Fakultät für Wirtschaftswissenschaften
Facoltà di Economia
Faculty of Economics and Management

BEMPS –
Bozen Economics & Management Paper Series

NO 25 / 2015

In Medio Stat Virtus: Does a Mixed Economy increase Welfare?

Alessandro Fedele and Sara Depedri

In Medio Stat Virtus: Does a Mixed Economy Increase Welfare?*

Alessandro Fedele[a] Sara Depedri[b]

October 2015

Abstract

Over the past few decades, social enterprises have grown remarkably. This paper investigates how social enterprises affect access to social services (e.g., education and health-care) and utilitarian welfare. To this end, two economic systems are compared: a market economy system, where all firms are profit maximizers, and a mixed economy system, where both for-profit businesses and social enterprises are present. Findings show that individuals are more likely to have access to social services within mixed economy. Moreover, conditions are derived under which utilitarian welfare is larger within mixed economy. Public policies in support of social enterprises (e.g., subsidies) are shown to result in the following trade-off: access to social services is further enhanced but utilitarian welfare is more likely to be lower than that within market economy.

JEL classification: L33 (Comparison of Public and Private Enterprises and Nonprofit Institutions); L38 (Nonprofit Organizations and Public Enterprise: Public Policy); L13 (Oligopoly and Other Imperfect Markets); P51 (Comparative Analysis of Economic Systems).

Keywords: market economy; mixed economy; access to social services; utilitarian welfare; public policies.

*We thank the seminar audience at the 5th CIRIEC International Research Conference on Social Economy (Lisbon, 2015), 55th Riunione Scientifica Annuale SIE (Trento, 2014), 54th Congress of the European Regional Science Association (St. Petersburg, 2014), IV EMES International Research Conference on Social Enterprise (Liége, 2013), and VI Colloquio sull'Impresa Sociale (Brescia, 2012) for useful comments.

[a] *Corresponding author*, Faculty of Economics and Management, Free University of Bozen/Bolzano, Italy; *e-mail:* alessandro.fedele@unibz.it.

[b] European Research Institute on Cooperative and Social Enterprise and University of Trento, Italy; e-mail: sara.depedri@unitn.it.

Introduction

In the aftermath of the economic crisis, Stiglitz (2009) remarked that productive organizations such as cooperative and socially oriented enterprises may play a key role in restoring people's confidence. Indeed, these productive organizations "are less inclined to exploit those with whom they interact: their workers, their customers, and their suppliers" (p. 357). Accordingly, Stiglitz argued that an economy is more likely to be successful if it is able to "find a balance between markets, government, and other institutions, including not-for-profits and cooperatives" (p. 348) and, as it can be inferred from his reasoning, that cooperative and socially oriented enterprises may help increase both the wellbeing of individuals and economic efficiency. Put differently, the welfare of citizens and producers may be positively affected by the presence of different firm types in the same sector of production.

Casual empirical observation of several services sectors across countries suggests that socially oriented enterprises play an increasingly important role. Some recent contributions have thus developed frameworks to explain the co-existence of mixed forms in the same market. For instance, Marwell and McInerney (2005) study the dynamic relationships that arise in a market where when for-profit, nonprofit, and government providers coexist. Te'eni and Young (2003) focus on the resilience of nonprofit firms due to their relative advantages in the network economy.

To the best of our knowledge, however, no economics paper has relied on a formal theoretical analysis to investigate how the co-existence of diverse firm types in the same sector affects access to social services and economic efficiency. The current work aims to fill this gap by comparing two different economic systems. (i) An economy where all firms are profit maximziers. This system is referred to as a market economy. (ii) An economy where both for-profit businesses and socially oriented enterprises are present. This system is defined as a *mixed economy.* Throughout this paper, we refer to socially oriented organizations as social enterprises. According to the literature (e.g., Borzaga and Defourny, 2001) and recent instructions of the European Commission, as reported in the Social Europe Guide (European Commission, 2013a, 2013b), social enterprises are defined as hybrid organizations that balance their social mission with their entrepreneurial activity. In addition, our study aims to contribute to the analysis of public policies supporting the presence of social enterprises.

The remainder of the paper is organized as follows. In Section 1, we describe how the meaning of "mixed economy" and the role of social enterprises have evolved over time; this will help contextualize the analysis. In Section 2, we describe the setup and main assumptions of the model. In Section 3, we study the equilibrium properties of the market economy and mixed economy. The two economies are compared in Section 4 to identify the conditions under which the presence of social enterprises in the production of social services enhances both the access of individuals to social service and utilitarian welfare. The social enterprises are supposed to be financially self-sustainable in that they are subject to a break-even constraint; public policies aimed at directly supporting social enterprises are disregarded. This part of the analysis could, therefore, provide normative insights into the access to the service and the efficiency guaranteed by a mixed economy in countries where social enterprises are less likely to be directly supported by governments. Sections 5 and 6 provide two extensions of the model. First, we introduce individuals who are influenced by ideological concerns when choosing which organization, either the for-profit or social enterprise, resort to. Second, we explicitly consider policies supporting the presence of social enterprises through, for example, subsidies. Section 7 concludes with policy recommendations. Computations and proofs of our results are in the online Appendix (attached to this submission).

1 Mixed Economy and Role of Social Enterprises

The term mixed economy can be used to define the presence of different economic actors (e.g., private and public firms), that produce a good or service. This definition has evolved over time, following the evolution of welfare systems and the role of a welfare state.[1]

Focusing on social services sectors, we can remark that in most European economies, social services were supplied directly by public bodies until the 1970s, while private for-profit businesses supplied integrative services through accreditation systems. The role of nonprofit organizations in the direct production of social services was somewhat marginal; nonprofit organizations were confined to perform an advocacy function and supply social services only to the poorest people. In this context, the term mixed economy was referred to as a mix of for-profit businesses and public bodies in the provision of social services (e.g., Kazepov, 2009).

In the 1980s, due to the crisis of welfare state, a growing demand for social services related to new social needs (e.g., drug addiction and alcoholism), and the increasing participation of women in the labor market, the number of nonprofit organizations rose and their productive role became more relevant. These productive nonprofits were institutionalized through the introduction of new organizational forms. Solidarity co-ops in Québec, sociétés coopératives d'intérêt collectif in France, social cooperatives in Italy, and, more generally and recently, social enterprises. Furthermore, the increasing relevance of social enterprises induced many governments to consider them as an active part of social policies and to establish new forms of public-private relationship, where private nonprofit organizations directly supply social services (e.g., Ostrander, 1989).

[1] Some contributions use the alternative term mixed-form markets (e.g., Marwell and McInerney, 2005).

The new century has witnessed a further evolution in welfare systems. An increasing presence of social enterprises and potential competition between social enterprises and for-profit businesses have been observed in social services sectors (e.g., Ben-Ner, 2002). Accordingly, the term mixed economy is now often used to define situations in which services are provided by different productive entities, including not-for-profits (e.g., Beckford, 1991). Following this evolution of welfare systems and mixed economies, a new stream of theoretical economics literature on mixed oligopolies has flourished. Initially, the benchmark was the analysis of competition between state-owned welfare-maximizing public firms and profit-maximizing private firms (see De Fraja and Delbono, 1990, for a survey). Subsequently, the focus has shifted to efficiency generated by cooperative firms (e.g., Delbono and Reggiani, 2013; and Marini et al., 2015). Our work is focused on social enterprises and contributes to this literature.

From an empirical point of view, several cross-country studies have investigated the growth of social enterprises. A seminal contribution is the Johns Hopkins Comparative Nonprofit Sector Project, conducted in 22 countries (European countries, Australia, Japan, the United States, and some Latin American countries) in the 1990s. As reported in this study, the expenditure of the nonprofit economic sector was about $1.1 trillion, equivalent to 4.6% of the total GDP of the sample countries (Salamon and Anheier, 1997). Further studies at the country level (e.g., CIRIEC, 2007) find that about 130,000 nonprofit enterprises are active in France with more than 1.4 million employees; about 37,000 units in Portugal with 160,000 employees; 127,000 enterprises in Spain with 380,000 employees; 31,400 organizations in Sweden with 95,000 employees; about 13,000 organizations in Denmark with 121,000 people; 506,000 units in Germany, with more than 1.4 million employees. According to Salamon (2006, p. 402), "nonprofits account for 40 percent of all hospital patient days in Germany, 55 percent of all residents in residential care facilities in French, three-fourths of all students in higher education in Japan, and much of the social service provision in Italy". Today, many nonprofit enterprises are commercial institutions that sell their products and services in the marketplace. According to Kerlin (2006), the commercial revenues of nonprofit enterprises in the United States increased on average by 219% from 1982 to 2002; similarly, commercial revenues accounted for 57.6% of nonprofit firms' total revenues in 2002 compared with the 48.1% in 1982.

2 Setup

We introduce a hypothetical economy made up of two industrial sectors. In Sector A, a good is produced by for-profit firms. For example, consider a car industry where producers are typically for-profit. Sector B supplies instead a social service (e.g., education and health-care). Each Sector $j = A, B$ is characterized by a segment of length 1 where two firms, indexed by i, j with $i = 0, 1$, are located at the extremes, firm $0, j$ is at $x = 0$ and firm $1, j$ at $x = 1$ (Figure 1). This type of segment is named Hotelling-type segment, after statistician and economist Harold Hotelling, and captures horizontal product differentiation. Potential buyers of mass one are uniformly distributed along the segment. In each sector, each individual demands at most one unit of the commodity, the good in Sector A or the social service in Sector B.

[Figure 1 here]

Individuals derive utility s_j from one unit of the commodity produced in Sector $j = A, B$. We refer to the difference between utility and total purchase costs as surplus of an individual. More precisely, the surplus of an individual located at point $x \in [0, 1]$ is equal to

$$\begin{array}{lll} s_j - p_{0,j} - tx & \text{when buying the commodity from firm } 0, j, & (a) \\ s_j - p_{1,j} - t(1-x) & \text{when buying the commodity from firm } 1, j, & (b) \\ 0 & \text{when not buying a commodity,} & (c) \end{array} \qquad (1)$$

where $p_{0,j}$ ($p_{1,j}$) is per unit of commodity price charged by firm 0 (1) in Sector $j = A, B$. Expressions tx and $t(1-x)$ denote a further cost borne by the individual located at x when buying from firm $0, j$ and $1, j$, respectively. The Hotelling framework fits our analysis because of its "flexibility". Indeed, the segment where firms compete by producing horizontally differentiated commodities can be interpreted in several different ways.

Following the traditional interpretation, the segment can be thought of as a physical space: individuals bear transportation costs when moving along the segment to make their purchases. Location x of an individual denotes her/his geographical distance from the two firms. In Figure 1, for instance, x and $1 - x$ are the distances travelled by the individual located at x when going to firm $0, j$ and $1, j$, respectively. In addition, parameter $t > 0$ denotes the per unit of distance cost of transportation. Overall, tx and $t(1-x)$ are the transportation cost borne by the individual located at x when buying from firm $0, j$ and $1, j$, respectively. In Sector B, where social services are traded, tx and $t(1-x)$ represents, for example, the cost of transporting children to school and day nursery or the elderly to hospitals.

An alternative and fairly innovative interpretation is compatible with the flexible Hotelling framework and proposed in Section 5. Individuals are assumed to have heterogeneous tastes in firm types (i.e., for-profit versus social enterprise). In this case, individuals bear ideological costs for not purchasing from the preferred type of firm when "travelling" along the ideological space. This interpretation is rather natural when different types of firms, not only for-profits, coexist and when a social service is traded in the market. Indeed, the users' choice of social services providers is based on the perception of risk, confidence, and trust, in which case the location x can describe the proximity in terms of identity and organizational fit, as described by the psychology and behavioral economics literature (e.g., Van Dyne and Pierce, 2004).[2]

We define two additional aspects, which are key to our analysis: firms' profits and surplus of all individuals. Firm i, j, is assumed to incur constant per unit of commodity production cost $c_j \geq 0$. Accordingly, its profit function is

$$\Pi_{i,j} = (p_{i,j} - c_j)\, D_{i,j}, \tag{2}$$

where $D_{i,j}$ denotes the share of individuals who decide to buy from firm i, j, i.e., the demand for the commodity supplied by firm i, j. Surplus of firm i, j's customers is given by

$$CS_{i,j} = D_{i,j}\left(s - p_{i,j} - \frac{t}{2} D_{i,j}\right): \tag{3}$$

see online Appendix A.1 for computations. Surplus of individuals who do not buy is obviously zero:

$$CS_{H,j} = 0. \tag{4}$$

Expression (3) is (negatively) affected by the unit transportation cost t, which plays a crucial role in our framework. To illustrate this role, we denote with $x_{I,j}$ the location of an individual who obtains the same surplus when purchasing the commodity from firm $0, j$ or firm $1, j$. This location is obtained after solving equality (1 - a) = (1 - b) by x:

$$x_{I,j} = \frac{1}{2} + \frac{p_{1,j} - p_{0,j}}{2t}. \tag{5}$$

We then plug $x_{I,j}$ into either (1 - a) or (1 - b) to get the surplus of the indifferent individual, denoted by $\sigma_{I,j}$. In symbols,

$$\sigma_{I,j} = s_j - \frac{t}{2} - \frac{p_{1,j} + p_{0,j}}{2}. \tag{6}$$

Not surprisingly, $\sigma_{I,j}$ decreases when the unit transportation cost t increases.

In Figure 2, we provide a graphical representation of individuals' surplus as a function of their location x, i.e., we depict (1 - a) and (1 - b). Intuitively, both expressions are decreasing in the distance travelled by the individuals, x when buying from firm $0, j$ and $(1 - x)$ when buying from firm $1, j$. We also depict the surplus of the indifferent individual, $\sigma_{I,j}$, by assuming it is positive. In this case, the indifferent individual is willing to buy either from firm $0, j$ or firm $1, j$. As a result, all individuals located to the left of $x_{I,j}$ buy from firm $0, j$, while those located to the right of $x_{I,j}$ buy from firm $1, j$. The demand shares of the two firms are $D_{0,j} = [0, x_{I,j}]$ and $D_{1,j} = [x_{I,j}, 1]$. Sector $j = A, B$ is said to

[2] A similar ideological interpretation of the Hotelling segment is proposed by Becchetti et al. (2014), who assume that different individuals' locations in the segment implies differences in the psychological perceptions of the ethical value of a good. Yet their focus is on vertical differentiation, where any individual prefers more ethical goods, even if to a different extent, thus incurring utility costs only when going from a more ethical to a less ethical point in the segment.

be fully covered because all individuals buy. Note that this is likely to occur when the unit transportation cost t is low, i.e., when the two downward-sloping bold lines are relatively flat.

