purchase additional securities Y_i^P to repo. Such a chain of repo transactions takes place at date 1 and determines, through the repo technology specified in the next subsection, the number of pledgeable securities an informed investor holds until the following date.

The bearish informed investor sells $Y_i^S = a - Y_i^H$ to both noise and informed traders who want to increase their exposure to the pledgeable security. He can then store the proceeds pY_i^S for future consumption.

Investors are rational and lever their endowment efficiently. They Sell, Purchase, Hold or Repo quantities that maximize their expected final utilities, given all information they have available. Each repo seller takes into account the facts that (1) all liquidity he raises may be used to buy additional securities; (2) the latter can be pledged against further liquidity he can use to buy additional securities; (3) such a leverage spiral can go on until he purchases Y_i^{P*} securities – having optimally exploited the pledgeability of his initial endowment.

An investor has no incentive to both buy and sell the pledgeable security, since the outcome would be zero. Because the model is solved using risk-neutral valuation formulas, traders never repo and hold liquidity at the same time either. In fact, both assets are risk-free and one of them always dominates the other. In summary, the set of strategies an informed investor chooses among is

$$\{\{Y_i^H, Y_i^R, Y_i^P\}, \{Y_i^H, Y_i^S\}\}. \quad (2)$$

3.2. Investment technologies

Investors' initial decision translates into final payoffs through the following investment technologies:

- The risk-free technology is one wherein investors can store their holding of the consumption good over one period. The risk-free return is normalized to zero;
- The market for the risky pledgeable security is a standard spot market where each informed investor $i \in [0, 1]$ submits an order to purchase Y_i^P (sell Y_i^S) units of the pledgeable security at price p ;
- Repos are collateralized loans between repo traders $i \in (\mu, 1]$. Relatively bearish investors can sell their securities spot and use the proceeds to reverse-repo the securities of bullish traders. The latter act as borrowers: they sell through repos the pledgeable security to receive liquidity at the initial date. At the same time they commit to repurchase it, one

period later, paying the repo interest rate r .² If a borrower does not execute the second leg of the agreement, his repo lender has the opportunity to sell the collateral and cash its current spot price. Repos are settled through a competitive clearing house that sets the repo margin $h \in (0, 1)$ under a risk constraint. The “haircut” h protects lenders against market risk. It determines the gap between the spot price p at inception and the liquidity – *i.e.* the asset pledgeable value – investor i borrows.

Lemma 1. *An investor’s activity in the repo market determines his trading in the spot market:*

$$Y_i^P = Y_i^R \frac{1-h}{h}. \quad (3)$$

Proof. See Appendix A □

3.3. Investors’ payoff

After observing his private signal, spot investor $i \in [0, \mu]$ chooses the optimal quantity to sell (buy) in the spot market. His final payoff is

$$C_i^S = Y_i^S p + Y_i^H v \quad (4)$$

$$= w[p + (0.5 - y_i^S)(v - p)]. \quad (5)$$

Equality (4) is the sum of the liquidity an investor receives – or pays – through spot trades, and the liquidation value of the endowment he holds to maturity. This is subject to resource constraints on both liquidity and securities:

$$m + pY_i^S \geq 0 \quad (6)$$

$$a - Y_i^S \geq 0 \quad (7)$$

The chain of repo transactions that allows an investor to lever his endowment takes place all at once, at the initial date. It allows repo traders $i \in (\mu, 1]$ to afford additional pledgeable securities Y_i^P that are pledged together with his initially repoed endowment Y_i^R .

If a repo trader wants to increase his holding of the risky security, the final payoff depends on whether he will repurchase his collateral. In the Good scenario, where he repurchase the pledged

²Although a repo is the sale of securities with the commitment to buy them back at a higher price, its economic interpretation is that of a collateralized loan. Once the loan rate r is compounded on the principal over the maturity of the contract, it determines the difference between repurchase price at time 2 and sell price at date 1. Since repos are only one among other loans and both borrowers and lenders are atomistic, I assume for the sake of tractability that r is set exogenously.

assets, he receives:

$$C_i^G = (0.5w - Y_i^R)v + (Y_i^R + Y_i^P)p(1 - h) + (0.5w - Y_i^P)p + (Y_i^R + Y_i^P)p(1 - h)(1 + r) + (Y_i^R + Y_i^P)v; \quad (8)$$

whereas in the Bad scenario, where a borrower defaults on the loan, his payoff is:

$$C_i^B = (0.5w - Y_i^R)v + (Y_i^R + Y_i^P)p(1 - h) + (0.5w - Y_i^P)p. \quad (9)$$

In equalities (8) and (9), a repo investor receives the value $(a - Y_i^R)v$ of the assets he holds until maturity. He repos Y_i^R , together with the amount of assets Y_i^P he decides to purchase and pledge. That allows the repo buyer to receive the pledgeable value of his collateral, $(Y_i^R + Y_i^P)p(1 - h)$, he use to pay the Y_i^P new securities.

The difference between the two payoffs is that in equality (8) investor i repays his loan, in order to get the liquidation value $(Y_i^R + Y_i^P)v$ of his pledged securities. In the *Bad* scenario (9) the investor defaults on his repos: he does not repay the loan but has to forgo the liquidation value v on all securities $(Y_i^R + Y_i^P)$ he pledged.

Lemma 2. *The second scenario is defined Bad because the payoff an investor receive is always lower than it is in the Good scenario: $C_i^B < C_i^G$.*

Proof. See Appendix A □

Margin h plays a primary role in investors' decision. Its value depends on the VaR constraint. In the bad scenario, repo lenders may lose part of their endowment because $h < (v - p)/p$ - i.e. the repo margin is not sufficient to hedge the lender's position against the depreciation of collateral.

Geanakoplos (1996) points out that "Collateral has the advantage that the lender needs not bother with the reliability or even the identity of the borrower, but can concentrate entirely on the future value of the collateral. Collateral thus retain anonymity in market transactions". That allows central clearing of repo transactions, with consequent advantages in terms of market liquidity.³ To keep the model as simple as possible, I assume the VaR constraint requires repayment of the whole loan principal - i.e. $k = 0$.⁴ This assumption comes with the advantage that h can be set by the clearing house to a level that does not depend on the borrower's risk profile - a feature that fits

³The Dodd-Frank legislation and reform proposals by the Bank for International Settlements, as well as the French "Collateral Basket With Pledge", make central clearing the standard settlement method for repos.

