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Abstract

In a model of Originate-To-Distribute (OTD) banking, I show that contagion may spread before any preference shock, fire sale, or change in haircuts takes place. The drivers of contagion are opaqueness of collateral and roll-over frequency. Complexity of structured finance and poor screening of borrowers induce both originators and investors at different stages of the OTD chain to develop heterogeneous expectations on the future value of securitized debt. When new information on the value of collateral is sufficiently bad, creditors in the money market have to write down their loans to overoptimistic banks and shrink liquidity supply in the short-term. Banks with accurate pricing models are unable to roll over and go bankrupt for illiquidity reasons. I provide a set of conditions under which the industry is able to prevent contagion and policy makers shall commit to limit their intevention.

Keywords: Financial crises, Originate to distribute, Opaque securities, Shadow banking, Crisis resolution

JEL: G01, G32

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1. Introduction

There is broad consensus that contagion in the subprime crisis spread through a reduction in the liquidity available against collateral held by financial institutions. An extensive body of literature suggests that the causes are to be found in lending margins and fire sales. New evidence shows that the role of haircuts and repurchase agreements has been overplayed (see Comotto (2012) and Krishnamurthy et al. (2014)), whereas fire sales worsened contagion only after the crisis was underway, when wholesale liquidity providers sold the collateral posted by defaulting banks[2]

The aim of this paper is to fill the gap between theoretical underpinnings of contagion in financial markets and the materialization of the subprime crisis. I develop a model that accounts for financial contagion regardless of preference shocks, margin adjustments, balance sheets interconnection, or fire sales. The main finding is that, when banks raise financing through short-term loans against opaque structured securities, contagion spreads among banks because of indirect linkages through wholesale liquidity providers. The latter are jeopardized by overleveraged banks and decrease their supply of liquidity in the short term. Lenders of solvent banks may initially rely on the intrinsic value of collateral and keep margins unchanged. However, other lenders are affected by banks' default on their loans and risk constraint and bla. The pledgeable value of collateral falls below its intrinsic value for lack of demand, and solvent banks may default for illiquidity reasons. I show a set of conditions under which policy makers may prevent a liquidity crisis by committing to not intervene. As contagion spreads, the liquidity shortage worsens and collateral value falls. Fire sales by funds that risk breaking the buck follows, to recoup part of their credit. The drop in pledgeable value worsens and more institutions default, so that crisis resolution becomes harder to attain with private money.

One key feature of modern banking is the integration between traditional banks and unregulated financial institutions constituting the diverse shadow banking system (see Pozsar et al. (2010)). That allowed traditional banks, subject to prudential regulation and blessed with more or less explicit government backstops, to adopt the Originate-To-Distribute (OTD) business model and exploit profit opportunities outside the regulatory umbrella. Loan brokers at the final stage of the OTD chain grant noninstitutional borrowers loans provided by warehouse lenders, with the sole purpose of transferring their assets to other financial institutions. Noninstitutional loans are pooled and sold to a traditional bank, who use Special Purpose Vehicles (SPVs) to get rid of their risk weight and securitize them into Mortgage-Backed Securities (MBSs) like Collateralized Debt Obligations (CDOs) and CDOs squared.[3] The opaqueness of structured MBSs determined heterogeneous valuations and portfolio allocations on the eve of the crisis. Specialists of structured finance developed pricing models whose outcome depends on arbitrary parameters whereas wholesale liq-

[2]The main business newspapers on 6/21/2007 report that fire sales begun just one day earlier, when two Bear Stearns' vehicles defaulted on loans collateralized by subrime loans. See for instance The Financial Times or The Wall Street Journal.

[3]Gorton and Metrick (2011) report that subprime mortgage origination was about $1.2 trillion in 2005 and 2006, and 80% of it was securitized.

uidity providers did not understand the complexity of the subprime chain and had to rely on credit ratings (see Gorton (2008)).

Banks create – or sponsor – SPVs to both attain leverage outside prudential regulation and smooth liquidity before noninstitutional loans pay out. In order to allow risky SPVs to raise cheap liquidity, parent banks provide backstops if the cash flow delivered by the pool of loans is not sufficient to repay SPVs' counterparties. Reputational credit lines commanded no capital charge before the crisis, whereas the difference between interest earned on noninstitutional loans and that required by wholesale lenders to the SPV yielded a positive return. The OTD model succeeded until MBSs could serve as collateral, either through Asset Backed Commercial Paper (ABCP) or in repurchase agreements (repos), to borrow liquidity under favourable terms in the money market.