[Figure 2 here]

In Figure 3, instead, the surplus of the indifferent individual, $\sigma_{I,j}$, is assumed to be negative, in which case the individual located at $x_{I,j}$ does not buy. As a result, the demand shares of firms $0, j$ and $1, j$ become $D_{0,j} = [0, x_{0,j}]$ and $D_{1,j} = [x_{1,j}, 1]$, with $x_{0,j} < x_{I,j} < x_{1,j}$. Sector $j = A, B$ is said to be partially covered because individuals located in $(x_{0,j}, x_{1,j})$ do not buy. This is likely to occur when the unit transportation cost t is high, i.e., when the two downward-sloping bold lines are relatively steep.

[Figure 3 here]

The analysis proceeds by comparing two different economies.

(i) An economy where each firm $i = 0, 1$ in each Sector $j = A, B$ is for-profit. By definition, a for-profit firm i, j aims at maximizing its own profit $\Pi_{i,j}$.

(ii) An economy where both firms are profit maximziers in Industry A, while Sector B is made up of the following mixed duopoly: firm $0, B$ maximizes the surplus of its customers, $CS_{0,B}$, and it is referred to as a social enterprise, whereas firm $1, B$ is a standard profit maximizer, which targets its own profit $\Pi_{1,B}$.

We introduce the following:

Definition 1 *An economy where all firms are profit maximizers is defined as a market economy. An economy where both firms are profit maximizers in Sector A and Sector B has a social enterprise is defined as a mixed economy.*

In Sector B of the mixed economy, the unit production costs incurred by the two types of firms are identical and equal to c_B. In other words, the social enterprise and for-profit firm are supposed to have access to the same production technology.

The timing of events in our framework is as follows.

- At $t = 0$, in each Sector $j = A, B$ of each economy, either market or mixed, firms $0, j$ and $1, j$ simultaneously choose prices $p_{0,j}$ and $p_{1,j}$ to maximize their objective functions.

- At $t = 1$, profits accrue to the firms.

We make the following reasonable hypothesis: price competition occurs only between firms belonging to the same industrial Sector, either A or B, and not between firms across sectors. This is due to the different nature of, and thus the different demand for, the commodities supplied in the two industries. One good is typically supplied by for-profit companies, for example, cars. The other is a social service, which is offered by both for-profit and nonprofit entities. Finally, we let the unit consumption utility s_j be higher than the unit production cost c_j in both industries. This is a necessary condition for trade between individuals and firms to occur.

The analysis proceeds as follows. In the next Section we study the (Nash) equilibrium of the price competition game taking place at $t = 0$. We consider separately the market economy and mixed economy. In Section 4, we move to a welfare analysis.

3 Equilibrium Analysis

Market economy. We compute the equilibrium of the market economy and, then, study how the equilibrium is affected by different values of the unit transportation cost t. All firms set prices $p_{i,j}$ to maximize profit $\Pi_{i,j}$, subject to the following constraint: all their customers must get a non-negative surplus, otherwise they would not buy.

The equilibrium prices in the market economy are computed in online Appendix A.2, where we show that in each Sector $j = A, B$, the two firms set the same price, denoted by p_j^*. This is because the two firms are symmetric, i.e., they maximize the same profit function $\Pi_{i,j}$. In line with the intuition provided by Figure 2, we also prove that the full coverage of Sector $j = A, B$ occurs only when the unit transportation cost t is relatively low (for the sake of precision, not larger than $s_j - c_j$). For higher values of t, instead, those individuals living close to $x = \frac{1}{2}$ prefer not to buy: partial coverage occurs, as depicted in Figure 3. These results come as no surprise. A higher t makes it more difficult to serve all the individuals since, ceteris paribus, their surplus is negatively affected, as testified by expressions (1 - a) and (1 - b).

Mixed economy. We turn our focus on the mixed economy. According to Definition 1, Sector A is still made up of two for-profit firms, $0, A$ and $1, A$, whose symmetric equilibrium price, p_A^*, has been computed in online Appendix A.2.

By contrast, in Sector B, firm $0, B$ is a customer surplus maximizer rather than profit maximizer. Accordingly, it aims at maximizing the surplus of its customers, $CS_{0,B}$, subject to the following constraint: its profits must be non-negative, $\Pi_{0,B} \geq 0$. This break-even constraint ensures the financial self-sustainability of the social enterprise. Since the surplus of its customers is negatively affected by price, at equilibrium, the social enterprise sets the price $p_{0,B}^{**}$ as low as possible, i.e., equal to the production cost c_B, with the effect that its equilibrium profit is zero, $\Pi_{0,B}^{**} = 0$. The price $p_{1,B}^{**}$ set by the for-profit firm $1, B$ is instead higher than c_B: for computations, see online Appendix A.3, where we show that the full coverage of Sector B, depicted in Figure 2, occurs only when transportation cost t is not larger than $\frac{3(s_B - c_B)}{2}$.

To conclude this Section, we remark that an interesting aspect concerning coverage of Sector B arises when comparing the two different economies. The parametric interval where full coverage occurs is larger under the mixed economy, $t \leq \frac{3(s_B - c_B)}{2}$, rather than $t \leq s_B - c_B$. In other words, the social service market is more likely to be fully covered under the mixed economy. This is because the sum of equilibrium prices in the social Sector B is lower if the economy is mixed, $p_{0,B}^{**} + p_{1,B}^{**} < p_{0,B}^{*} + p_{1,B}^{*} = 2p_B^{*}$ (see online Appendix A.4), which eases the purchase also for individuals who live far away. By setting its price as low as possible, the social enterprise forces the for-profit rival to reduce its own price in the mixed economy:[3] a side-effect of the presence of a social enterprise is to make competition tougher.

We sum up these findings in the following Proposition (see online Appendix A.5 for a formal proof):

Proposition 1 *(i) When $t \leq s_B - c_B$, all individuals have access to the social service under both types of economy. (ii) When $t \in \left(s_B - c_B, \frac{3(s_B - c_B)}{2}\right]$, all individuals have access to the social service only if the economy is mixed. (iii) When $t > \frac{3(s_B - c_B)}{2}$, there is no full coverage but more individuals have access to the social service under the mixed economy.*

4 Welfare Analysis

This Section compares two levels of welfare: (i) the welfare under the market economy, where competition occurs between for-profit firms; (ii) the welfare under the mixed economy, where firm 0 in Sector B is customer surplus maximizer. We adopt a utilitarian approach by defining welfare as the sum of firms' profits and surplus of all individuals in the two sectors.

Let us first consider the market economy. Since both firms within each sector set the same price p_j^*, at equilibrium, they end with the same profit, which we denote by Π_j^*. Similarly, customers of each firm obtain the same total surplus within each sector, i.e., $CS_{0,j}^* = CS_{1,j}^* = CS_j^*$. See online Appendix A.2 for the mathematical values of Π_j^* and CS_j^*. Summing up, the equilibrium utilitarian welfare is given by $2\Pi_A^* + 2\Pi_B^* + 2CS_A^* + 2CS_B^* + CS_{H,j}$, where we recall that $CS_{H,j} = 0$ denotes the surplus of individuals who do not buy.

[3] It is well known that the prices are strategic complements in Hotelling-like models. If firm $0, B$ reduces its price, firm $1, B$ does the same to maximize profits.

Let us turn our attention to the mixed economy, where the equilibrium in Sector A is as that in the market economy. Each firm makes profit Π_A^* and the surplus of all customers of each firm is CS_A^*. Things are different in Sector B, where a social enterprise is active. Recall that the social enterprise makes zero profit, $\Pi_{0,B}^{**} = 0$. We denote with $\Pi_{1,B}^{**}$ the profit made by firm $1,B$. Similarly, the surplus of all customers of firm $0,B$ and $1,B$ are indicated with $CS_{0,B}^{**}$ and $CS_{1,B}^{**}$. The equilibrium utilitarian welfare in the mixed economy can, thus, be written as $2\Pi_A^* + \Pi_{0,B}^{**} + \Pi_{1,B}^{**} + 2CS_A^* + CS_{0,B}^{**} + CS_{1,B}^{**} + CS_{H,j}$.

To proceed with our welfare analysis, we compare the welfare values arising in the two scenarios, market and mixed, by computing their difference:

$$(2\Pi_B^* + 2CS_B^*) - \left(\Pi_{0,B}^{**} + \Pi_{1,B}^{**} + CS_{0,B}^{**} + CS_{1,B}^{**}\right). \tag{7}$$

The surplus of individuals who do not buy, $CS_{H,j}$, is zero; hence, it does not appear in (7). Similarly, (7) does not depend on the equilibrium values in Sector A, $2\Pi_A^*$ and $2CS_A^*$. These values are indeed equal across both economies, market and mixed. As a consequence, we can disregard Sector A and focus our attention on what happens in Sector B. The role played by Sector A is twofold. On the one hand, the presence of Sector A gives a more complete picture of the real-world economy, where different types of goods and services are supplied; on the other hand, Sector A produces profits that are partially transferred to the social enterprise. This second aspect will be analyzed in Section 6, where we allow the social enterprise to sell below cost.

In Figure 4, we depict the two terms of difference (7) as a function of unit transportation cost t. Welfare in Sector B of the market economy, $2\Pi_B^* + 2CS_B^*$, is represented by the dashed line, while welfare in Sector B of the mixed economy, $\Pi_{0,B}^{**} + \Pi_{1,B}^{**} + CS_{0,B}^{**} + CS_{1,B}^{**}$, is denoted by the solid line.

As is apparent from Figure 4, welfare is larger in the market economy only when unit transportation cost t is relatively low. More precisely, in online Appendix A.6, we prove the following:

Proposition 2 *(i) When $t \leq \frac{6(s_B - c_B)}{5}$ welfare is larger under the market economy. (ii) When $t > \frac{6(s_B - c_B)}{5}$ welfare is larger under the mixed economy.*

First note that both lines in Figure 4 are negatively affected by t. As transportation cost t increases welfare decreases.[4] We discuss the results of Proposition 2 by considering the relevant intervals of t separately.

[Figure 4 here]

[4] The only exception is given by the solid line in interval $\frac{4(s_B - c_B)}{3} \leq t \leq \frac{3(s_B - c_B)}{2}$, which exhibits an inverted-U relationship with t. The intuition for this technical result is given in online Appendix A.6.

When $t \leq s_B - c_B$, there is full coverage of Sector B under both types of economy according to Proposition 1. Equilibrium prices increase when moving from a mixed to a market system. This has a positive effect on firms' profits and a negative effect on individuals' surplus in Sector B because the sum of equilibrium prices in Sector B is larger under the market economy. In symbols, $2\Pi_B^* > \Pi_{0,B}^{**} + \Pi_{1,B}^{**}$ and $2CS_B^* < CS_{0,B}^{**} + CS_{1,B}^{**}$. This trade-off is standard when prices increase. More interestingly, the profit gain turns out to be larger than surplus loss (in absolute value). In symbols, $2\Pi_B^* - \left(\Pi_{0,B}^{**} + \Pi_{1,B}^{**}\right) > \left(CS_{0,B}^{**} + CS_{1,B}^{**}\right) - 2CS_B^*$. As a result, the welfare is greater under the market economy. This is because customers incur lower total transportation costs: see online Appendix A.6 for a formal proof. Indeed, such costs are minimized when the indifferent individual is located in the middle of the segment, such that firm $0, B$ $(1, B)$ serves the left (right) half of the market. This is what occurs in the market economy, where firms set the same equilibrium price in Sector B, p_B^*. Plugging $p_{0,B}^* = p_{1,B}^* = p_B^*$ into the location of the indifferent individual (5) with $j = B$ yields $x_{I,B} = \frac{1}{2}$. By contrast, the social enterprise $0, B$ sets a lower price than the rival in Sector B of the mixed economy, $p_{0,B}^{**} < p_{1,B}^{**}$. Plugging $p_{0,B}^{**} < p_{1,B}^{**}$ into (5) with $j = B$ yields $x_{I,B} > \frac{1}{2}$. As a result, the indifferent individual lies closer to firm $1, B$ and total transportation costs become larger.

When $s_B - c_B < t < \frac{4(s_B - c_B)}{3}$, the full coverage of Sector B occurs only in the mixed system according to Proposition 1. Figure 4 shows the downward-sloping dashed line is steeper than the downward-sloping solid line, meaning that the negative effect of t on the welfare of the market economy is larger than that on the welfare of the mixed economy. This is due to the reduction in demand in the market economy. Welfare becomes greater under the mixed economy at $t = \frac{6(s_B - c_B)}{5}$. This result is confirmed in interval $\frac{4(s_B - c_B)}{3} \leq t \leq \frac{3(s_B - c_B)}{2}$, where again the full coverage of Sector B occurs only in the mixed system.

Finally, when $t > \frac{3(s_B - c_B)}{2}$ there is partial coverage under both systems, but demand is greater in the mixed economy according to Proposition 1. As a consequence, welfare is enhanced under the mixed economy.

Summing up, the full coverage of the social service market is more likely to occur in the mixed economy because the equilibrium prices are lower. Yet, when t is relatively low, full coverage occurs under the market economy as well. In this case, welfare is enhanced when all firms are profit maximzіers because individuals bear lower total transportation costs. As t increases, instead, welfare becomes larger in the mixed economy because a greater fraction of individuals have access to the social service.

5 Extension I: t as an Ideological Cost

In line with the original Hotelling (1929) framework, the segment denotes a physical space and parameter t denotes a transportation cost in the above analysis. In this Section, we check the robustness of our findings by proposing an alternative interpretation.

We assume that Sector A, where only for-profit companies operate, is still represented by a physical space. Instead, we disregard transportation costs in the social service Sector B and suppose that the unit segment represents a space of firm types. More precisely, a hypothetical firm located at point $x \in [0, 1]$ is assumed to maximize the following objective function: a convex combination of its profits and surplus of its customers, where x is the weight attached to profits and $1-x$ is attached to customers' surplus. Accordingly, a social enterprise attaching maximum weight to customers' surplus is located at the extreme left of the unit segment, $x = 0$. By contrast, a for-profit firm lies on the extreme right, $x = 1$, because it puts weight 1 on its profits.