⁴The VaR constraint can be applied to the principal repayment, with $k = 0$, supporting the result of a constant h ; to the full repayment, with $k = 0$, yielding a pledgeability function $h(r)$ that depends on the interest earned on market-wide counterparty risk; to the loan principal, with $k \neq 0$, yielding a $h_i(x_i^R)$ that depends on a lender's risk exposure out of his endowment; or to the full repayment, with $k \neq 0$, yielding $h_i(x_i^R, r)$.

to transactions taking place in a centralized exchange rather than to Over The Counter (OTC) trades – or on the nominal cost r of borrowed liquidity.⁵

The possibility that a borrower defaults on his loan introduces nonlinearity in the relationship between the value of a security and repos payoff. Since a borrower defaults on his loan when the security liquidation value is low, his payoff at the final date is random only when the final value of the pledged security is high enough. When the security underperforms, its market value goes to the lender and the payoff of the latter is random. Borrower i 's payoff in equalities (8 – 9) can then be stated as

$$C_i^R = (0.5w - Y_i^R)v + (0.5w - Y_i^P)p + (Y_i^R + Y_i^P) \left[p(1 - h) + \max\{v - p(1 - h)(1 + r); 0\} \right]. \quad (10)$$

The repo payoff of both counterparties distribute as a *truncated* log-Normal, where the truncation is the security liquidation value that makes the repo seller switch between default and execution of the second leg.

The truncation of distributions other than the simple uniform one implies transcendental functions in the moments characterizing them, this function being the Gaussian error function in the special case of the family of Normal distributions. Rather than settling for numerical optimization, I find an analytic solution for the pricing function by using option-pricing formulas, building on the following Lemma.

Lemma 3. *The position of a repo seller is akin to a portfolio of call options and the underlying pledgeable security:*

$$C_i^R = w \left[\frac{Call}{h} + (0.5 - y_i^R) \left(v - \frac{Call}{h} \right) \right], \quad (11)$$

where $Call \equiv \max\{v - p(1 - h)(1 + r); 0\}$ is the payoff of the call option a repo seller is implicitly long in.

Proof. See Appendix A. □

Treating repo transactions as portfolios of a call option and the underlying pledgeable security allows to handle non-linear payoffs relatively easily, as it was noticed in Section 2.

⁵? points out that “Any excess collateral/cash remaining after the sale or purchase of collateral securities has made whole the position of a non-defaulting party cannot be retained by that party and so does not represent contingent additional compensation for bearing [counterparty] default risk”. The size of haircuts should only be a function of market liquidity risk, operational risk, legal risk, and default risk on the collateral securities. It “should not vary with counterparty credit risk”.

Lemma 4. *A repo investor pledges part of his endowment if and only if $h < \frac{Call}{p}$. Otherwise, the repo market shuts down and the pledgeability of a security does not affect its spot price.⁶*

Proof. See Appendix A. □

4. Price Formation in the spot Market

This section derives the equilibrium spot price in rational expectations. After providing the definition of equilibrium, investors' optimal allocation to the pledgeable security is derived. The equilibrium spot price is found as a self-fulfilling regret-free function that depends on informative signals and pledgeability of the security.

4.1. Definition of equilibrium

The definition of REE in the economy is standard. It is a measurable price function

$$\ln \tilde{p} = \ln p \left(\int_{\mu}^1 y_i^R di, \int_{\mu}^1 y_i^P di, \int_0^{\mu} y_i^S di, \ln u \right)$$

that maps the state of the world $\{v, u\}$ into prices; together with a set of strategies $\{y_i^{R*}, y_i^{P*}, y_i^{S*}\}$ such that:

- the spot market clears

$$\ln u + \int_{\mu}^1 Y_i^P(Y_i^R(s_i, p)) di = \int_0^{\mu} Y_i^S(s_i, p) di, \quad (12)$$

- the repo market clears

$$\ln u + \int_{\mu}^1 Y_i^R(s_i, p, Y_i^P(s_i, p)) di = 0, \quad (13)$$

- each investor maximizes his expected utility, given his information set

$$\{y_i^{R*}, y_i^{P*}, m_i^*, y_i^{S*}\} = \operatorname{argmax} \mathbb{E} [U(C_i) | \Omega_i].$$

4.2. Investors' optimization problem

4.2.1. Spot traders

The expected utility of a spot trader is not affected directly by the pledgeability of the security.

⁶This confirms a result Gromb and Vayanos (2002) and Garleanu and Pedersen (2011) find in different frameworks: funding conditions induce price distortion only when the leverage constraint binds.

Lemma 5. *The optimal share of initial endowment sold by spot traders is*

$$y_i^{S*} = \frac{1}{2} + \frac{\ln p - \mathbb{E}[\ln v|\Omega_i]}{\gamma\tau_{\ln v}^{-1}} - \frac{1}{2\gamma}. \quad (14)$$

Proof. See Appendix A □

A spot trader is either a buyer or a seller, depending on his expectation on the security liquidation value. He acts as a seller if, given the observed spot price and his private signal:

$$\mathbb{E}[\ln v|\Omega_i] < \ln p + (2\tau_{\ln v})^{-1}(\gamma - 1) \quad (15)$$

whereas he is a buyer otherwise.

4.2.2. Repo traders

The expected utility of repo trader $i \in [\mu, 1]$ is determined by plugging his expected final payoff (11) into the utility function (1):

$$\mathbb{E}[U(C_i^R)] = \mathbb{E}\left[\frac{\{w[\frac{Call}{h} + (0.5 - y_i^R)(v - \frac{Call}{h})]\}^{1-\gamma}}{1-\gamma} \middle| \Omega_i\right]. \quad (16)$$

Lemma 6. *The repo investor pledges an amount of securities*

$$y_i^{R*} = \frac{\ln \frac{Call}{h} - \mathbb{E}[\ln v|\Omega_i]}{\gamma\tau_{\ln v}^{-1}} + \frac{\gamma - 1}{2\gamma}. \quad (17)$$

The spot demand by a repo trader is

$$y_i^{P*} = \left(\frac{1}{2} + \frac{\ln \frac{Call}{h} - \mathbb{E}[\ln v|\Omega_i]}{\gamma\tau_{\ln v}^{-1}} - \frac{1}{2\gamma}\right) \frac{1-h}{h} \quad (18)$$

Proof. See Appendix A. □

Three consequences of the above Lemma are worth noticing.