The delinquency rate of US subprime borrowers rose dramatically by mid 2007 and credit ratings assigned to many MBSs proved wrong.[4] When the adverse shock hit, SPVs had to use depreciating MBSs to borrow the short-term liquidity necessary to fulfill their commitments. The opaqueness of structured MBSs had induced much heterogeneity in their valuation on the eve of the crisis. Pricing models adopted by some SPVs had overvalued collateral, thus its pledgeable value, and parent banks had to activate liquidity backstops that proved the economic integration of the entity formed by a parent bank and its SPVs.

The default of overleveraged institutions is not bad *per se*, since its prospect acts as a disciplining device on top management. It becomes a threat to financial stability when cross-liabilities among peer banks cause a domino effect (see Bhattacharya and Gale (1987), Allen and Gale (2000), Brusco and Castiglionesi (2007)) or the aforementioned fire sale depresses the value of collateral held by otherwise solvent institutions.

Only after the $62.5bn Primary Fund defaulted, in September 2008, the US government understood how important wholesale liquidity providers are for the stability of the financial industry and insured money market funds until September 2009. However, lenders were hit by writedowns of SPVs already in 2007. By June, Bear Stearns had to shut two of its hedge funds and Wharton's fund Y2K Finance lost 25% of its value, amid rating downgrades on their MBSs. Funding conditions tightened and the SPVs Cheyne Finance and Rhinebridge defaulted on more that $7bn of debt in October. The $67bn worth Columbia Cash Reserves fund held almost $1bn of Cheyne Finance's ABCP [thus supply fell]. In November, Bank of America had to shift $600mn of assets to monetary market funds who risked breaking the buck and HSBC took $45bn of MBSs on its own balance sheet to bail out one of its SPVs.

Fresh liquidity providers had good reasons to not step in the troubled money market. To begin with, funds were limited in their holding of illiquid assets maturing in more than seven days. The limit was 10% of total assets and implied that a conceivable default *en masse* by many SPVs would force wholesale liquidity providers to sell an abnormal amount of collateral. Moreover, amid economic uncertainty on the value of

[4]"In the second half of 2007, Moody's downgraded more bonds that it had over the previous 19 years combined". The Financial Times, 18 October 2008.

opaque securities and their holding by traditional and shadow institutions, investors shunned funds exposed to MBSs. The stigma was so strong that the Legg Mason Fund, which previously held billions of dollars in SPVs holdings, advertised that it was “SPV-free”. The US insurance on money market funds was meant to re-establish confidence and liquidity supply, but it was provided only when the spiral of fire sales and margin calls had materialized.

In section 2 I motivate the present paper by giving an account of some aspects of the Subprime crisis the extant literature does not stress. In section 3, the model of asset-side contagion is introduced and banks' optimal portfolio allocation is characterized. Section 4 describes the impact interim information has on the banking industry, showing how the misvaluation by some banks propagates as a liquidity crisis. Section 5 discusses policy implications and section 6 concludes.

2. Modelling the subprime crisis

The channel is the market for short-term collateralized lending among different stages of the OTD chain. When new information on the value of collateral is released, some liquidity providers leave the money market.[5] Liquidity supply was not readily replaced by new entrants in the money market, the pledgeable value of collateral falls before any fire sale.

The complexity of structured finance techniques employed to securitize loans, together with the poor screening of borrowers and the reliance of subprime loans' repayment rate on the US real estate bubble, suggest that both originators and traders had to use their discretion in working out the future value of securitized loans. The amount of securities a bank originates depends on the pricing model it uses.[6] Some banks adopt a conservative pricing model and engage moderatly in the OTD business, whereas some others originate a greater number of securities and attain more leverage. I show that conservative banks are unable to fulfill their commitments after new information on the fundamental value of collateral makes wholesale liquidity providers write down their loans to overleveraged banks and exit the market. Contagion happens even when information on collateral affects the fundamental value of considerate banks' portfolio positively. In fact, write-downs lower liquidity supply in the money market in the short run and thus the pledgeable value of collateral at roll-over. That happens before any fire sale, whereas wholesale liquidity providers may worsen the drop in banks' funding ability by increasing haircuts. The widening of haircuts is neglected, and it would reinforce the effect of asset misevaulation if it was accounted for. Lending margins are held at their initial level The key drivers of contagion are opaqueness of collateral, which induce highly heterogeneous valuations, and roll-over frequency, which impedes the entrance of new liquidity providers.

[5]Although few funds in the monetary market broke the buck, many of them had to be bailed out by their sponsors to prevent redemptions by external investors.

[6]To abstract from issues of moral hazard I assume the owner of a security is its originator, but the results of the model do not change if one adds more steps to the OTD chain and the owner bought its MBSs from other banks. Moreover, heterogeneity of investment strategies among banks at a stage of the chain relies on genuinely different valuations of the security.