Similarly, the location of individuals along the segment denotes their ideological position towards firm types. The ideal type for an individual located in $x \in [0, 1]$ consists in a firm attaching weight x to its profits and weight $1 - x$ to the surplus of its customers. Thus, this individual incurs the ideological cost tx when buying from a social enterprise located at 0 and $t(1 - x)$ when buying from a for-profit firm located at 1, where t denotes the per unit of distance cost to fill the ideological distance between an individual's ideal type of firm and the actual type she/he buys from. This is an example of single-peaked preferences in the spirit of the median voter framework. One might think of an individual who takes into account both the social responsibility of companies and commercial and business aspects. If the individual gives higher importance to the former (latter), her/his ideological location is closer to the social enterprise (for-profit firm).

This alternative interpretation of our framework does not affect the strategic interaction in Sector B of the mixed economy. Indeed, the two rivals are still located at the extremes of the segment, the social enterprise $0, B$ at $x = 0$ and the for-profit firm $1, B$ at $x = 1$. Accordingly, the mixed economy equilibrium is as that described in Section 3.

By contrast, the strategic interaction in Sector B of the market economy is affected because the two rivals are profit maximziers and, therefore, both located at $x = 1$, rather than lying at the extremes of the segment. One can easily check that, given the same extreme-right location of the two firms, their services are not horizontally differentiated, hence, their strategic behavior boils down to Bertrand competition, where both firms set the equilibrium price equal to the unit production cost. In symbols, $p_B^I = c_B$, where superscript I stands for ideological.

Interestingly, in online Appendix A.7 we prove what follows. First, the full coverage of Sector B (of the market economy) occurs only if $t \leq s_B - c_B$, as in Proposition 1. Second, when $t > s_B - c_B$, the share of individuals who have access to the social service does not change in comparison with the result for the standard Hotelling framework. These two findings may appear surprising since individuals pay a lower price for the social service, $p_B^I < p_B^*$. Yet, they also incur larger ideological costs when they decide to buy because all firms are located at the extreme right of the segment. These two opposite effects compensate each other. Finally, equilibrium welfare in Sector B of the market economy, which we denote with $2\Pi_B^I + 2CS_B^I$, is equal to $2\Pi_B^* + 2CS_B^*$, i.e., its value is not affected by the interpretation of parameter t as an ideological cost. The intuition is as follows. The two for-profit firms charge a lower equilibrium price than that set in the standard Hotelling framework, $p_B^I = c_B < p_B^*$. This affects negatively their profits, $2\Pi_B^I < 2\Pi_B^*$, and positively the surplus of their customers, $2CS_B^I > 2CS_B^*$.[5] These two opposite effects compensate each other.

Bearing in mind that the mixed economy equilibrium is as that described in Section 3, we can write the following

Proposition 3 *When parameter t denotes an ideological rather than a transportation cost, the results of Propositions 1 and 2 stand.*

The above Proposition proves that our findings are robust to the alternative specification of parameter t as an ideological cost.

6 Extension II: Mixed Economy with Transfers

In this Section, we enrich our analysis by considering an alternative form of mixed economy, where the social enterprise in Sector B is allowed to set the price below its production cost. Accordingly, we modify the timing of events introduced in Section 2 by assuming that at $t = 1$, the social enterprise receives a lump-sum transfer k on top of the profits realized. Thus, we refer to this system as a *mixed economy with transfers.* The amount k is taken from profits of firms operating in Sector A and can be thought of as a non-distortionary lump-sum tax paid by the for-profits to subsidize the social enterprise.[6]

First, note that the strategic behavior of the two for-profit firms in Sector A is not affected since the transfer k is lump-sum. Equilibrium profits of each firm in Sector A are, thus, denoted by Π_A^* minus the transfer to the social enterprise. Similarly, the equilibrium surplus of customers of each firm is still equal to CS_A^*.

[5] For the sake of precision, there are two effects on individuals' surplus. On the one hand, it is increased by the reduction in prices. On the other hand, it is reduced by the larger ideological costs borne by individuals. Indeed, all individuals located in $x \in \left[0, \frac{1}{2}\right)$ have to "travel ideologically" more than half of the segment when they decide to buy. The positive effect of lower prices is shown to prevail over the negative one of higher ideological costs, with the effect that individuals' surplus increases in comparison with that in the market economy, where t denotes transportation costs.

[6] Including rival firm $1, B$'s profits as a source of transfers to the nonprofit firm would complicate the computations without adding any additional insight.

By contrast, in Sector B, the social enterprise $0, B$ solves a new problem. It still aims at maximizing the surplus of its customers, $CS_{0,B}$, but subject to a different constraint. Given that firm $0, B$ is now allowed to set the price below cost, the break-even constraint $\Pi_{0,B} \geq 0$ is substituted with a price non-negativity constraint, $p_{0,B} \geq 0$.

In online Appendix A.8, we show that the equilibrium in Sector B takes the following features. First, the social enterprise sets the equilibrium price, denoted by $p^{\circ}_{0,B}$, as low as possible, i.e., equal to 0. This price is lower than that in the mixed economy (with no transfers): $p^{\circ}_{0,B} = 0 < p^{**}_{0,B} = c_B$. Second, the equilibrium price set by the for-profit firm $1, B$, denoted by $p^{\circ}_{1,B}$, is higher than $p^{\circ}_{0,B}$. Finally, the full coverage of Sector B occurs only when transportation cost t is not larger than $\frac{3s_B - c_B}{2}$.

Following the analysis in Section 3, we are interested in comparing the coverage of Sector B of the mixed economy with transfers vis-a-vis the coverage of Sector B of the market economy. To this effect we first verify that the sum of equilibrium prices in the mixed economy with transfers is lower than that in the market economy, $p^{\circ}_{0,B} + p^{\circ}_{1,B} < 2p^{*}_{B}$. The intuition for this result is as follows. By setting its price equal to zero, the social enterprise forces the for-profit rival to reduce its own price in the mixed economy. As a result, the social service market is more likely to be fully covered under the mixed economy with transfers. More precisely, in online Appendix A.9, we prove the following:

Proposition 4 *(i) When $t \leq s_B - c_B$, all individuals have access to the social service under both types of economy, market and mixed with transfers. (ii) When $t \in \left(s_B - c_B, \frac{3s_B - c_B}{2}\right]$, all individuals have access to the social service only under the mixed economy with transfers. (iii) When $t > \frac{3s_B - c_B}{2}$, there is no full coverage but more individuals have access to the social service under the mixed economy with transfers.*

The findings of Proposition 4 are similar to those of Proposition 1. Yet, note that $\frac{3s_B - c_B}{2} > \frac{3(s_B - c_B)}{2}$, which implies that the parametric interval in which partial coverage occurs only under the market economy enlarges from $\left(s_B - c_B, \frac{3(s_B - c_B)}{2}\right]$ to $\left(s_B - c_B, \frac{3s_B - c_B}{2}\right]$. This is because the social enterprise sets an even lower price when transfers are allowed, 0 instead of c_B, thus further easing the purchase for individuals who live far away. In other words, a side-effect of allowing for transfers to the social enterprise is to make price competition even tougher: the coverage of the social service sector is further enhanced under the mixed economy with transfers.

We proceed by providing a welfare analysis, as in Section 4. To this aim, we compute the welfare arising in the mixed economy with transfers. The overall profits made by the firms are denoted with $(2\Pi^{*}_{A} - k) + \left(\Pi^{\circ}_{0,B} + k\right) + \Pi^{\circ}_{1,B}$. Recall that the amount k denotes the lump-sum transfer from for-profit firms i, A to the social enterprise $1, B$. Instead, the surplus of all individuals is denoted with $2CS^{*}_{A} + CS^{\circ}_{0,B} + CS^{\circ}_{1,B} + CS_{H,j}$. See online

Appendix A.8 for the mathematical values of $\Pi^{\circ}_{0,B}$, $\Pi^{\circ}_{1,B}$, $CS^{\circ}_{0,B}$, and $CS^{\circ}_{1,B}$. Instead, recall that $CS_{H,j} = 0$ denotes the surplus of individuals who do not buy. Summing up, the equilibrium welfare in the mixed economy with transfers is given by $2\Pi^*_A + \Pi^{\circ}_{0,B} + \Pi^{\circ}_{1,B} + 2CS^*_A + CS^{\circ}_{0,B} + CS^{\circ}_{1,B}$.

With the aim of making a welfare comparison, we write the difference in welfare between a market economy and mixed economy with transfers,

$$(2\Pi^*_B + 2CS^*_B) - \left(\Pi^{\circ}_{0,B} + \Pi^{\circ}_{1,B} + CS^{\circ}_{0,B} + CS^{\circ}_{1,B}\right), \tag{8}$$

and, to simplify the reading of our results, we introduce the following notation:

$$t^{\circ} = \begin{cases} \frac{4s_B - c_B}{3} & \text{if} \quad c_B \in \left[0, \frac{3\sqrt{2}-1}{17} s_B\right], \\ \frac{4s_B - 2c_B - \sqrt{2s_B^2 - 4s_B c_B - 2c_B^2}}{2} & \text{if} \quad c_B \in \left(\frac{3\sqrt{2}-1}{17} s_B, \frac{s_B}{3}\right). \end{cases} \tag{9}$$

In online Appendix A.10, we prove the following

Proposition 5 *(i) When $c_B \leq \frac{s_B}{3}$, welfare is larger (lower) under the market economy if and only if $t \leq (>)\, t^{\circ}$. (ii) When $\frac{s_B}{3} \leq c_B \leq s_B$, welfare is larger under the market economy for any t.*

We depict t° in plane (c_B, t) with $c_B \in [0, s_B)$ to illustrate the results of Proposition 5: see Figure 5.

[Figure 5 here]

As in Proposition 2, the market economy enhances welfare for relatively low values of t (i.e., in the south portion of plane (c_B, t)), in which case all individuals have access to the social service under both types of economy. Again, this result emerges because total transportation costs are minimized under the market economy: see online Appendix A.10 for a formal proof.

Differently from Proposition 2, for relatively large values of t, the mixed economy with transfers enhances welfare only if the unit production cost c_B is low relative to s_B (i.e., in the shaded area of Figure 5). In other words, when transfers are allowed, the area in which the mixed economy enhances the welfare reduces from $t > \frac{6(s_B - c_B)}{5}$ and $c_B < s_B$ to $t > t^{\circ}$ and $c_B \leq \frac{s_B}{3}$, with $t^{\circ} > \frac{6(s_B - c_B)}{5}$.

To understand why, note that the coverage of Sector B is strictly greater under the mixed economy with transfers when $t > t^{\circ}\left(\geq \frac{4s_B - c_B}{3}\right)$. The reason why larger coverage enhances welfare only if c_B is relatively low is as follows. Recalling that the social enterprise $0, B$ sets $p^{\circ}_{0,B} = 0$, we plug such a value into the profit function (2) and get $\Pi^{\circ}_{0,B} = -c_B D^{\circ}_{0,B}$, where $D^{\circ}_{0,B}$ denotes the equilibrium demand of firm $0, B$: see online Appendix A.9 for further details. The before-transfer profits $\Pi^{\circ}_{0,B}$ made by firm $0, B$ are unsurprisingly negative. If c_B is relatively low, loss $\Pi^{\circ}_{0,B}$ is relatively low as well. Only in this case, the welfare turns out to be larger under the mixed economy with transfers because the surplus gain enjoyed by individuals when moving from the market to the mixed economy with transfers outdoes the profit loss (in absolute value) incurred by firm $0, B$: see online Appendix A.10 for a formal proof.

We sum up the results of this Section. An interesting trade-off arises when the social enterprise in Sector B is allowed to sell below cost. On the one hand, the coverage of the social service market is further enhanced. This happens because the social enterprise sets an even lower price than in the mixed economy without transfers. On the other hand, the parametric area, where welfare is lower under the market economy, shrinks. This is because the before-transfer losses incurred by the social enterprise negatively affects welfare.

7 Policy Implications and Conclusion

In this paper, we investigated the impact of social enterprises on the individuals' access to social services and the level of utilitarian welfare. To this end, two different economic systems have been compared. A market economy where all firms are profit maximziers; a mixed economy where one firm, a social enterprise, maximizes the surplus of its customers and competes with a for-profit firm in the supply of a social service. Our results have potentially relevant policy implications, which we detail in the following paragraphs.

In the basic setup of Sections 2, 3, and 4, no ideological costs have been considered. Monetary and transportation costs and prices are the only determinants of individuals' and suppliers' trading behavior. Even in this stylized framework, the presence of a social enterprise has been shown to have a positive impact. Indeed, Proposition 1 ensures that individuals are more likely to have access to the social service under the mixed economy. Public policies should, therefore, promote the entry of social enterprises given the likely positive impact on individuals' surplus.

Interestingly, Proposition 2 ensures that the utilitarian welfare - defined as the sum of profits of all firms and surplus of all individuals - is larger under the mixed economy when transportation costs are relatively high. An important implication of this result is that conditions exist under which the presence of social enterprises turns out to be not only effective, because of the enhanced access to the social service, but also efficient. Put differently, no efficiency/effectiveness trade-off arises here: governments should encourage the entry of social enterprises in social services sectors where individuals and families bear large transportation costs. However, when transportation costs are less relevant, market economies are more efficient, even if less effective. In this case, public policies encouraging the entry of social enterprises should be adopted only by governments whose main aim is to enable more people to access the social service, i.e., enhance the redistributive effect of social enterprises.

The first extension of the basic model helped explain the importance of mixed economies when ideological aspects drive individuals' choice between the different types of firms supplying the social service. Interestingly, Proposition 3 confirms the results of Propositions 1 and 2. This means that mixed economies are both more effective and efficient than market economies when ideological costs are relatively high, i.e., when individuals' preferences for different types of firms are particularly heterogeneous. However, when individuals care less about the type of firms, mixed economies are still more effective in terms of enhanced access to the service but less efficient than market economies. Implications for public policies are, mutatis mutandis, as above.

The second extension of the model investigated the case of a mixed economy with transfers, where governments play an active role by transferring monetary resources from for-profit firms to social enterprises. An interesting trade-off has been shown to arise. On the one hand, Proposition 4 states that the coverage of the social service market is further enhanced. This means that the presence of subsidized social enterprises in the marketplace is even more effective and their redistributive impact is magnified. On the other hand, Proposition 5 states that the parametric area in which utilitarian welfare is lower under the market economy shrinks. In other words, the possible negative impact of mixed economies on efficiency is exacerbated.