(1) The demand function of an investor in the spot market is a positive multiple of his repo supply. Therefore, the spot *demand* of a repo trader is actually a *supply* function. The higher an investor's expectation on the value of the traded security is, the less he buys. This means that, under the feasibility condition set in Lemma 4, the leverage effect dominates the speculative effect.

(2) The pledgeability of a security – measured as the ratio between the value of the call option a repo investor implicitly holds and the haircut he faces – increases the investor's optimal risky investment.

(3) Although the optimal strategy of a repo trader seems independent from the security price, spot demand increases with the spot price. This is showed in the following Proposition.

Proposition 7. *The call option implicitly bought by repo sellers does not depend on the price of the underlying pledgeable securities and can be written as*

$$\chi \equiv \frac{Call}{p}, \quad (19)$$

χ is the value of a call option on the pledgeable security when the latter has an initial value $S = 1$ and strike price $(1 - h)(1 + r)$.

Proof. See Appendix A □

Therefore, the demand function of a repo seller writes

$$y_i^{P*} = \left(\frac{1}{2} + \frac{\ln \frac{\chi}{h} + \ln p - \mathbb{E}[\ln v | \Omega_i]}{\gamma \tau_{\ln v}^{-1}} - \frac{1}{2\gamma} \right) \frac{1 - h}{h} \quad (20)$$

The ratio $\frac{\chi}{h}$ affects the quantities repo investors trade in the spot market.

The effect pledgeability has on traders optimal investment depends on the value of χ . According to Black and Scholes (1973), the latter increases with the asset volatility σ^2 and decreases with the repo rate r . Both effect decrease with the haircut h repo borrowers face.

4.3. Rational expectations equilibrium in the spot market

Having identified traders' optimal investment decision, the spot price function is derived by imposing market clearing conditions. The optimal demand by repo investors shows that the security pledgeability value $\ln \frac{\chi}{h}$ has an impact on spot trading. To find the REE, assume all traders believe pledgeability has an impact on the spot price and conjecture the following price function

$$\begin{aligned} \ln \tilde{p} &= \beta_0 + \beta_1 \int_0^1 \ln \tilde{s}_i di + \beta_2 \ln \tilde{u} + \beta_3 \ln \frac{\chi}{h}, \\ &= \beta_0 + \beta_1 \ln \tilde{v} + \beta_2 \ln \tilde{u} + \beta_3 \ln \frac{\chi}{h}, \end{aligned} \quad (21)$$

where the values of parameters β_n are derived in equilibrium, and 21 follows from the law of large numbers and the independence of ϵ_i .

Most models of price formation under rational expectations assume the price function is linear.⁷ Price function (21) is linear in the logarithm of factors affecting the security price in equilibrium. CRRA investors aggregate the information contained in lognormal variables and, if the conjectured

⁷See for instance Easley and O'Hara (2004), Hellwig (1980) and Verrecchia (1982).

pricing function is fulfilled in equilibrium, the logarithm of the security price is a linear combination of normally distributed random variables.

To make the computation of conditional distributions easier, it is useful to define an “observable information” variable, in logarithms:

$$\ln \theta = \frac{\ln p - \beta_0 - \beta_2 k - \beta_3 \ln \frac{\chi}{h}}{\beta_1} \quad (22)$$

where $k \equiv \ln \bar{u} + \left(\frac{\beta_2}{\beta_1} - 1\right) (2\tau_{\ln u})^{-1}$. The observable $\ln \theta$ delivers imperfect information on v , as it contains the effect of noise trading. Plugging the conjectured price function (21) into equality (22) yields

$$\ln \theta = \ln v + \frac{\beta_2}{\beta_1} [\ln u - k]. \quad (23)$$

Therefore, the observable information variable is normally distributed, with conditional mean and variance

$$\mathbb{E}[\ln \tilde{\theta} | \ln v] = \ln v - \frac{\beta_2^2}{\beta_1^2} (2\tau_{\ln u})^{-1} \quad (24)$$

$$\text{VAR} [\ln \tilde{\theta} | \ln v] = \frac{\beta_2^2}{\beta_1^2} \tau_{\ln u}^{-1} \equiv \tau_{\ln \theta}^{-1}. \quad (25)$$

Informed investors use both their private signal $\ln s_i$ and public information $\ln \theta$ to update their prior beliefs and optimal spot trades.

Proposition 8. *The assumption of a linear pricing is fulfilled in equilibrium. The price function is the one conjectured in equation (21). It is uniquely identified by the following set of coefficients:*

$$\begin{aligned} \beta_0 &= \frac{\tau_{\ln v} (\ln \bar{v} - (2\tau_{\ln v})^{-1}) - \tau_{\ln s} (2\tau_{\ln \epsilon})^{-1} - \tau_{\ln \theta} (2\tau_{\ln \delta})^{-1}}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}} + \\ &\quad - (\gamma - 1) (2\tau_{\ln v})^{-1} \\ \beta_1 &= \frac{\tau_{\ln s} + \tau_{\ln \theta}}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}} \\ \beta_2 &= \frac{h\gamma\tau_{\ln v}^{-1}}{w[\mu - (1 - h)]} \\ \beta_3 &= \frac{\mu h}{\mu - (1 - h)} - 1. \end{aligned}$$

Proof. See Appendix A □

Proposition 8 shows the impact of pledgeability on the spot price of a security. The coefficient β_3 measures such effect. A security spot price is more affected by its pledgeability when the repo

haircut is lower.

Coefficient β_0 confirms the standard results that a security spot price: (1) increases with the prior expectation on its terminal value; (2) decreases with its volatility; and (3) the latter effect is stronger when investors' risk aversion is higher.

Inspection of β_1 shows that the security price increases when the realization of its liquidation value is higher, and such effect is weaker when private and public signals $\ln s$ and $\ln \theta$ are less precise.

The spot price increases when noise traders' demand increases. According to the formula of β_2 such effect is lower when informed investors' wealth w is higher, so that noise trades are more diluted among informed ones. It is stronger when the volatility of the security final value is higher, since it is more likely that a surge in demand for the security is due to a higher liquidation value.