In this context, one can think of alternative policies that encourage voluntary transfers to social enterprises rather than imposing (coercive) taxation on for-profit firms. It has been argued (e.g., Rose-Ackerman, 1996) that an individual's overall utility is not negatively affected in case she/he decides to donate. In other words, the amount of a donation positively enters into the utilitarian welfare function. This means that relying on voluntary contributions to social enterprises rather than, for example, taxation on for-profit firms may increase the mixed economy effectiveness without compromising efficiency.

In conclusion, we remark that the issue of encouraging donations and citizens' involvement is extremely up-to-date: see, for example, some of the contents of the Social Business Initiative (European Commission, 2011). This issue is one of the main pillars of a "Big Society", where people participate in the creation and management of social enterprises and where an increasing "organizational biodiversity" in the marketplace might positively affect economic efficiency.

References

[1] L. Becchetti, A. Palestini, N. Solferino, and E. Tessitore (2014) 'The socially responsible choice in a duopolistic market: A dynamic model of "ethical product" differentiation', *Economic Modelling*, 43(C): 114-123.

[2] Beckford, J.A. (1991), 'Great Britain: Voluntarism and Sectional Interests' in R. Wuthnow (ed.) *Between States and Markets: The Voluntary Sector in Comparative Perspective*, Princeton, NJ: Princeton University Press, 30-63.

[3] Ben-Ner, A. (2002), 'The shifting boundaries of the mixed economy and the future of the nonprofit sector', *Annals of Public and Cooperative Economics*, 73:1, 5-40.

[4] Borzaga, C. and Defourny, J. (2001), *The Emergence of Social Enterprises*, Routledge, London.

[5] CIRIEC (2007) The social economy in the European Union, N°. CESE/COMM/05/2005 *The European Economic and Social Committee (EESC).*

[6] De Fraja, G. and Delbono, F. (1990), 'Game theoretic Models of mixed Oligopoly', *Journal of Economic Surveys*, 4:1, 1-17.

[7] Delbono, F. and Reggiani, C. (2013), 'Cooperative firms and the crisis: evidence from some Italian mixed oligopolies', *Annals of Public and Cooperative Economics*, 84:4, 383-397.

[8] European Commission (2011), Social Business Initiative, COM(2011) 682, http://ec.europa.eu/internal_market/social_business/ docs/COM2011_682_en.pdf

[9] European Commission (2013a) Social Economy and Social Entrepeneurship, *Social Europe Guide* 2013 vol.4, Luxembourg: Publications Office of the European Union http://ec.europa.eu/social/main.jsp?catId=738&langId=it&pubId=7523&type=2&furtherPubs=yes

[10] European Commission (2013b) Social policies, *Social Europe Guide* 2013 vol.5, Luxembourg: Publications Office of the European Union http://ec.europa.eu/social/main.jsp?catId=738&langId=en&pubId=7683

[11] Kazepov, Y. (2009), 'The subsidiarization of social policies: processes and impacts', *European Societies*, 10:2, 247-273.

[12] Kerlin, J.A. (2006), 'Social Enterprise in the United States and Europe: Understanding and Learning from the Differences', *Voluntas*, 17, 247-263.

[13] Marini, M.A., Polidori, P., Teobaldelli, D. and Zevi, A. (2015), 'Welfare enhancing coordination in consumer cooperatives under mixed oligopoly', *Annals of Public and Cooperative Economics*, 86:3, 505-527

[14] Marwell N.P. and McInerney P.B. (2005), 'The Nonprofit/For-Profit Continuum: Theorizing the Dynamics of Mixed-Form Markets', *Nonprofit and Voluntary Sector Quarterly*, 34:1, 7-28.

[15] Ostrander, S.A. (1989), 'Private Social Services: Obstacles to the Welfare State?', *Nonprofit and Voluntary Sector Quarterly*, 18:1, 25-45.

[16] Rose-Ackerman, S. (1996), 'Altruism, Nonprofits, and Economic Theory', *Journal of Economic Literature*, 34:2, 701-728.

[17] Salamon, L.M. (2006), 'Government-nonprofit relations from an international perspective', in J. Steven Ott and L. Dicke (eds.), *The Nature of the Nonprofit Sector*, Westview Press, USA, 367-384.

[18] Salamon, L.M. and Anheier, H.K. (1997), Defining the nonprofit sector. A cross-national analysis, *John Hopkins Nonprofit Sector Series*, Manchester University Press, Manchester, UK.

[19] Stiglitz, J. (2009), 'Moving beyond market fundamentalism to a more balanced economy', *Annals of Public and Cooperative Economics*, 80:3, 345-360.

[20] Te'eni D. and Young D.R. (2003) 'The Changing Role of Nonprofits in the Network Economy', *Nonprofit and Voluntary Sector Quarterly*, 32:3, 397-414.

[21] Van Dyne, L. and Pierce, J.L. (2004), 'Psychological ownership and feelings of possession: three field studies predicting employee attitudes and organizational citizenship behavior', *Journal of Organizational Behavior*, 25:4, 439-459.

A Online Appendix

A.1 Individuals' Surplus

Denoting with $D_{0,j} = [0, x_{0,j}]$ and $D_{1,j} = [x_{1,j}, 1]$, $x_{0,j} \leq x_{1,j}$, the demand shares of firms $0, j$ and $1, j$, $x_{0,j}$ ($x_{1,j}$) being the furthest individual who buys from firm $0, j$ $(1, j)$, the surplus of firm $0, j$'s customers can be computed as follows:

$$CS_{0,j} = \int_0^{x_{0,j}} (s - p_{0,j} - ty)\, dy = x_{0,j} \left(s - p_{0,j} - \frac{t}{2} x_{0,j} \right). \tag{10}$$

The surplus of firm $1, j$'s customers is

$$CS_{1,j} = \int_{x_{1,j}}^{1} [s - p_{1,j} - t(1-y)]\, dy = (1 - x_{1,j}) \left[s - p_{1,j} - \frac{t}{2} (1 - x_{1,j}) \right]. \tag{11}$$

A.2 Market Economy Equilibrium

Since all firms choose prices in order to maximize profit $\Pi_{i,j}$ and are symmetric within each Sector $j = A, B$, we can consider firm $0, j$ as the representative one and study its strategic behavior. Firm $0, j$ solves the following program

$$\begin{gathered} \max_{p_{0,j}, x_{0,j}} (p_{0,j} - c_j)\, x_{0,j} \\ \text{s.t. } x_{0,j} \leq x_{I,j} \text{ and } s_j - p_{0,j} - t x_{0,j} \geq 0, \end{gathered} \tag{12}$$

The objective function of firm $0, j$ is its profit. Recalling that firm $0, j$ lies at the leftmost point of the unit segment, the first constraint in (12) ensures the furthest customer of firm $0, j$ is at most the indifferent individual located at $x_{I,j}$, see (5). When such a constraint is binding, all individuals buy either from firm $0, j$ or firm $1, j$: there is full coverage as in Figure 2. In symbols, $x_{0,j} = x_{I,j}$ (and $x_{I,j} = x_{1,j}$ for symmetry of firms). Instead, when the constraint is not active, $x_{0,j} < x_{I,j}$ (and $x_{I,j} < x_{1,j}$ for symmetry of firms), there is no full coverage as in Figure 3. The second constraint in (12) requires that all customers of firm $0, j$ get a non-negative surplus.

By virtue of (5), firm $0, j$ problem (12) can be rewritten as

$$\begin{gathered} \max_{p_{0,j}, x_{0,j}} \Pi_{0,j} = (p_{0,j} - c_j)\, x_{0,j} \\ \text{s.t. } x_{0,j} \leq \tfrac{1}{2} + \tfrac{p_{1,j} - p_{0,j}}{2t} \text{ and } x_{0,j} \leq \tfrac{s_j - p_{0,j}}{t}. \end{gathered} \tag{13}$$

The Lagrangian is

$$\mathcal{L} = (p_{0,j} - c_j)\, x_{0,j} - \lambda \left(x_{0,j} - \frac{p_{1,j} - p_{0,j} + t}{2t} \right) - \mu \left(x_{0,j} - \frac{s_j - p_{0,j}}{t} \right). \tag{14}$$

FOCs are

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial p_{0,j}} = x_{0,j} - \frac{\lambda}{2t} - \frac{\mu}{t} = 0, \\ \frac{\partial \mathcal{L}}{\partial x_{0,j}} = p_{0,j} - c_j - \lambda - \mu = 0. \end{cases} \tag{15}$$

Since $\Pi_{0,j}$ increases with $x_{0,j}$ at least one of the two constraints must be binding at a solution to (13). We thus study three alternative scenarios.

1) $\mu = 0$ and $\lambda > 0$, then only the first constraint in (13) is binding, *i.e.*,

$$\frac{p_{1,j} - p_{0,j} + t}{2t} < \frac{s_j - p_{0,j}}{t}. \tag{16}$$

This means that the market is fully covered and that the indifferent individual gets positive surplus when buying. Plugging $\mu = 0$ into the FOCs yields

$$\begin{cases} x_{0,j} = \frac{\lambda}{2t}, \\ p_{0,j} - c_j = \lambda, \end{cases} \tag{17}$$

whereby $x_{0,j} = \frac{p_{0,j}-c_j}{2t}$. Solving

$$\begin{cases} x_{0,j} = \frac{p_{0,j}-c_j}{2t}, \\ x_{0,j} = \frac{p_{1,j}-p_{0,j}+t}{2t}, \end{cases} \tag{18}$$

by $p_{0,j}$ and $x_{0,j}$ yields

$$p_{0,j} = \frac{p_{1,j} + c_j + t}{2} \text{ and } x_{0,j} = \frac{p_{1,j} + t - c_j}{4t}. \tag{19}$$

Plugging $p_{0,j} = \frac{p_{1,j}+c_j+t}{2}$ into (16) yields

$$p_{1,j} < \frac{4s_j - 3t - c_j}{3}. \tag{20}$$

Finally, note that λ is positive iff $p_{0,j} - c_j > 0$. Substituting $p_{0,j} = \frac{p_{1,j}+c_j+t}{2}$ yields

$$p_{1,j} > c_j - t. \tag{21}$$

2) $\mu \geq 0$ and $\lambda \geq 0$, then both constraints are binding, *i.e.*, $\frac{p_{1,j}-p_{0,j}+t}{2t} = \frac{s_j-p_{0,j}}{t}$ or

$$p_{0,j} = 2s_j - t - p_{1,j} \text{ and } x_{0,j} = \frac{t + p_{1,j} - s_j}{t}. \tag{22}$$

This means that the market is fully covered and that the indifferent individual gets zero surplus when buying. FOCs are as in (15), whereby

$$\begin{cases} \mu = 2t\frac{t+p_{1,j}-s_j}{t} - (2s_j - t - p_{1,j}) + c_j, \\ \lambda = 2\left(2s_j - t - p_{1,j}\right) - 2c_j - 2t\frac{t+p_{1,j}-s_j}{t}. \end{cases} \tag{23}$$

after plugging (22). Both μ and λ must be non-negative:

$$\frac{4s_j - c_j - 3t}{3} \leq p_{1,j} \leq \frac{3s_j - c_j - 2t}{2}. \tag{24}$$

3) $\mu > 0$ and $\lambda = 0$, then only the second constraint in (13) is binding, *i.e.*,

$$\frac{p_{1,j} - p_{0,j} + t}{2t} > \frac{s_j - p_{0,j}}{t}. \tag{25}$$

This implies that firm 0 is local monopolist. Plugging $\mu > 0$ and $\lambda = 0$ into the FOCs yields

$$\begin{cases} x_{0,j} - \frac{\mu}{t} = 0, \\ p_{0,j} - c_j - \mu = 0, \end{cases} \tag{26}$$

whereby $\mu = p_{0,j} - c_j = tx_{0,j}$. Taking into account the constraint $x_{0,j} = \frac{s_j-p_{0,j}}{t}$ we have

$$p_{0,j} = \frac{c_j + s_j}{2} \text{ and } x_{0,j} = \frac{\frac{c_j+s_j}{2} - c_j}{t} = \frac{s_j - c_j}{2t}. \tag{27}$$

Substituting $p_{0,j} = \frac{c_j+s_j}{2}$ into (25) yields

$$p_{1,j} > \frac{3s_j - 2t - c_j}{2}. \tag{28}$$

Finally, we check that $\mu > 0$, *i.e.*, $p_{0,j} - c_j = tx_{0,j} > 0$: $\frac{s_j-c_j}{2} > 0$, which holds true given that $s_j > c_j$.

Summing up the three scenarios yields firm 0 best response:

$$p_{0,j} = \begin{cases} \frac{p_{1,j}+c_j+t}{2} & \text{if } c_j - t \le p_{1,j} < \frac{4s_j-3t-c_j}{3}, \\ 2s_j - t - p_{1,j} & \text{if } \frac{4s_j-c_j-3t}{3} \le p_{1,j} \le \frac{3s_j-c_j-2t}{2}, \\ \frac{c_j+s_j}{2} & \text{if } p_{1,j} > \frac{3s_j-2t-c_j}{2}, \end{cases} \tag{29}$$

and

$$x_{0,j} = \begin{cases} \frac{p_{1,j}+t-c_j}{4t} & \text{if } c_j - t \le p_{1,j} < \frac{4s_j-3t-c_j}{3}, \\ \frac{t+p_{1,j}-s_j}{t} & \text{if } \frac{4s_j-c_j-3t}{3} \le p_{1,j} \le \frac{3s_j-c_j-2t}{2}, \\ \frac{s_j-c_j}{2t} & \text{if } p_{1,j} > \frac{3s_j-2t-c_j}{2}. \end{cases} \tag{30}$$

The symmetric equilibrium is hence as follows.

a) $p_j^* = \frac{p_j^*+c_j+t}{2}$, hence $p_j^* = c_j + t$ and $x_{0,j}^* = \frac{c_j+t+t-c_j}{4t} = \frac{1}{2}$. The corresponding interval becomes $c_j - t < c_j + t < \frac{4s_j-3t-c_j}{3}$ or

$$t < \frac{2}{3}(s_j - c_j). \tag{31}$$

b) $p_j^* = 2s_j - t - p_j^*$, hence $p_j^* = s_j - \frac{t}{2}$ and $x_{0,j}^* = \frac{-s_j+t+s_j-\frac{t}{2}}{t} = \frac{1}{2}$. The corresponding interval becomes $\frac{4s_j-c_j-3t}{3} \le \frac{2s_j-t}{2} \le \frac{3s_j-c_j-2t}{2}$ or

$$\frac{2s_j - 2c_j}{3} \le t \le s_j - c_j. \tag{32}$$

c) If $p_{1,j} > \frac{3s_j-2t-c_j}{2}$ we have local monopolies. Since $p_j^* = \frac{c_j+s_j}{2}$ and $x_j^* = \frac{s_j-c_j}{2t}$, the relevant interval is $\frac{c_j+s_j}{2} > \frac{3s_j-2t-c_j}{2}$ or $t > s_j - c_j$.