The price function of a pledgeable security nests that of the same security without pledgeability. It reduces to the latter when the haircut is 100% and there are no repo traders. When the repo market is shut, the price function is the same conjectured for the pledgeable security, with:

$$\begin{aligned}\beta'_0 &= \frac{\tau_{\ln v} (\ln \bar{v} - (2\tau_{\ln v})^{-1}) - \tau_{\ln s}(2\tau_{\ln \epsilon})^{-1} - \tau_{\ln \theta}(2\tau_{\ln \delta})^{-1}}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}} + \\ &\quad + (2\tau_{\ln v})^{-1}(\gamma - \frac{3}{2}) \\ \beta'_1 &= \frac{\tau_{\ln s} + \tau_{\ln \theta}}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}} \\ \beta'_2 &= \frac{\gamma\tau_{\ln v}^{-1}}{w} \\ \beta'_3 &= 0.\end{aligned}$$

The conditional volatility of the price function is captured by the noise trading coefficient β_2 . Comparing the equilibrium value of that term with and without pledgeability allows to point out the information bias brought by repo trading:

Remark 1. *The impact of noise trading on the spot price is higher when a security is pledgeable. This can be seen by comparing β_2 and β'_2 . Under Lemma (2) and since $\mu < 1$:*

$$\frac{h\gamma\tau_{\ln v}^{-1}}{w[\mu - 1 + h]} > \frac{\gamma\tau_{\ln v}^{-1}}{w}.$$

The additional volatility increases as h decreases:

$$\frac{\partial(\beta_2 - \beta'_2)}{\partial h} = -\frac{\gamma(1 - \mu)\tau_{\ln v}^{-1}}{w[\mu - 1 + h]^2} < 0.$$

4.4. Impact of repo margins on the price bias

The option approach allows disentangling the contribution of margin constraint and repo rate to the pledgeability bias. Since the latter is given by the ratio $\frac{\chi}{h}$ between the implicit repo option value and haircut, both elements together contribute to the total effect of pledgeability on the spot price.

Concerning the denominator, it is clear that a lower haircut has a positive effect on price. However, as the following lemma demonstrates, the relationship between margins and the implicit repo option value at the numerator is not trivial.

Lemma 9. *A lower haircut lowers the value of the implicit option χ .*

The derivative of a generic European call option c with respect to a determinant h of its strike price is:

$$\frac{\partial \ln c}{\partial h} = \frac{1}{c} \left[\frac{\partial K(h)}{\partial h} e^{-r_f \Delta t} N(d_2) \right] \quad (26)$$

In the case of pledgeable securities considered in the model, the derivative of the implicit repo option χ with respect to repo margin is:

$$\frac{\partial \ln \chi}{\partial h} = \frac{(1+r)N(d_2)}{\chi} > 0 \quad (27)$$

Proof. See Appendix A □

It is possible to derive what effect prevails between the direct one (haircut) and the indirect one (implicit repo option). That allows to point out a result which, in its simplicity, has a great impact on policy makers: low margins on repo transactions inflate the value of the underlying pledgeable securities only when the repo rate is below a threshold value \bar{r} , which is strictly positive. Thus, repo trading inflates the spot price of pledgeable securities when liquidity is cheap.

Proposition 10. *Low repo margins inflate the spot price of pledgeable securities when the repo rate is sufficiently low:*

$$r < \bar{r} \quad (28)$$

The threshold \bar{r} is always positive, and it increases with the volatility of the pledgeable security.

Proof. See Appendix A □

The above propositions implies that the pledgeability bias becomes ineffective when liquidity is scarce. That is exactly the time when policy makers are more interested in sustaining the value of pledgeable securities. Thus, whereas low margins inflate the price of pledgeable securities and

may cause a bubble in a thriving market, they are ineffective against vicious liquidity spirals in a downturn.

4.5. Implications for a multi-asset economy

In this subsection I introduce multiple pledgeable securities indexed by $n = 1, \dots, N$. Final values are independently distributed as in Easley and O'Hara (2004), in order to focus on the implications of pledgeability for asset prices through the process of information aggregation. The equilibrium price function derived in the first part of the paper is unaffected and, in a multi-asset economy, contributes to explain empirical findings such as excess comovement and failures of the law of one price.

The price function for security n is just an indexed version of the basic pricing formula (21) and Proposition (9) above:

$$\ln \tilde{p}_n = \beta_{0n} + \beta_{1n} \ln \tilde{v}_n + \beta_{2n} \ln \tilde{u}_n + \beta_{3n} \ln \frac{\chi_n}{h_n}, \quad (29)$$

where

$$\begin{aligned} \beta_{0n} &= \frac{\tau_{\ln v_n} (\ln \bar{v}_n - (2\tau_{\ln v_n})^{-1}) - \tau_{\ln s_n} (2\tau_{\ln \epsilon_n})^{-1} - \tau_{\ln \theta_n} (2\tau_{\ln \delta_n})^{-1}}{\tau_{\ln v_n} + \tau_{\ln s_n} + \tau_{\ln \theta_n}} \\ &\quad - (\gamma - 1)(2\tau_{\ln v_n})^{-1} \\ \beta_{1n} &= \frac{\tau_{\ln s_n} + \tau_{\ln \theta_n}}{\tau_{\ln v_n} + \tau_{\ln s_n} + \tau_{\ln \theta_n}} \\ \beta_{2n} &= \frac{h_n \gamma \tau_{\ln v_n}^{-1}}{w[\mu - (1 - h_n)]} \\ \beta_{3n} &= \frac{\mu h_n}{\mu - (1 - h_n)} - 1. \end{aligned}$$

Factors such as perceived systemic risk, regulatory provisions, and market sentiment, affect the VaR of different securities and the haircuts a CCP applies on all repo transactions. To the extent that haircuts h_n ($n = 1, \dots, N$) move in the same direction for a subset of securities, Proposition 9 suggests that those securities exhibit price comovements which are not explained by information on their future cash flows.

The contribution of the model to explain failures of the law of one price goes along a similar argument: two securities which are expected to pay the same cash flows are priced differently, if they are traded under different terms in the repo market. A security which is not accepted for repo transactions is priced according to $\beta'_0 - \beta'_3$, whereas pledgeable securities follow Proposition 9 and yield different prices depending on their haircut. Securities with different levels of pledgeability (haircuts) are not equivalent, although the underlying real assets create the same expected cash flows.

5. Concluding remarks

The role collateralized loans played during the latest financial crisis makes our understanding of the pricing of pledgeable securities of primary importance. In a setting where investors have heterogeneous information, I confirm the result that margin policies affect the value of securities (see Garleanu and Pedersen (2011), among others).

However, looking at information aggregation allows to get additional insights. Pledgeability changes the incentives of investors, since collateral givers are at the same time spot buyers. The opportunity to attain leverage by pledging a security may increase spot demand for the latter and inflate prices in equilibrium. Such effect depends on market conditions: low haircuts inflate spot prices only if the interest on loans is under a threshold level. As regulators are engaged in an effort to migrate repo transactions on CCPs, it is worth noting that countercyclical haircut policies are useful to prevent price bubbles when liquidity is plentiful but they have little effect in a crisis. When the price of pledgeable securities falls because liquidity is becoming scarce, loose haircut policies are ineffective.

Pledgeability does not affect only the level of prices in equilibrium. It also has an impact on market efficiency. I show that the margin premium is a bias in terms of informativeness. The weight of noise trading in the price function increases when margins on collateral are low, so that what we call margin premium is actually a bias in terms of informational efficiency.

The fact that option pricing allows to embed pledgeable securities in a model of REE is a promising contribution to future research, as it allows to find equilibrium prices of collateral without numerical approximation of non-algebraic functions. In particular, when the base model is extended to a multi-asset economy it contributes to explain empirical findings such as failures of the law of one price and excess comovement of asset prices.

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Appendix A.

Proof of Lemma 1. Repo contracts entail the same risk as cash, since the model is solved using risk neutral valuation. Rational investors pledge their endowment optimally. That means the proceeds of a round of repo transaction are not held as cash, but rather reinvested in the purchase of additional securities to be pledged in the following rounds of leverage. Label $Y_{i,\xi}^P$ the additional amount of securities bought at round ξ using cash raised in $\xi - 1$. Then, pledging Y_i^R at round 0, an investor is able to buy the pledgeable value of its holding divided by the market price of the security:

$$\begin{aligned} Y_{i,1}^P &= Y_i^R \frac{p(1-h)}{p} \\ Y_{i,2}^P &= Y_{i,1}^P \frac{Y_{i,1}^P p(1-h)}{p} = \frac{Y_i^R p^2 (1-h)^2}{p^2} \\ &\dots \\ Y_{i,\xi}^P &= Y_{i,\xi-1}^P \frac{p(1-h)}{p} = Y_i^R \frac{p^\xi (1-h)^\xi}{p^\xi} \end{aligned}$$

Since the haircut is $0 < h < 1$, the sum of all securities purchased in different rounds is

$$\sum_{\xi=0}^{\infty} Y_{i,\xi}^P = \sum_{\xi=0}^{\infty} Y_i^R (1-h)(1-h)^\xi \approx Y_i^R \frac{1-h}{h} \quad (\text{A.1})$$

□

Proof of Lemma 2. For the borrower to prefer losing a unit of collateral – with known liquidation value v – rather than paying $p(1-h)(1+r)$, it must be true that $v < p(1-h)(1+r)$. Thus $\frac{v}{p(1-h)} < 1+r$, which proves that (9) < (8). □

Proof of Lemma 3. The position any informed investor holds in the pledgeable security is the sum of the pledgeable value of the endowment he optimally decides to repo and the pledgeable value of every additional pledgeable security he purchases, net of the liquidity m_i^* he decides to store.

Formally, a repo seller faces the budget constraint

$$(Y_i^P - 0.5w)p + m_i^* \leq (Y_i^R + Y_i^P)p(1-h) \quad (\text{A.2})$$

where the right-hand side of the equality is the liquidity an investor has available when he pledges Y_i^R , together with the newly bought securities Y_i^P as collateral in repos. On the left-hand side is the cost $Y_i^P p$ of the incremental position in the pledgeable security, plus the amount m_i^* of liquidity the investor prefers to keep risk-free. The optimal investment decision of any repo investor is either to hold no liquidity – i.e. $m_i^* = 0$ – and be a repo seller, or to refrain from repoing any security –

i.e. $Y_i^R = 0$ – and hold liquidity.⁸ Thus, the budget constraint holds as equality:

$$Y_i^P = \frac{Y_i^R(1-h) + 0.5w}{h}. \quad (\text{A.3})$$

Substituting this into (10) yields:

$$C_i^R = w \left[(0.5 - y_i^R)v + \frac{0.5 + y_i^R}{h} \max\{v - p(1-h)(1+r); 0\} \right] \quad (\text{A.4})$$

That is the same as (11) in lemma 3 if one labels as *Call* the payoff of a call option on the security, with strike price equal to the loan repayment $p(1-h)(1+r)$. \square

Proof of Lemma 4. By inspection of equalities (5) and (11) one notices that, when $p > \frac{Call}{h}$, an investor who has access to the repo market receives a higher payoff by acting as a spot seller. The two trades entail the same level of risk. Thus, an investor with access to the repo market has incentive to trade repos if and only if $h < \frac{Call}{p}$. \square

Proof of Lemma 5. Substituting the investor's payoff (5) into the expected utility function (1) yields

$$\begin{aligned} \mathbb{E} [U(C_i^S)] &= \mathbb{E} \left[\frac{C_i^{1-\gamma}}{1-\gamma} \middle| \Omega_i \right] \\ &= \mathbb{E} \left[\frac{\left(w [p + (0.5 - y^S)(v-p)] \right)^{1-\gamma}}{1-\gamma} \middle| \Omega_i \right]. \end{aligned} \quad (\text{A.5})$$

Label the investor return on his endowment, given any chosen y_i^S , as

$$\phi_i^S = p + (0.5 - y_i^S)(v-p).$$

Maximising (A.5) is the same as

$$\max_{y_i^S} \mathbb{E} \left[\frac{\left(w \phi_i^S \right)^{1-\gamma}}{1-\gamma} \middle| \Omega_i \right]. \quad (\text{A.6})$$

Campbell and Viceira (2002) show that in this standard problem, over short time intervals, an

⁸See Lemma 4.