We can conclude that in the market economy the symmetric equilibrium of each industrial Sector $j = A, B$ takes the following features. The equilibrium prices are $p_{0,j}^* = p_{1,j}^* = p_j^*$, where

$$p_j^* = \begin{cases} t + c_j & \text{if } t < \frac{2}{3}(s_j - c_j), \\ s_j - \frac{t}{2} & \text{if } \frac{2}{3}(s_j - c_j) \le t \le s_j - c_j, \\ \frac{s_j+c_j}{2} & \text{if } t > s_j - c_j, \end{cases} \tag{33}$$

The equilibrium demands are instead $D_{0,j}^* = D_{1,j}^* = D_j^*$, where

$$D_j^* = \begin{cases} \frac{1}{2} & \text{if } t \le s_j - c_j, \\ \frac{s_j-c_j}{2t} & \text{if } t > s_j - c_j. \end{cases} \tag{34}$$

Before explaining the above result, we compute the equilibrium values of firms' profits and individuals' surplus. Plugging (33) and (34) into (2) gives the symmetric equilibrium profits $\Pi_{0,j}^* = \Pi_{1,j}^* = \Pi_j^*$ of each firm in the two sectors,

$$\Pi_j^* = \begin{cases} \frac{t}{2} & \text{if } t < \frac{2}{3}(s_j - c_j), \\ \frac{2s_j-t-2c_j}{4} & \text{if } \frac{2}{3}(s_j - c_j) \le t \le s_j - c_j, \\ \frac{(s_j-c_j)^2}{4t} & \text{if } t > s_j - c_j. \end{cases} \tag{35}$$

Similarly, substituting (33) and (34) into (10) and (11) yields the symmetric equilibrium individuals' surplus $CS^*_{0,j} = CS^*_{1,j} = CS^*_j$ in the two sectors,

$$CS^*_j = \begin{cases} \frac{4s_j-5t-4c_j}{8} & \text{if } t < \frac{2}{3}\left(s_j - c_j\right), \\ \frac{t}{8} & \text{if } \frac{2}{3}\left(s_j - c_j\right) \le t \le s_j - c_j, \\ \frac{(s_j-c_j)^2}{8t} & \text{if } t > s_j - c_j. \end{cases} \tag{36}$$

We discuss the market economy equilibrium results by studying the three relevant intervals of t separately. When the transportation costs are relatively low, $t < \frac{2}{3}\left(s_j - c_j\right)$, only the first constraint in (12) is binding. This means that all individuals purchase and that the indifferent individual gets positive surplus. In that case, the equilibrium price $p^*_j = t + c_j$ is increasing in t but the equilibrium demand $D^*_j = \frac{1}{2}$ is unaffected because larger transportation costs make customers more captive, giving firms larger market power. As a result, firms' profits $\Pi^*_j = \frac{t}{2}$ (individuals' surplus $CS^*_j = \frac{4s_j-5t-4c_j}{8}$) are positively (is negatively) affected by t.

When transportation costs are larger, $\frac{2(s_j-c_j)}{3} \le t \le s_j - c_j$, both constraints of program (12) are binding, which implies that there is still full coverage, but the indifferent individual gets zero surplus. In that case, the equilibrium price $p^*_j = s_j - \frac{t}{2}$ becomes decreasing in t. The intuition is as follows. Plugging $x_{0,j} = x_{I,j}$ into $s_j - p_{0,j} - tx_{0,j} = 0$ with $p^*_{0,j} = p^*_{1,j} = p^*_j$ yields the zero-surplus condition of the indifferent individual,

$$\sigma_{I,j} = s_j - \frac{t}{2} - \frac{p^*_{0,j} + p^*_{1,j}}{2} = s_j - \frac{t}{2} - p^*_j = 0. \tag{37}$$

As t increases, p^*_j must decrease in order for (37) to be fulfilled. Since the equilibrium demand $D^*_j = \frac{1}{2}$ is instead unaffected, firms' profits $\Pi^*_j = \frac{2s_j-t-2c_j}{4}$ (individuals' surplus $CS^*_j = \frac{t}{8}$) are negatively (is positively) affected by t.

Finally, for relatively high transportation costs, $t > s_j - c_j$, only the second constraint in (12) is active. In that case, Sector j is not fully covered, $D^*_{0,j} + D^*_{1,j} = \frac{s_j-c_j}{t} < 1$. Put differently, a fraction $1 - \frac{s_j-c_j}{t}$ of individuals is not served. Note that the equilibrium price $p^*_j = \frac{s_j+c_j}{2}$ does not depend on t. Yet both $\Pi^*_j = \frac{(s_j-c_j)^2}{4t}$ and $CS^*_j = \frac{(s-c_j)^2}{8t}$ decrease with t because the demand $D^*_j = \frac{s_j-c_j}{2t}$ is negatively affected.

A.3 Mixed Economy Equilibrium

In the mixed economy the symmetric equilibrium of Sector A is as in Appendix A.2. By contrast, in Sector B firm $0, B$ solves the following problem:

$$\begin{gathered} \max_{p_{0,B}, x_{0,B}} x_{0,B}\left(s_B - p_{0,B} - \tfrac{t}{2}x_{0,B}\right) \\ \text{s.t.} \\ x_{0,B} \le x_{I,B}, \\ s_B - p_{0,B} - tx_{0,B} \ge 0, \\ \Pi_{0,B} = \left(p_{0,B} - c_j\right) D_{0,B} \ge 0. \end{gathered} \tag{38}$$

The first two constraints are as in (12), with $j = B$, while the third constraint ensures that firm $0, B$ profits are non-negative. The objective function consists in the surplus of firm $0, B$ customers, $CS_{0,B}$.

By virtue of (10), the social enterprise $0, B$'s problem is:

$$\begin{gathered} \max_{p_{0,B}, x_{0,B}} x_{0,B}\left(s - p_{0,B} - \tfrac{t}{2}x_{0,B}\right), \\ \text{s.t. } x_{0,B} \le \tfrac{p_{1,B}-p_{0,B}+t}{2t},\ x_{0,B} \le \tfrac{s_B-p_{0,B}}{t}, \text{ and } p_{0,B} \ge c_B \end{gathered} \tag{39}$$

First notice that the objective function is decreasing in $p_{0,B}$, hence $p_{0,B} = c_B$ is optimal. In such a case the two constraints can be rewritten as

$$x_{0,B} \leq \frac{p_{1,B} - c_B + t}{2t} \text{ and } x_{0,B} \leq \frac{s_B - c_B}{t} \tag{40}$$

and the objective function as $x_{0,B}\left(s_B - c_B - \frac{t}{2}x_{0,B}\right)$. F.O.C. is

$$s_B - c_B - tx_{0,B} = 0 \tag{41}$$

and S.O.C. is verified since $-t < 0$. As a consequence, solution to (39) is

$$x_{0,B} = \frac{s_B - c_B}{t} \text{ if } \frac{p_{1,B} - c_B + t}{2t} \geq \frac{s_B - c_B}{t}, \tag{42}$$

in which case only the second constraint in (39) is binding. This means that firms are local monopolists. By contrast, solution to (39) is

$$x_{0,B} = \frac{p_{1,B} - c_B + t}{2t} \text{ if } \frac{p_{1,B} - c_B + t}{2t} < \frac{s_B - c_B}{t}, \tag{43}$$

in which case only the first constraint in (39) is binding. This means that the market is fully covered and the indifferent individual gets non-negative surplus. Summing up yields social enterprise 0 best response:

$$p_{0,B} = c_B;\ D_{0,B} = \begin{cases} \frac{p_{1,B} - c_B + t}{2t} & \text{if } p_{1,B} < 2s_B - c_B - t, \\ \frac{s_B - c_B}{t} & \text{if } p_{1,B} \geq 2s_B - c_B - t. \end{cases} \tag{44}$$

For-profit firm $1, B$ best response functions are given by (29) and (30), *mutatis mutandis*:

$$p_{1,B} = \begin{cases} \frac{p_{0,B} + c_B + t}{2} & \text{if } c_B - t \leq p_{0,B} < \frac{4s_B - 3t - c_B}{3}, \\ 2s_B - t - p_{0,B} & \text{if } \frac{4s_B - c_B - 3t}{3} \leq p_{0,B} \leq \frac{3s_B - c_B - 2t}{2}, \\ \frac{c_B + s_B}{2} & \text{if } p_{0,B} > \frac{3s_B - 2t - c_B}{2}, \end{cases} \tag{45}$$

and

$$D_{1,B} = \begin{cases} \frac{p_{0,B} + t - c_B}{4t} & \text{if } c_B - t \leq p_{0,B} < \frac{4s_B - 3t - c_B}{3}, \\ \frac{t + p_{0,B} - s_B}{t} & \text{if } \frac{4s_B - c_B - 3t}{3} \leq p_{0,B} \leq \frac{3s_B - c_B - 2t}{2}, \\ \frac{s_B - c_B}{2t} & \text{if } p_{0,B} > \frac{3s_B - 2t - c_B}{2}. \end{cases} \tag{46}$$

The equilibrium is hence as follows.

a) If $t < \frac{4}{3}(s_B - c_B)$, $p^{**}_{1,B} = \frac{2c_B + t}{2}$, $D^{**}_{1,B} = \frac{1}{4}$, $p^{**}_{0,B} = c_B$ and $D^{**}_{0,B} = \frac{\frac{2c_B + t}{2} - c_B + t}{2t} = \frac{3}{4}$.Notice that both $p^{**}_{1,B} < 2s_B - c_B - t$ and $D^{**}_{0,B} < \frac{s_B - c_B}{t}$ are equivalent to $t < \frac{4}{3}(s_B - c_B)$, which is true.

c) If $\frac{4}{3}(s_B - c_B) \leq t \leq \frac{3}{2}(s_B - c_B)$, then $p^{**}_{1,B} = 2s_B - t - c_B$, $D^{**}_{1,B} = 1 - \frac{s_B - c_B}{t}$, $p^{**}_{0,B} = c_B$ and $D^{**}_{0,B} = \frac{2s_B - t - c_B - c_B + t}{2t} = \frac{s_B - c_B}{t}$. Notice that $\frac{p^{**}_{1,B} - c_B + t}{2t} = \frac{s_B - c_B}{t}$.

d) If $t > \frac{3}{2}(s_B - c_B)$, then $p^{**}_{1,B} = \frac{c_B + s_B}{2}$, $D^{**}_{1,B} = \frac{s_B - c_B}{2t}$, $p^{**}_{0,B} = c_B$ and $D^{**}_{0,B} = \frac{s_B - c_B}{t}$. Notice that $p^{**}_{1,B} > 2s_B - t - c$ and $\frac{s_B - c_B}{t} + \frac{s_B - c_B}{2t} < 1$ are equivalent to $t > \frac{3}{2}(s_B - c_B)$, which is true.

We can conclude that in the mixed economy the equilibrium of Sector B takes the following features. The equilibrium prices are

$$p_{0,B}^{**} = c_B;\ p_{1,B}^{**} = \begin{cases} \frac{t}{2} + c_B & \text{if } t < \frac{4}{3}(s_B - c_B), \\ 2s_B - c_B - t & \text{if } \frac{4}{3}(s_B - c_B) \le t \le \frac{3}{2}(s_B - c_B), \\ \frac{s_B + c_B}{2} & \text{if } t > \frac{3}{2}(s_B - c_B). \end{cases} \tag{47}$$

The equilibrium demands are

$$D_{0,B}^{**} = \begin{cases} \frac{3}{4} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{s_B - c_B}{t} & \text{if } t \ge \frac{4(s_B - c_B)}{3}. \end{cases} \quad D_{1,B}^{**} = \begin{cases} \frac{1}{4} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ 1 - \frac{s_B - c_B}{t} & \text{if } \frac{4(s_B - c_B)}{3} \le t \le \frac{3(s_B - c_B)}{2}, \\ \frac{s_B - c_B}{2t} & \text{if } t > \frac{3(s_B - c_B)}{2}. \end{cases} \tag{48}$$

Before commenting on the above results, we calculate equilibrium firms' profits in Sector B. To this aim, we plug (47) and (48) into (2) with $j = B$:

$$\begin{aligned} &\Pi_{0,B}^{**} = 0; \\ &\Pi_{1,B}^{**} = \begin{cases} \frac{t}{8} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{(2s_B - 2c_B - t)(t + c_B - s_B)}{t} & \text{if } \frac{4(s_B - c_B)}{3} \le t \le \frac{3(s_B - c_B)}{2}, \\ \frac{(s_B - c_B)^2}{4t} & \text{if } t > \frac{3(s_B - c_B)}{2}. \end{cases} \end{aligned} \tag{49}$$

Similarly, we plug (47) and (48) into (10) and (11) to compute the equilibrium individuals' surplus,

$$\begin{aligned} CS_{0,B}^{**} &= \begin{cases} \frac{3(8s_B - 3t - 8c_B)}{32} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{(s_B - c_B)^2}{2t} & \text{if } t \ge \frac{4(s_B - c_B)}{3}. \end{cases} \\ CS_{1,B}^{**} &= \begin{cases} \frac{8s_B - 5t - 8c_B}{32} & \text{if } t < \frac{4(s_B - c_B)}{3}, \\ \frac{(s_B - c_B - t)^2}{2t} & \text{if } \frac{4(s_B - c_B)}{3} \le t \le \frac{3(s_B - c_B)}{2}, \\ \frac{(s_B - c_B)^2}{8t} & \text{if } t > \frac{3(s_B - c_B)}{2}. \end{cases} \end{aligned} \tag{50}$$

We have all the elements to explain the mixed economy equilibrium results in Sector B. First remark that the customer surplus maximizer $0, B$ charges $p_{0,B}^{**} = c_B$ and ends up with zero profits. This is because customer surplus $CS_{0,B}$ is negatively affected by the price, as one can check by inspecting the objective function of program (39). On the contrary, the for-profit firm $1, B$ sets a larger price, $p_{1,B}^{**} > p_{0,B}^{**}$ for any t. As a result, the demand share of firm $0, B$ is greater.