acceptable approximation polynomial is:

$$\begin{aligned}\rho_i^S &= \ln p + (0.5 - y_i^S)(\ln v - \ln p) + (0.5 - y_i^S)(0.5 + y_i^S)(2\tau_{\ln v})^{-1} \\ &\approx \ln \phi_i^S.\end{aligned}$$

Thus, the objective function in (A.6) is approximately the same as

$$\begin{aligned}\mathbb{E} \left[\frac{(we^{\rho_i^S})^{1-\gamma}}{1-\gamma} \middle| \Omega_i \right] &= \\ \mathbb{E} \left[\frac{(wpe^{(0.5-y_i^S)(\ln \frac{v}{p})+(0.5-y_i^S)(0.5+y_i^S)(2\tau_{\ln v})^{-1}})^{1-\gamma}}{1-\gamma} \middle| \Omega_i \right] & \\ = \frac{(wp)^{1-\gamma} e^{(0.5-y_i^S)(0.5+y_i^S)(2\tau_{\ln v})^{-1}(1-\gamma)} \mathbb{E} \left[e^{(0.5-y_i^S)(\ln \frac{v}{p})(1-\gamma)} \middle| \Omega_i \right]}{1-\gamma}.\end{aligned}\tag{A.7}$$

Given the distribution of v , the argument of the above expectation is log-Normal:

$$\begin{aligned}e^{(0.5-y_i^S)(\ln v - \ln p)(1-\gamma)} &\sim \log N(\mu_S, \tau_S^{-1}) \\ \text{where } \mu_P &= (0.5 - y_i^S)(1 - \gamma) (\mathbb{E}[\ln v | \Omega_i] - \ln p), \\ \text{and } \tau_P^{-1} &= (0.5 - y_i^S)^2 (1 - \gamma)^2 \tau_{\ln v}^{-1}.\end{aligned}$$

Taking logs of the numerator and simplifying, problem (A.6) becomes⁹

$$\begin{aligned}\max_{y_i^S} \ln(wp) + 0.5(1 - 0.5\gamma)(2\tau_{\ln v})^{-1} + y_i^S [(\gamma - 1) - \gamma y_i^S] (2\tau_{\ln v})^{-1} + \\ + (0.5 - y_i^S) (\mathbb{E}[\ln v | \Omega_i] - \ln p).\end{aligned}$$

This yields the first order condition

$$FOC_{y_i^S} : (\gamma - 1)(2\tau_{\ln v})^{-1} - 2\gamma y_i^S (2\tau_{\ln v})^{-1} - \mathbb{E}[\ln v | \Omega_i] + \ln p = 0\tag{A.8}$$

and an optimal value for the share of endowment to be sold

$$y_i^{S*} = \frac{\ln p - \mathbb{E}[\ln v | \Omega_i]}{\gamma \tau_{\ln v}^{-1}} - \frac{1 - \gamma}{2\gamma}.\tag{A.9}$$

□

⁹For any lognormally distributed random variable x , $\log \mathbb{E}[x] = \mathbb{E}[\log(x)] + \frac{1}{2} \text{VAR}[\log(x)]$.

Proof of Lemma 6. To maximize (16) is the same as

$$\max_{y_i^R} \mathbb{E} \left[\frac{(w\phi_i^R)^{1-\gamma}}{1-\gamma} \middle| \Omega_i \right] \quad (\text{A.10})$$

where

$$\phi_i^R = \frac{Call}{h} + (0.5 - y_i^R) \left(v - \frac{Call}{h} \right). \quad (\text{A.11})$$

The approximation by Campbell and Viceira (2002) allows to rewrite the formula of repo traders' return above as

$$\begin{aligned} \rho_i^R &= \ln \frac{Call}{h} + (0.5 - y_i^R) \left(\ln v - \ln \frac{Call}{h} \right) + \\ &+ (0.5 - y_i^R)(0.5 + y_i^R)(2\tau_{\ln v})^{-1} \approx \ln \phi_i^R. \end{aligned} \quad (\text{A.12})$$

Thus, the objective function in (A.10) is approximately the same as

$$\begin{aligned} \mathbb{E} \left[\frac{(w e^{\rho_i^R})^{1-\gamma}}{1-\gamma} \middle| \Omega_i \right] &= \\ \frac{(w \frac{Call}{h})^{(1-\gamma)} e^{(0.5-y_i^R)(0.5+y_i^R)(2\tau_{\ln v})^{-1}(1-\gamma)} \mathbb{E} \left[e^{(0.5-y_i^R)(1-\gamma)(\ln v - \ln \frac{Call}{h})} \middle| \Omega_i \right]}{1-\gamma}. \end{aligned} \quad (\text{A.13})$$

Given the assumed distribution of v ,

$$e^{(0.5-y_i^R)(1-\gamma)(\ln v - \ln \frac{Call}{h})} \phi_i^R \sim \log N(\mu_R, \tau_R^{-1}) \quad (\text{A.14})$$

where

$$\begin{aligned} \mu_R &= (0.5 - y_i^R)(1 - \gamma) \left(\mathbb{E}[\ln v | \Omega_i] - \ln \frac{Call}{h} \right), \\ \tau_R^{-1} &= (0.5 - y_i^R)^2 (1 - \gamma)^2 \tau_{\ln v}^{-1}. \end{aligned}$$

Taking logs of the numerator in (A.13) and simplifying, the optimization problem of a repo seller writes

$$\begin{aligned} \max_{y_i^R} & \left[\ln \left(w \frac{Call}{h} \right) + 0.5(1 - 0.5\gamma) + y_i^R [(\gamma - 1) - \gamma y_i^R] \right] (2\tau_{\ln v})^{-1} + \\ & + (0.5 - y_i^R) \left(\mathbb{E}[\ln v | \Omega_i] - \ln \frac{Call}{h} \right). \end{aligned} \quad (\text{A.15})$$

The First Order Condition for optimization is

$$FOC_{y_i^R} : (\gamma - 1 - 2\gamma y_i^R)(2\tau_{\ln v})^{-1} - \left(\mathbb{E} \left[\ln v \middle| \Omega_i \right] - \ln \frac{Call}{h} \right) = 0. \quad (\text{A.16})$$