More precisely, when $t < \frac{4(s_B - c_B)}{3}$ only the first constraint of program (39) is binding, *i.e.*, full coverage occurs and the indifferent individual gets positive surplus. The equilibrium price charged by the for-profit firm $1, B$, $p_{1,B}^{**} = \frac{t}{2} + c_B$, is increasing in t as in Section 3. Notwithstanding, its demand $D_{1,B}^{**} = \frac{1}{4}$ does not decrease because customers become more captive as t increases. As a result, firm $1, B$'s profits, $\Pi_{1,B}^{**} = \frac{t}{8}$, are positively affected by t, while individuals' surplus, $CS_{1,B}^{**} = \frac{8s - 5t - 8c_B}{32}$, is negatively affected. Price and demand share of the social enterprise, $p_{0,B}^{**} = c_B$ and $D_{0,B}^{**} = \frac{3}{4}$, respectively, do not depend on t. Yet the surplus of its customers, $CS_{0,B}^{**} = \frac{3(8s_B - 3t - 8c_B)}{32}$, is negatively affected by t simply because moving along the segment becomes more costly, see formula (10) with $j = B$.

When $t \in \left[\frac{4(s_B - c_B)}{3}, \frac{3(s_B - c_B)}{2}\right]$, there is full coverage, while the indifferent individual gets zero surplus. The price charged by the for-profit firm, $p_{1,B}^{**} = 2s_B - c_B - t$, is decreasing in t. The reason is as for the equilibrium price $p_j^* = s_j - \frac{t}{2}$ arising in interval $\frac{2(s_j - c_j)}{3} \le t \le s_j - c_j$ of the market economy: see Appendix A.2. Accordingly, its demand

$D^{**}_{1,B} = 1 - \frac{s_B - c_B}{t}$ increases with t. Firm $1, B$'s profit, $\Pi^{**}_{1,B} = \frac{(2s_B - 2c_B - t)(t + c_B - s_B)}{t}$, is first increasing and then decreasing in t, while the surplus of its customers, $CS^{**}_{1,B} = \frac{(s_B - c_B - t)^2}{2t}$, is positively affected by t. Conversely, the demand share of the social enterprise, $D^{**}_{0,B} = \frac{s_B - c_B}{t}$, decreases with t and so does the surplus of its customers, $CS^{**}_{0,B} = \frac{(s_B - c_B)^2}{2t}$.

Finally, when $t > \frac{3(s_B - c_B)}{2}$ there is no full coverage. The equilibrium prices, $p^{**}_{0,B} = c_B$ and $p^{**}_{1,B} = \frac{s_B + c_B}{2}$ are unaffected by t, while the demand shares, $D^{**}_{0,B} = \frac{s_B - c_B}{t}$ and $D^{**}_{1,B} = \frac{s_B - c_B}{2t}$, decrease with t. We can conclude that: (i) firm $1, B$ profits, $\Pi^{**}_{1,B} = \frac{(s_B - c_B)^2}{4t}$, decreases with t; (i) the surplus of customers of both firms are negatively affected by t.

A.4 Equilibrium Prices

We check that the sum of equilibrium prices in Sector B is lower under the mixed economy. We consider five relevant intervals of t.

1. If $t < \frac{2}{3}(s_B - c_B)$, $p^*_{0,B} + p^*_{1,B} = 2p^*_B = 2t + 2c_B > p^{**}_{0,B} + p^{**}_{1,B} = c_B + \frac{t}{2} + c_B \Leftrightarrow 2t > \frac{t}{2}$, which is fulfilled.
2. $\frac{2}{3}(s_B - c_B) \leq t \leq s_B - c_B$, $2p^*_B = 2s_B - t > p^{**}_{0,B} + p^{**}_{1,B} = c_B + \frac{t}{2} + c_B \Leftrightarrow 2s_B - 2c_B > \frac{3t}{2} \Leftrightarrow \frac{4}{3}(s_B - c_B) > t$, which is fulfilled.
3. $s_B - c_B < t < \frac{4}{3}(s_B - c_B)$, $2p^*_B = s_B + c_B > p^{**}_{0,B} + p^{**}_{1,B} = c_B + \frac{t}{2} + c_B \Leftrightarrow s_B > c_B + \frac{t}{2} \Leftrightarrow 2(s_B - c_B) > t$, which is fulfilled.
4. $\frac{4}{3}(s_B - c_B) \leq t \leq \frac{3}{2}(s_B - c_B)$, $2p^*_B = s_B + c_B > p^{**}_{0,B} + p^{**}_{1,B} = c_B + 2s_B - c_B - t \Leftrightarrow s_B + c_B > 2s_B - t \Leftrightarrow t > s_B - c_B$, which is fulfilled.
5. $t > \frac{3}{2}(s_B - c_B)$, $2p^*_B = s_B + c_B > p^{**}_{0,B} + p^{**}_{1,B} = c_B + \frac{s_B + c_B}{2} \Leftrightarrow \frac{2s_B + 2c_B}{2} > \frac{s_B + 2c_B}{2}$, which is fulfilled.

A.5 Proposition 1

We observe that $D^*_{0,B} + D^*_{1,B} = 2D^*_B = 1$ if and only if $t \leq s_B - c_B$, see (34), and that $D^{**}_{0,B} + D^{**}_{1,B} = 1$ if and only if $t \leq \frac{3(s_B - c_B)}{2}$, see (48). When $t > \frac{3(s_B - c_B)}{2}$, $D^{**}_{0,B} + D^{**}_{1,B} > 2D^*_B \Leftrightarrow \frac{s_B - c_B}{t} + \frac{s_B - c_B}{2t} - \frac{s_B - c_B}{t} = \frac{s_B - c_B}{2t} > 0$, which is true. These results prove Proposition 1.

A.6 Proposition 2

Relying on (35) and (36) with $j = B$, we can write

$$2\Pi^*_B + 2CS^*_B = \begin{cases} \frac{4(s_B - c_B) - t}{4} & \text{if } t \leq s_B - c_B, \\ \frac{3}{4}\frac{(s_B - c_B)^2}{t} & \text{if } t > s_B - c_B. \end{cases} \quad (51)$$

Note that (51) is negatively affected by t. The intuition is as follows. According to (35) and (36) firms' profits $\Pi^*_B = \frac{t}{2}$ (individuals' surplus $CS^*_B = \frac{4s_B - 5t - 4c_B}{8}$) are positively (is negatively) affected by t when $t \leq \frac{2}{3}(s_B - c_B)$, while firms' profits $\Pi^*_B = \frac{2s_B - t - 2c_B}{4}$ (individuals' surplus $CS^*_B = \frac{t}{8}$) are negatively (is positively) affected by t. The negative effect on individuals' surplus in the first case and on profits in the second case is shown to prevail. By contrast, the market is not fully covered when $t > s_B - c_B$ and (51) decreases with t because total demand $2D^*_B = \frac{s_B - c_B}{t}$ is negatively affected.

Similarly, we can write the sum of firms' profits and individuals' surplus in Sector B of the mixed economy. Recalling that $\Pi_{0,B}^{**}=0$ we have

$$\Pi_{1,B}^{**}+CS_{0,B}^{**}+CS_{1,B}^{**}=\begin{cases}\frac{16(s_B-c_B)-5t}{16} & \text{if } t<\frac{4(s_B-c_B)}{3},\\ \frac{4(s_B-c_B)-t}{2}-\frac{(s_B-c_B)^2}{t} & \text{if } \frac{4(s_B-c_B)}{3}\leq t\leq\frac{3(s_B-c_B)}{2},\\ \frac{7}{8}\frac{(s_B-c_B)^2}{t} & \text{if } t>\frac{3(s_B-c_B)}{2}.\end{cases} \tag{52}$$

When $t<\frac{4(s_B-c_B)}{3}$, summation (52) is decreasing t because the negative effect of t on $CS_{0,B}^{**}=\frac{3(8s_B-3t-8c_B)}{32}$ and $CS_{1,B}^{**}=\frac{8s_B-5t-8c_B}{32}$ outdoes the positive one on $\Pi_{1,B}^{**}=\frac{t}{8}$. When $\frac{4(s_B-c_B)}{3}\leq t\leq\frac{3(s_B-c_B)}{2}$, summation (52) is first increasing and then decreasing in t and reaches its maximum at $t=\sqrt{2}\,(s_B-c_B)$. This is driven by the fact that $\Pi_{1,B}^{**}=\frac{(2s_B-2c_B-t)(t+c_B-s_B)}{t}$ reaches its maximum at $t=\sqrt{2}\,(s_B-c_B)$ too. Finally, the market is not fully covered when $t>\frac{3(s_B-c_B)}{2}$ and (52) decreases with t because total demand $D_{0,B}^{**}+D_{1,B}^{**}=\frac{3(s_B-c_B)}{2t}$ is negatively affected.

We denote with ΔW the difference in (7) and study its sign as parameter t increases. When $\Delta W>0\,(<0)$, welfare is larger (lower) under the market economy. First note that the ordering of relevant t-cutoffs for equilibrium profits and individuals' surplus under the two economies is as follows:

$$\frac{2\,(s_B-c_B)}{3}<s_B-c_B<\frac{4\,(s_B-c_B)}{3}<\frac{3\,(s_B-c_B)}{2}. \tag{53}$$

1. If $t\leq s_B-c_B$, $\Delta W=\frac{4(s_B-c_B)-t}{4}-\frac{16(s_B-c_B)-5t}{16}=\frac{t}{16}>0$. Note that $\frac{t}{16}$ is exactly the difference in the total transportation costs borne by customers under the two scenarios. Indeed, in case of full coverage, total transportation costs are given by

$$\int_0^{\hat{x}_{I,B}} tx dx+\int_{\hat{x}_{I,B}}^1 t\,(1-x)\,dx=\frac{1}{2}t\,(\hat{x}_{I,B})^2+\frac{1}{2}t\,(1-\hat{x}_{I,B})^2, \tag{54}$$

   where $\hat{x}_{I,B}$ is the location of the indifferent individual in Sector B. Differentiating the right hand side of (54) with respect to x yields $t\,(2x-1)$. It is then easy to check that (54) is minimum for $\hat{x}_{I,B}=\frac{1}{2}$ and equal to $\frac{t}{4}$. This is the equilibrium location of the indifferent individual in Sector B of the market economy, see (34) with $j=B$ for $t\leq s_B-c_B$, because the two firms set the same price. By contrast $\hat{x}_{I,B}=\frac{3}{4}$ in Sector B of the mixed economy, see (48) for $t<\frac{4(s_B-c_B)}{3}$, because the social enterprise sets a lower price than the for-profit rival. Plugging $\hat{x}_{I,B}=\frac{3}{4}$ into (54) yields $\frac{5}{16}t$. Note that $\frac{5}{16}t-\frac{t}{4}=\frac{t}{16}$.

2. If $s_B-c_B<t<\frac{4(s_B-c_B)}{3}$, $\Delta W=\frac{3}{4}\frac{(s_B-c_B)^2}{t}-\frac{16(s_B-c_B)-5t}{16}=\frac{[2(s_B-c_B)-t][6(s_B-c_B)-5t]}{16t}$, where $2\,(s_B-c_B)-t>0$ and $6\,(s_B-c_B)-5t\geq 0$ iff $t\leq\frac{6(s_B-c_B)}{5}$.

3. If $\frac{4(s_B-c_B)}{3}\leq t\leq\frac{3(s_B-c_B)}{2}$, $\Delta W=\frac{3}{4}\frac{(s_B-c_B)^2}{t}-\left(\frac{4(s_B-c_B)-t}{2}-\frac{(s_B-c_B)^2}{t}\right)$
   $=\frac{8tc_B-8ts_B-14c_Bs_B+2t^2+7c_B^2+7s_B^2}{4t}<0$ iff $\left(2-\frac{\sqrt{2}}{2}\right)(s_B-c_B)<t<\left(2+\frac{\sqrt{2}}{2}\right)(s_B-c_B)$, which is fulfilled because $\left(2-\frac{\sqrt{2}}{2}\right)(s_B-c_B)<\frac{4(s_B-c_B)}{3}$ and $\left(2+\frac{\sqrt{2}}{2}\right)(s_B-c_B)>\frac{3(s_B-c_B)}{2}$.

4. If $t>\frac{3(s_B-c_B)}{2}$, $\Delta W=\frac{3}{4}\frac{(s_B-c_B)^2}{t}-\frac{7}{8}\frac{(s_B-c_B)^2}{t}=-\frac{1}{8}\frac{(s_B-c_B)^2}{t}<0$.

A.7 Proposition 3

The symmetric equilibrium price charged by the two for-profit firms in Sector B of the market economy is $p_B^I = c_B$. This Bertrand outcome is because there is no differentiation when the two firms are located at the same point, $x = 1$ in this case. Under the assumption that firms share equally the demand, the symmetric equilibrium demand of each firm is $D_B^I = \min\left\{\frac{1}{2}, \frac{s_B - c_B}{2t}\right\}$. This means that full coverage occurs if and only if $t \leq s_B - c_B$. In that case, the equilibrium surplus of each firm's clients is

$$CS_B^I = \frac{1}{2}\left(s_B - c_B - \frac{t}{2}\frac{1}{2}\right) = \frac{4(s_B - c_B) - t}{8}, \tag{55}$$

obtained after plugging $p_B^I = c_B$ and $D_B^I = \frac{1}{2}$ into (11). The market is not fully covered if $t > s_B - c_B$, in which case the symmetric equilibrium demand is as in (34) and the equilibrium surplus of each firm's clients is

$$CS_B^I = \frac{s_B - c_B}{2t}\left(s_B - c_B - \frac{t}{2}\frac{s_B - c_B}{2t}\right) = \frac{3}{8}\frac{(s_B - c_B)^2}{t}, \tag{56}$$

obtained after plugging $p_B^I = c_B$ and $D_B^I = \frac{s_B - c_B}{2t}$ into (11). Since the equilibrium profits are zero, welfare in Sector B is simply given by $2CS_B^I$. Substituting (55) and (56) into $2CS_B^I$ yields $2\Pi_B^* + 2CS_B^*$ in (51).

We also check that $2CS_B^I - 2CS_B^*$ is positive.