That yields an optimal decision

$$y_i^{R*} = \frac{1}{2} + \frac{\ln \frac{Call}{h} - \mathbb{E}[\ln v | \Omega_i]}{\gamma \tau_{\ln v}^{-1}} - \frac{1}{2\gamma} \quad (\text{A.17})$$

The investor's demand for securities in the spot market is therefore

$$\begin{aligned} y_i^{P*} &= y_i^{R*} \frac{1-h}{h} \\ &= \left(\frac{1}{2} + \frac{\ln \frac{Call}{h} - \mathbb{E}[\ln v | \Omega_i]}{\gamma \tau_{\ln v}^{-1}} - \frac{1}{2\gamma} \right) \frac{1-h}{h} \end{aligned} \quad (\text{A.18})$$

With no loss of generality, the value of the exogeneous parameter μ that determines the share of repo traders in the economy is limited to ensure that the maximum amount of securities that spot traders can offer on the repo market is sufficient to satisfy the maximum demand from repo traders. The number of investors $(1 - \mu)$ who can participate to the repo market is limited by the repo margin:

$$\begin{aligned} (1 - \mu)a \frac{1-h}{h} &< \mu a \\ 1 - \mu &< h \end{aligned} \quad (\text{A.19})$$

$(1 - \mu)a$ is the whole amount of securities repo traders are endowed with, whereas the whole left-hand side is the amount of securities that can be purchased by pledging such endowment. On the right-hand side is the greatest supply that is possible to find in the spot market, in the limit case where all spot investors want to sell. \square

Proof of Proposition 7. The standard pricing formulas for an European call option in to the Black-Scholes-Merton model are

$$c(S, \Delta t) = SN(d_1) - X e^{-r_f \Delta t} N(d_2) \quad (\text{A.20})$$

$$d_1 = \frac{\ln(S/K) + (r + \frac{\sigma^2}{2})\Delta t}{\sigma \sqrt{\Delta t}}, \quad (\text{A.21})$$

$$d_2 = d_1 - \sigma \sqrt{\Delta t}. \quad (\text{A.22})$$

where S is the price of the underlying asset, σ^2 its volatility, r_f the risk-free rate, and Δt the time to exercise. The strike price of the repo option $Call$ is $X = p(1+r)(1-h)$. The price function

(A.20) becomes

$$\begin{aligned} \text{Call}(p, \Delta t) &= pN(d_1) - p[(1-h)(1+r)]e^{-r_f\Delta t}N(d_2) \\ &= p \left(\underbrace{N(d_1) - (1-h)(1+r)e^{-r_f\Delta t}N(d_2)}_{\chi} \right) \end{aligned} \quad (\text{A.23})$$

Since the strike price $p(1-h)(1+r)$ of the original option *Call* is linear in the current spot price p , the spot prices at the numerator and at the denominator of the log operators in the d_1 and d_2 formulas of the Black and Scholes model cancel out:

$$d_1 = \frac{\ln\left(\frac{p}{p(1-h)(1+r)}\right) + \left(r + \frac{\sigma^2}{2}\right)\Delta t}{\sigma\sqrt{\Delta t}} \quad (\text{A.24})$$

$$= \frac{\ln\left(\frac{1}{(1-h)(1+r)}\right) + \left(r + \frac{\sigma^2}{2}\right)\Delta t}{\sigma\sqrt{\Delta t}}, \quad (\text{A.25})$$

$$d_2 = d_1 - \sigma\sqrt{\Delta t}. \quad (\text{A.26})$$

Thus, the repo option value $\chi = \text{Call}/p$ does not depend on the current price of the underlying pledgeable security. \square

Proof of Proposition 8. Looking at all observable information, investor i updates his prior belief and computes the posterior conditional distribution of the risky return. In order to form the updated expectation $\mathbb{E}[\ln v|\Omega_i]$, investors apply Bayes' rule to incorporate both the private signal and public information. The posterior distribution of asset returns is normal with

$$\begin{aligned} \mathbb{E}[\ln v|\Omega_i] &= \frac{\tau_{\ln v}(\ln \bar{v} - (2\tau_{\ln v})^{-1}) + \tau_{\ln s} \ln s_i + \tau_{\ln \theta} \ln \theta}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}} \\ &= \frac{\tau_{\ln v}(\ln \bar{v} - (2\tau_{\ln v})^{-1}) + \tau_{\ln s}(\ln v + \ln \frac{\epsilon_i}{\bar{\epsilon}}) + \tau_{\ln \theta}(\ln v + \ln \delta)}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}}, \end{aligned} \quad (\text{A.27})$$

where

$$\ln \delta = \frac{\beta_2}{\beta_1}[\ln u - k] \sim N\left(-\left(\frac{\beta_2}{\beta_1}\right)^2(2\tau_{\ln u})^{-1}, \left(\frac{\beta_2}{\beta_1}\right)^2\tau_{\ln u}^{-1}\right). \quad (\text{A.28})$$

And

$$\text{VAR}[\ln v|\Omega_i] = (\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta})^{-1}. \quad (\text{A.29})$$

Substituting investors' optimal decisions in the market clearing condition specified in equality (12) yields

$$\begin{aligned} \ln p = & (1 - \gamma)(2\tau_{\ln v})^{-1} + \frac{h\gamma\tau_{\ln v}^{-1}}{w(\mu - (1 - h))} \ln u + \left(\frac{\mu h}{\mu - (1 - h)} - 1 \right) \ln \frac{\chi}{h} \\ & + \frac{\left(h \int_0^1 \mathbb{E}[\ln v | \Omega_i] di - \int_\mu^1 \mathbb{E}[\ln v | \Omega_i] di \right)}{\mu - (1 - h)}. \end{aligned} \quad (\text{A.30})$$

Since signals received by spot and repo traders are the same, the integral of posterior expectations on $\ln v$ over all informed investors is a multiple of that over spot traders:

$$\begin{aligned} \int_0^1 \mathbb{E}[\ln v | \Omega_i] di &= \frac{1}{1 - \mu} \int_\mu^1 \mathbb{E}[\ln v | \Omega_i] di = \\ \frac{\tau_{\ln v}(\ln \bar{v} - (2\tau_{\ln v})^{-1}) + \tau_{\ln s}(\ln v - (2\tau_{\ln \epsilon})^{-1}) + \tau_{\ln \theta}(\ln v - (2\tau_{\ln \delta})^{-1})}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}}. \end{aligned} \quad (\text{A.31})$$