1. If $t \leq \frac{2}{3}(s_B - c_B)$, $2CS_B^I = \frac{4(s_B - c_B) - t}{4}$ and $2CS_B^* = \frac{4(s_B - c_B) - 5t}{4}$. We have $2CS_B^I - 2CS_B^* = t > 0$.
2. If $\frac{2}{3}(s_B - c_B) \leq t \leq s_B - c_B$, $2CS_B^I = \frac{4(s_B - c_B) - t}{4}$ and $2CS_B^* = \frac{t}{4}$. We have $2CS_B^I - 2CS_B^* = s_B - c_B - \frac{t}{2} > 0$.
3. If $t > s_B - c_B$, $2CS_B^I = \frac{3}{4}\frac{(s_B - c_B)^2}{t}$ and $2CS_B^* = \frac{(s_B - c_B)^2}{4t}$. We have $2CS_B^I - 2CS_B^* = \frac{(s_B - c_B)^2}{2t} > 0$.

A.8 Mixed Economy with Transfers Equilibrium

In the mixed economy with transfers firm $0, B$ solves the following problem:

$$\begin{gathered} \max_{p_{0,B}, x_{0,B}} D_{0,B}\left(s_B - p_{0,B} - \tfrac{t}{2} D_{0,B}\right) \text{ with } D_{0,B} = [0, x_{0,B}] \\ \text{s.t.} \\ x_{0,B} \leq x_{I,B}, \\ s_B - p_{0,B} - t x_{0,B} \geq 0, \\ p_{0,B} \geq 0. \end{gathered} \tag{57}$$

By repeating the proof of Appendix A.3, with $\mathring{p}_{0,B} = 0$ rather than $p_{0,B}^{**} = c_B$ one can show that the equilibrium of Sector B takes the following features. The equilibrium prices are

$$\mathring{p}_{0,B} = 0; \; \mathring{p}_{1,B} = \begin{cases} c_B & \text{if } t \leq c_B, \\ \frac{t + c_B}{2} & \text{if } c_B < t < \frac{4s_B - c_B}{3}, \\ 2s_B - t & \text{if } \frac{4s_B - c_B}{3} \leq t \leq \frac{3s_B - c_B}{2}, \\ \frac{s_B + c_B}{2} & \text{if } t > \frac{3s_B - c_B}{2}. \end{cases} \tag{58}$$

The equilibrium demands are

$$D^{\circ}_{0,B} = \begin{cases} 1 & \text{if } t \leq c_B, \\ \frac{3t+c_B}{4t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{s_B}{t} & \text{if } t \geq \frac{4s_B-c_B}{3}; \end{cases} \quad D^{\circ}_{1,B} = \begin{cases} 0 & \text{if } t \leq c_B, \\ \frac{t-c_B}{4t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{t-s_B}{t} & \text{if } \frac{4s_B-c_B}{3} \leq t \leq \frac{3s_B-c_B}{2}, \\ \frac{s_B-c_B}{2t} & \text{if } t > \frac{3s_B-c_B}{2}. \end{cases} \tag{59}$$

To calculate equilibrium firms' profits and individuals' surplus in Sector B of the mixed economy with transfers, we plug (58) and (59) into (2) with $j = B$:

$$\Pi^{\circ}_{0,B} + k = \begin{cases} -c_B + k & \text{if } t \leq c_B, \\ -c_B\frac{3t+c_B}{4t} + k & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ -c_B\frac{s_B}{t} + k & \text{if } t \geq \frac{4s_B-c_B}{3}; \end{cases}$$
$$\Pi^{\circ}_{1,B} = \begin{cases} 0 & \text{if } t \leq c_B, \\ \frac{(t-c_B)^2}{8t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{(t-s_B)(2s_B-c_B-t)}{t} & \text{if } \frac{4s_B-c_B}{3} \leq t \leq \frac{3s_B-c_B}{2}, \\ \frac{(s_B-c_B)^2}{4t} & \text{if } t > \frac{3s_B-c_B}{2}, \end{cases} \tag{60}$$

where recall that k is the amount transferred to the firm $0, B$ at $t = 1$. Observe that firm $0, B$ incurs losses for any t when $k = 0$, *i.e.*, $\Pi^{\circ}_{0,B} < 0$. We assume that the source of funding, total profits of Sector A, is sufficient to recover such losses, thus enabling the social enterprise to break-even for any t,

$$\textstyle\sum_i \Pi^*_{i,A} \geq k = \left|\Pi^{\circ}_{0,B}\right|. \tag{61}$$

Put differently,

$$\Pi^{\circ}_{0,B} + k = 0. \tag{62}$$

To compute the equilibrium individuals' surplus we plug (58) and (59) into (10) and (11),

$$CS^{\circ}_{0,B} = \begin{cases} s_B - \frac{t}{2} & \text{if } t \leq c_B, \\ \frac{(3t+c_B)(8s_B-3t-c_B)}{32t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{s_B^2}{2t} & \text{if } t \geq \frac{4s_B-c_B}{3}; \end{cases}$$
$$CS^{\circ}_{1,B} = \begin{cases} 0 & \text{if } t \leq c_B, \\ \frac{(t-c_B)(8s_B-5t-3c_B)}{32t} & \text{if } c_B < t < \frac{4s_B-c_B}{3}, \\ \frac{(s_B-t)^2}{2t} & \text{if } \frac{4s_B-c_B}{3} \leq t \leq \frac{3s_B-c_B}{2}, \\ \frac{(s_B-c_B)^2}{8t} & \text{if } t > \frac{3s_B-c_B}{2}. \end{cases} \tag{63}$$

We have all the elements to explain the equilibrium results. First remark that the social enterprise $0, B$ charges zero price. This is because customer surplus $CS_{0,B}$ is negatively affected by the price, as one can check by inspecting the objective function of program (57). On the contrary, the for-profit firm $1, B$ sets at least $p^{\circ}_{1,B} = c_B$ in order not to incur losses. As a result, the demand share of firm $0, B$ is greater, $D^{\circ}_{0,B} > D^{\circ}_{1,B}$ for any t.

More precisely, when $t \leq c_B$ individuals can move easily along the segment. In symbols, (1 -a) and (1 - b) depend mainly on $p_{i,B}$ when $j = B$, *i.e.*, individuals care about the price when they decide which firm they buy from. Since $p^{\circ}_{0,B} = 0$ and $p^{\circ}_{1,B} = c_B$, the individual located at point 1 ends up with $s_B - t$ when buying from the faraway social enterprise and with $s_B - c_B$ when resorting to the nearby for-profit firm. The former value is non-lower given $t \leq c_B$. As a result, all individuals resort to the non-profit firm, $D^{\circ}_{0,B} = 1$ and $D^{\circ}_{1,B} = 0$. Note that even if both price $p^{\circ}_{0,B} = 0$ and demand $D^{\circ}_{0,B} = 1$ are unaffected by t, the equilibrium surplus of clients of firm $0, B$, $CS^{\circ}_{0,B} = s_B - \frac{t}{2}$, is decreasing in t simply because they bear greater transportation costs.

At $t > c_B$ those individuals located far from firm $0, B$ prefer to resort to the for-profit rival, *i.e.*, $D^{\circ}_{1,B} > 0$, despite the price differential.

When $t \in \left(c_B, \frac{4s_B - c_B}{3}\right)$ full coverage occurs and the indifferent individual gets positive surplus. The equilibrium price charged by the for-profit firm $1, B$, $p^{\circ}_{1,B} = \frac{t + c_B}{2}$, is increasing in t as in Section 3. Notwithstanding, its demand $D^{\circ}_{1,B} = \frac{t - c_B}{4t}$ is increasing as well because its customers become more captive. As a result, firm $1, B$'s profits, $\Pi^{\circ}_{1,B} = \frac{(t - c_B)^2}{8t}$, are positively affected by t, while the effect on $CS^{\circ}_{1,B} = \frac{(t - c_B)(8s_B - 5t - 3c_B)}{32t}$ is ambiguous. Conversely, the demand share of the social enterprise, $D^{\circ}_{0,B} = \frac{3t + c_B}{4t}$, decreases with t. As a consequence its losses, $\left|\Pi^{\circ}_{0,B}\right| = c_B \frac{3t + c_B}{4t}$, as well as the surplus of its customers, $CS^{\circ}_{0,B} = \frac{(3t + c_B)(8s_B - 3t - c_B)}{32t}$, decrease with t.

When $t \in \left[\frac{4s_B - c_B}{3}, \frac{3s_B - c_B}{2}\right]$, there is full coverage, while the indifferent individual gets zero surplus. The price charged by the for-profit firm, $p^{\circ}_{1,B} = 2s_B - t$, is decreasing in t as in Section 3. Accordingly, its demand $D^{\circ}_{1,B} = \frac{t - s_B}{t}$ increases with t. The effect of t on firm $1, B$'s profit, $\Pi^{\circ}_{1,B} = \frac{(t - s_B)(2s_B - c_B - t)}{t}$, is ambiguous, while the surplus of its customers, $CS^{\circ}_{1,B} = \frac{(s_B - t)^2}{2t}$, is positively affected by t. Conversely, the demand share of the social enterprise, $D^{\circ}_{0,B} = \frac{s_B}{t}$, decreases with t. As a consequence its losses, $\left|\Pi^{\circ}_{0,B}\right| = c_B \frac{s_B}{t}$, as well as the surplus of its customers, $CS^{\circ}_{1,B} = \frac{s_B^2}{2t}$, decrease with t.

Finally, when $t > \frac{3s_B - c_B}{3}$ there is no full coverage. The equilibrium prices, $p^{\circ}_{0,B} = 0$ and $p^{\circ}_{1,B} = \frac{s_B + c_B}{2}$ are unaffected by t, while the demand shares, $D^{\circ}_{0,B} = \frac{s_B}{t}$ and $D^{\circ}_{1,B} = \frac{s_B - c_B}{2t}$, decrease with t. We can conclude that: (i) firm $1, B$ profits and firm $0, B$ losses, $\Pi^{\circ}_{1,B} = \frac{(s_B - c_B)^2}{4t}$ and $\left|\Pi^{\circ}_{0,B}\right| = c_B \frac{s_B}{t}$, respectively, decreases with t; (i) the surplus of customers of both firms are negatively affected by t.

A.9 Proposition 4

Relying on (34), we recall that $D^{*}_{0,B} + D^{*}_{1,B} = 2D^{*}_{B} = 1$ if and only if $t \leq s_B - c_B$. Relying on (59) we observe that $D^{\circ}_{0,B} + D^{\circ}_{1,B} = 1$ if and only if $t \leq \frac{3s_B - c_B}{2}$. Moreover, when $t > \frac{3s_B - c_B}{2}$, $D^{\circ}_{0,B} + D^{\circ}_{1,B} > 2D^{*}_{B} \Leftrightarrow \frac{s_B}{t} + \frac{s_B - c_B}{2t} - \frac{s_B - c_B}{t} = \frac{s_B + c_B}{2t} > 0$, which is true. Finally, note that $D^{\circ}_{0,B} + D^{\circ}_{1,B} - 2D^{*}_{B} = \frac{s_B + c_B}{2t} > \frac{s_B - c_B}{2t} = D^{**}_{0,B} + D^{**}_{1,B} > 2D^{*}_{B}$. These results prove Proposition 4.

A.10 Proposition 5

We denote with $\Delta W'$ the difference in (8) and study its sign as parameter t increases. When $\Delta W' > 0\,(< 0)$, welfare is larger (lower) under the market economy. It is useful to compute the sum of firms' profits and individuals' surplus in Sector B of the mixed economy with transfers. We have

$$\Pi^{\circ}_{0,B} + \Pi^{\circ}_{1,B} + CS^{\circ}_{0,B} + CS^{\circ}_{1,B} = \begin{cases} s_B - c_B - \frac{t}{2} & \text{if } t \leq c_B, \\ \frac{24ts_B - 22tc_B + -3c_B^2 + 8c_B s_B - 7t^2}{16t} & \text{if } c_B < t < \frac{4s_B - c_B}{3}, \\ \frac{4ts_B - 2tc_B - 2s_B^2 - t^2}{2t} & \text{if } \frac{4s_B - c_B}{3} \leq t \leq \frac{3s_B - c_B}{2}, \\ \frac{3c_B^2 + 7s_B^2 - 14c_B s_B}{8t} & \text{if } t > \frac{3s_B - c_B}{2}, \end{cases} \tag{64}$$

First note that the ordering of relevant t-cutoffs for equilibrium profits and individuals' surplus under the two economies depends on c_B and s_B: see (51) and (64).

a. if $c_B < \frac{s_B}{2}$, $c_B < s_B - c_B < \frac{4s_B - c_B}{3} < \frac{3s_B - c_B}{2}$

b. if $\frac{s_B}{2} < c_B < s_B$, $s_B - c_B < c_B < \frac{4s_B - c_B}{3} < \frac{3s_B - c_B}{2}$.

a.

1. If $t \leq c_B$, $\Delta W' = \frac{4(s_B - c_B) - t}{4} - \left(s_B - c_B - \frac{t}{2}\right) = \frac{t}{4} > 0$. Note that $\frac{t}{4}$ is exactly the difference in the total transportation costs borne by customers under the two scenarios. Recall that total transportation costs in case of full coverage are given by (54) and are minimum when the indifferent individual is located in the middle of the unit segment, $\hat{x}_{I,B} = \frac{1}{2}$, in which case they equal $\frac{t}{4}$. This is the equilibrium location of the indifferent individual in Sector B of the market economy. By contrast $\hat{x}_{I,B} = 1$ in Sector B of the mixed economy with transfers, see (59) for $t < c_B$. Plugging $\hat{x}_{I,B} = 1$ into (54) yields $\frac{t}{2}$. Note that $\frac{t}{2} - \frac{t}{4} = \frac{t}{4}$.

2. If $c_B < t < s_B - c_B$, $\Delta W' = \frac{4(s_B - c_B) - t}{4} - \left(-\frac{1}{16}\frac{18tc_B - 16ts_B + c_B^2 + 5t^2}{t}\right) = \frac{(c_B + t)^2}{16t} > 0$. The equilibrium location of the indifferent individual in Sector B of the mixed economy with transfers is $\hat{x}_{I,B} = \frac{3t + c_B}{4t}$, see (59) for $c_B < t < \frac{4s_B - c_B}{3}$. Plugging this value into (54) yields total transportation costs $\frac{2tc_B + c_B^2 + 5t^2}{16t}$. Note that $\frac{2tc_B + c_B^2 + 5t^2}{16t} - \frac{t}{4} = \frac{(c_B + t)^2}{16t}$.