Thus, having taken into account the process of information aggregation, the price function (A.30) of the pledgeable security becomes

$$\begin{aligned} \ln p = & \frac{\tau_{\ln v}(\ln \bar{v} - (2\tau_{\ln v})^{-1}) - \tau_{\ln s}(2\tau_{\ln \epsilon})^{-1} - \tau_{\ln \theta}(2\tau_{\ln \delta})^{-1}}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}} \\ & - (\gamma - 1)(2\tau_{\ln v})^{-1} + \frac{\tau_{\ln s} + \tau_{\ln \theta}}{\tau_{\ln v} + \tau_{\ln s} + \tau_{\ln \theta}} \ln v \\ & + \frac{h\gamma\tau_{\ln v}^{-1}}{w[\mu - (1 - h)]} \ln u + \left(\frac{\mu h}{\mu - (1 - h)} - 1 \right) \ln \frac{\chi}{h}. \end{aligned} \quad (\text{A.32})$$

□

Proof of Lemma 9. From the pricing formula of a generic call option (A.20)-(A.22), assuming that the strike price K depends on the lending margin h , the derivative of the option value with respect to h is

$$\frac{\partial \ln c}{\partial h} = \frac{\partial \ln \{c[K(h), d_1(K(h)), d_2(K(h))]\}}{\partial h} \quad (\text{A.33})$$

$$= \frac{1}{c} \left[\frac{\partial K(h)}{\partial h} \left(S \frac{\partial N(d_1)}{\partial d_1} \frac{\partial d_1(K)}{\partial K} - K e^{-r_f \Delta t} \frac{\partial N(d_2)}{\partial d_2} \frac{\partial d_2(K)}{\partial K} + e^{-r_f \Delta t} N(d_2) \right) \right] \quad (\text{A.34})$$

$$= \frac{1}{c} \left[\frac{\partial K(h)}{\partial h} \left(e^{\ln S} \frac{e^{-\frac{d_1^2}{2}}}{\sigma \sqrt{2\pi \Delta t}} \frac{-1}{K} - e^{\ln K} e^{-r_f \Delta t} \frac{e^{-\frac{d_2^2}{2}}}{\sigma \sqrt{2\pi \Delta t}} \frac{-1}{K} + e^{-r_f \Delta t} N(d_2) \right) \right] \quad (\text{A.35})$$

$$= \frac{1}{c} \left[\frac{\frac{\partial K(h)}{\partial h}}{K \sigma \sqrt{2\pi \Delta t}} \left(\underbrace{e^{\ln S} e^{-\frac{d_1^2}{2}} - e^{\ln K} e^{-r_f \Delta t} e^{-\frac{d_2^2}{2}}}_{=0} \right) + \frac{\partial K(h)}{\partial h} e^{-r_f \Delta t} N(d_2) \right] \quad (\text{A.36})$$

$$f(x) = \pi [(x^4 + 7x^3 + 2x^2 + 10x + 12)] \quad (\text{A.37})$$

Notice that the exponentials into brackets sum up to zero since, from the Black and Scholes (1973) equations in (A.21) and (A.22)

$$e^{-\frac{d_2^2}{2}} = e^{-\frac{(d_1 - \sigma \sqrt{\Delta t})^2}{2}} \quad (\text{A.38})$$

$$= e^{-\frac{d_1^2}{2}} e^{-\frac{\sigma^2 \Delta t}{2}} e^{d_1 \sigma \sqrt{\Delta t}} \quad (\text{A.39})$$

$$= e^{-\frac{d_1^2}{2}} e^{-\frac{\sigma^2 \Delta t}{2}} e^{\ln \frac{S}{K} (r_f + \frac{\sigma^2}{2}) \Delta t} \quad (\text{A.40})$$

$$= e^{-\frac{d_1^2}{2}} e^{\ln \frac{S}{K} r_f \Delta t} \quad (\text{A.41})$$

$$\Rightarrow e^{-\frac{d_2^2}{2}} e^{\ln K} e^{-r_f \Delta t} = e^{\ln S} e^{-\frac{d_1^2}{2}} \quad (\text{A.42})$$

The implicit repo option has $S = 1$, $K = (1 - h)(1 + r)$, $\sigma^2 = \sigma_{\ln v}^2$, $r_f = 0$, and $\Delta t = 1$. Therefore,

its derivative with respect to the repo margin is

$$\frac{\partial \ln \chi}{\partial h} = \frac{1}{c} \left[\frac{\partial K(h)}{\partial h} e^{-r_f \Delta t} N(d_2) \right] \quad (\text{A.43})$$

$$= \frac{(1+r)e^{-r_f \Delta t} N(d_2)}{\chi} \quad (\text{A.44})$$

$$= \frac{(1+r)N(d_2)}{\chi} \quad (\text{A.45})$$

that is positive since neither the repo rate, the Normal cumulative distribution function, or the call option value ever assume negative values. \square

Proof of Proposition 10. The derivative of the pledgeability bias $\beta_3(\ln \chi - \ln h)$ with respect to margins is

$$\begin{aligned} \frac{\partial \beta_3(\ln \chi - \ln h)}{\partial h} &= \frac{\partial \beta_3}{\partial h} (\ln \chi - \ln h) + \beta_3 \left(\frac{\partial \ln \chi}{\partial h} - \frac{\partial \ln h}{\partial h} \right) \\ &= \underbrace{\frac{\mu(\mu-1)}{(h-(1-\mu))^2}}_{<0} \underbrace{(\ln \chi - \ln h)}_{>0} + \underbrace{\frac{(1-h)(1-\mu)}{h-(1-\mu)}}_{>0} \left(\frac{(1+r)N(d_2)}{\chi} - \frac{1}{h} \right) \end{aligned} \quad (\text{A.46})$$

That is negative if (but not only if)

$$\frac{(1+r)N(d_2)}{N(d_1) - (1+r)N(d_2)} - \frac{1}{h} < 0 \quad (\text{A.47})$$

$$r < \bar{r} \equiv \frac{N(d_1)}{N(d_2)} - 1 \quad (\text{A.48})$$

Substituting (A.22) in the formula for \bar{r} shows that the threshold is strictly positive and increases with the variance of collateral value $\tau_{\ln v}^{-1}$. \square