3. If $s_B - c_B < t < \frac{4s_B - c_B}{3}$, $\Delta W' = \frac{3}{4}\frac{(s_B - c_B)^2}{t} - \left(-\frac{1}{16}\frac{18tc_B - 16ts_B + c_B^2 + 5t^2}{t}\right) = \frac{18c_Bt - 16s_Bt - 24s_Bc_B + 12s_B^2 + 13c_B^2 + 5t^2}{16t}$. If $\frac{3-\sqrt{5}}{4}s_B < c_B < \frac{s_B}{2}$, $\Delta W' > 0$ because the discriminant of the numerator, $s_B^2 - 6s_Bc_B + 4c_B^2$, is negative. If $c_B \leq \frac{3-\sqrt{5}}{4}s_B$, $\Delta W' > 0$ if
$$t < \frac{8}{5}s_B - \frac{9}{5}c_B - \frac{2}{5}\sqrt{s_B^2 - 6s_Bc_B + 4c_B^2}. \tag{65}$$
$\frac{8}{5}s_B - \frac{9}{5}c_B - \frac{2}{5}\sqrt{s_B^2 - 6s_Bc_B + 4c_B^2} > \frac{4s_B - c_B}{3}$ for $c_B \leq \frac{3-\sqrt{5}}{4}s_B$. It follows that $\Delta W' > 0$.

4. If $\frac{4s_B - c_B}{3} \leq t \leq \frac{3s_B - c_B}{2}$, $\Delta W' = \frac{3}{4}\frac{(s_B - c_B)^2}{t} - \frac{4ts_B - 2tc_B - 2s_B^2 - t^2}{2t} = \frac{4c_Bt - 8s_Bt - 6s_Bc_B + 7s_B^2 + 3c_B^2 + 2t^2}{4t}$, which is negative iff
$$2s_B - c_B - \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2} < t < 2s_B - c_B + \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2}. \tag{66}$$
$2s_B - c_B - \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2} < \frac{4s_B - c_B}{3}$ iff $c_B < \frac{3\sqrt{2}-1}{17}s_B$, $2s_B - c_B - \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2} < \frac{3s_B - c_B}{2}$ iff $c_B > \frac{s_B}{3}$, and $2s_B - c_B + \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2} > \frac{3s_B - c_B}{2}$. Summing up: $\Delta W' < 0$ if $c_B < \frac{3\sqrt{2}-1}{17}s_B$ or $2s_B - c_B - \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2} < t \leq \frac{3s_B - c_B}{2} \cup \frac{3\sqrt{2}-1}{17}s_B < c_B < \frac{s_B}{3}$; $\Delta W' > 0$ if $\frac{4s_B - c_B}{3} < t < 2s_B - c_B - \frac{\sqrt{2s_B^2 - 4s_Bc_B - 2c_B^2}}{2} \cup \frac{3\sqrt{2}-1}{17}s_B < c_B < \frac{s_B}{3}$ or $\frac{s_B}{3} \leq c_B \leq \frac{s_B}{2}$.

5. If $t > \frac{3s_B - c_B}{2}$, $\Delta W' = \frac{3}{4}\frac{(s_B - c_B)^2}{t} - \frac{3c_B^2 + 7s_B^2 - 14c_Bs_B}{8t} = \frac{(s_B + c_B)(3c_B - s_B)}{8t}$, which is negative iff $c_B < \frac{s_B}{3}$.

b.

1. If $t < s_B - c_B$, $\Delta W' = \frac{4(s_B - c_B) - t}{4} - \left(s_B - c_B - \frac{t}{2}\right) = \frac{t}{4} > 0$.

2. If $s_B - c_B < t \leq c_B$, $\Delta W' = \frac{3}{4}\frac{(s_B-c_B)^2}{t} - \left(s_B - c_B - \frac{t}{2}\right) = \frac{2t^2 - 4t(s_B-c_B) + \left(3s_B^2 - 6s_Bc_B + 3c_B^2\right)}{4t}$, which is positive since the discriminant of the polynomial is negative.

3. If $c_B < t < \frac{4s_B - c_B}{3}$, $\Delta W' = \frac{3}{4}\frac{(s_B-c_B)^2}{t} + \frac{1}{16}\frac{18tc_B - 16ts_B + c_B^2 + 5t^2}{t} = \frac{18c_Bt - 16s_Bt - 24s_Bc_B + 12s_B^2 + 13c_B^2 + 5t^2}{16t}$, which is positive since the discriminant of the polynomial is negative.

4. If $\frac{4s_B - c_B}{3} \leq t \leq \frac{3s_B - c_B}{2}$, $\Delta W' = \frac{4c_Bt - 8s_Bt - 6s_Bc_B + 7s_B^2 + 3c_B^2 + 2t^2}{4t} > 0$ because $\frac{s_B}{2} < c_B < s_B$ implies $c_B \geq \frac{s_B}{3}$.

5. If $t > \frac{3s_B - c_B}{2}$, $\Delta W' = \frac{(s_B + c_B)(3c_B - s_B)}{8t} > 0$ given $\frac{1}{2}s_B < c_B < s_B$.

To explain why welfare is larger under the mixed with transfers in $t > t^\circ \cup c_B \in \left[0, \frac{s_B}{3}\right)$ we focus on interval $t > \frac{3s_B - c_B}{2} \geq t^\circ$, where

$$\Delta W' = (2\Pi_B^* + 2CS_B^*) - \left(\overset{\circ}{\Pi}_{0,B} + \overset{\circ}{\Pi}_{1,B} + \overset{\circ}{CS}_{0,B} + \overset{\circ}{CS}_{1,B}\right) =$$

$$\left(2\frac{(s_B - c_B)^2}{4t} + 2\frac{(s_B - c_B)^2}{8t}\right) - \left(-c_B\frac{s_B}{t} + \frac{(s_B - c_B)^2}{4t} + \frac{s_B^2}{2t} + \frac{(s_B - c_B)^2}{8t}\right) \quad (67)$$

$$= -\frac{(s_B + c_B)(s_B - 3c_B)}{8t}.$$

It is worth observing that both minuend and subtrahend of subtraction (67) are decreasing in c_B. When $c_B < \frac{s_B}{3}$, the subtrahend is larger than the minuend because the losses incurred by the social enterprise, $-c_B\frac{s_B}{t}$, are relatively low: welfare is enhanced under the mixed economy with transfers. More precisely, the surplus gain enjoyed by customers of the social enterprise, $\frac{s_B^2}{2t} - \frac{(s_B-c_B)^2}{8t}$, outdoes the profit loss in absolute value incurred by firm $0, B$ when moving from the market to the mixed economy with transfers. Indeed, $\frac{s_B^2}{2t} - \frac{(s_B-c_B)^2}{8t} > \left|-c_B\frac{s_B}{t} - \frac{(s_B-c_B)^2}{4t}\right| \Leftrightarrow c_B < \frac{s_B}{3}$. As c_B increases, instead, the losses of the social enterprise becomes significant, with the effect that $\Delta W'$ becomes positive: welfare is enhanced under the market economy. In symbols, the negative effect of c_B on $\left(\overset{\circ}{\Pi}_{0,B} + \overset{\circ}{\Pi}_{1,B} + \overset{\circ}{CS}_{0,B} + \overset{\circ}{CS}_{1,B}\right)$ is larger in absolute value than that on $(2\Pi_B^* + 2CS_B^*)$,

$$\left|\frac{\partial\left(\overset{\circ}{\Pi}_{0,B} + \overset{\circ}{\Pi}_{1,B} + \overset{\circ}{CS}_{0,B} + \overset{\circ}{CS}_{1,B}\right)}{\partial c_B}\right| = \frac{7s_B - 3c_B}{4t} > \left|\frac{\partial(2\Pi_B^* + 2CS_B^*)}{\partial c_B}\right| = \frac{3(s_B - c_B)}{2t}.$$

A similar reasoning can be applied to explain why welfare is larger in the mixed economy with transfers when $t^\circ < t \leq \frac{3s_B - c_B}{2} \cup c_B \in \left[0, \frac{s_B}{3}\right)$.

In Medio Stat Virtus: Does a Mixed Economy Increase Welfare?
Figures

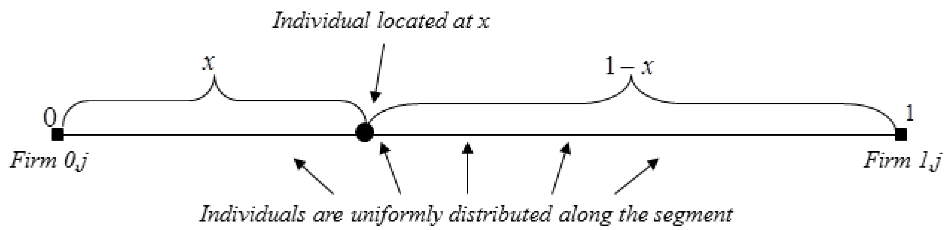

Figure 1: the Hotelling-type linear segment describing Sector $j = A, B$

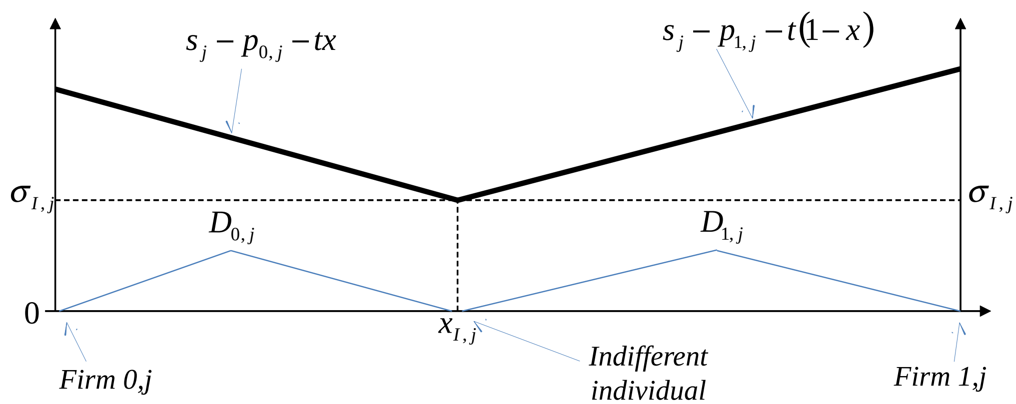

Figure 2: full coverage in Sector $j = A, B$

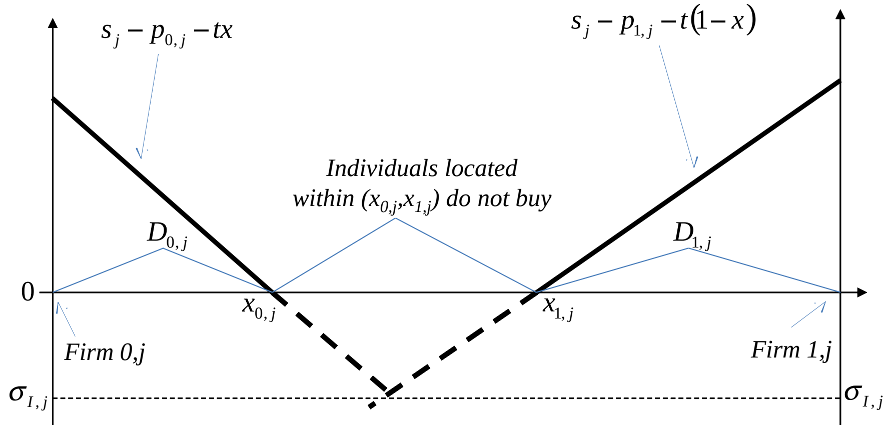

FIGURE 3: PARTIAL COVERAGE IN SECTOR $j = A, B$

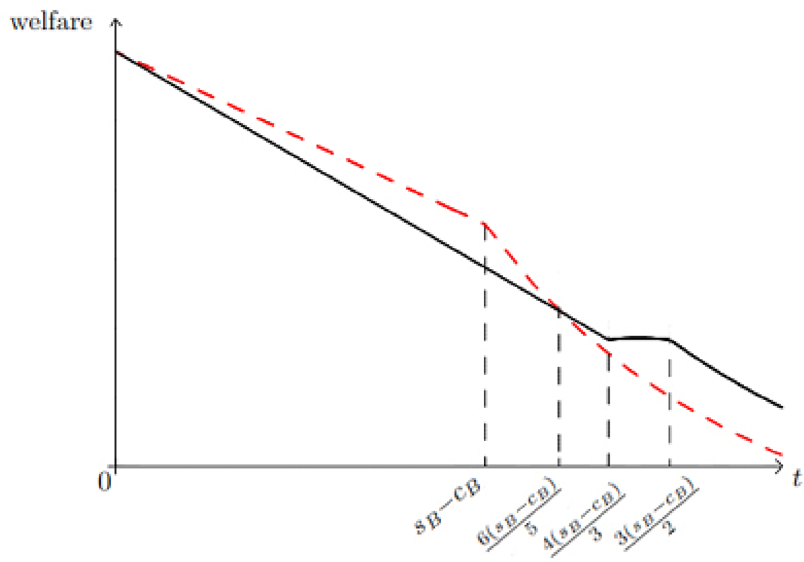

FIGURE 4: WELFARE IN THE MARKET ECONOMY VERSUS WELFARE IN THE MIXED ECONOMY

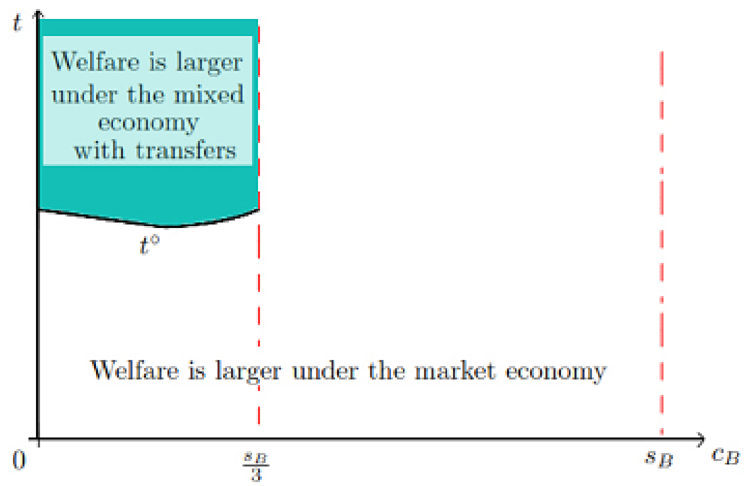

FIGURE 5: MARKET ECONOMY VERSUS MIXED ECONOMY WITH TRANSFERS IN TERMS OF WELFARE