Central banks showed they are ready to protect households' deposits against banks' failure.[7] Thus, preference shocks typical of the literature on "early diers"are kept off the model of contagion. Banks experienced difficulties before anything happened to liquidity demand, and they kept such information hidden as long as they could.[8] Episodes of bank runs were isolated, and investors withdrew their liquidity only after the latter appeared to be clearly endangered. Moreover, central banks and governments have succeeded in building up credibility as institutions ready to do anything for the sake of retail depositors and financial stability. Therefore a consumer panic, either driven by prophecies, interim signals or asymmetric information, does not give a satisfactory explanation to the occurrence of the crisis. Even if you look at MMF investors as depositors, it is not panic otherwise insurance would help.

2.1. Methodology

The misvaluation of assets by a few originators forces some lenders to exit the market for liquidity. Solvent banks may then go bankrupt for lack of cash and the consequent financial crisis is inefficient. Finally, the threat of contagion can be exploited by a regulator who wants to limit the cost of its intervention.

To account for the OTD business model that reached its peak before with the crisis, each bank that collects depositors' liquidity is recast as a complex economic entity made of an investment bank and a SPV. Troubles spread uninterrupted, from delinquent mortgage borrowers up to wholesale liquidity providers in the money market, through the flow of transactions among institutions standing at the different stages of the OTD chain.

At the initial date, banks take their investment decisions and commit to payments they must deliver over future dates to avoid bankruptcy. Every institution competes for its regional depositors by setting a periodic payment. Competition ensures the latter equals the whole liquidity a bank expects to hold at any payment date, according to its information set.

The liquidity a bank can borrow against its pledgeable assets depends on the market value of the latter. Thus, the prevailing price of securitized loans determines the amount of cash a bank can raise to satisfy its commitment. At the interim date, banks discover the true final return on loans. Such information is either a positive shock – for banks who acted relatively conservatively – or a negative shock – for relatively overly leveraged banks, who then default.

Lenders in the money market do not have the skill to understand interim information immediately and become aware of the actual repayment at the liquidation date.[9]

[7] EU countries insure 100% of deposits up to euro 100,000. Switzerland covers up to CHF 100,000, whereas the US insure USD 100,000-250,000 depending on the account.

[8] Shin (2008) reports on the run to Northern Rock: "The Bank of England was informed [of Northern Rock's funding problems] on August 14th. From that time until the fateful announcement on September 14th that triggered the deposit run (i.e. for a full month), the Financial Service Authority and Bank of England sought to resolve the crisis behind the scenes, possibly arranging a takeover by another UK bank."

[9] The delay in lenders' awareness has the unique purpose of keeping margins unchanged. This allows to focus on wholesale lenders' default and isolate from the margin spiral.

When a bank defaults, its lender liquidates the collateral to recoup any loss. The new fair value of collateral can be too low for some lenders to avoid breaching their VaR limits and results in their exit. This, in a market with limited participation, may determine cash-in-the-market pricing and an asset price below its fundamental value. As other lenders breach their VaR limit, the cash available in the market supports an even lower asset price, possibly affecting banks who undertook conservative investment policies. The model shows that, when banks rely on the market value of securitized loans to raise cash at the interim date against the value of their illiquid asset, the wrong assessment made by a bank may jeopardise the whole industry because otherwise solvent institutions are unable to gather the liquidity they need.

The pledgeability of opaque asset-backed securities – *i.e.* the amount of liquidity a bank can borrow by posting securitized loans as collateral – is based on the Value at Risk (VaR) measure used by financial regulators to both assess and limit risk exposure.

The source of risk in the model is uncertainty, at the initial date, on the repayment of noninstitutional loans issued by banks. Each bank relies on a different pricing model to decide the optimal amount of loans to be issued and securitized. Although I assume there is no agency problem between the banking industry and wholesale lenders in the money market, asymmetric information allows some banks to leverage and commit to future payments they cannot afford ex post. Uncertainty is partially resolved at an interim date, when new information let all banks and the regulator discover the actual amount of loans that are repaid at the final date.

If repayment on loans is sufficiently low to make some banks default on their short term borrowing, providers of liquidity in the money market may bear a loss. That decreases the amount of liquidity banks can borrow against pledgeable assets for the following period. Pledgeable assets are then underpriced on the basis of cash-in-the-market pricing *á la* Allen and Gale (1994). Any fall in the price of collateral lowers banks ability to fund their liquidity needs. Thus, the initial insolvency of some overleveraged financial institutions makes those that implemented cautious pricing models illiquid.

If the heterogeneity of banks' initial evaluation prevents an aggregate shortage of liquidity when assets are fairly priced, the policy maker may refrain from intervening. If there is lack of liquidity in the industry, it is necessary to fill such a gap. Yet, solvent banks do not have incentive to hoard fresh liquidity. They use it to bail out the assets of insolvent banks, in their own interest.

3. A Model of Indirect Contagion in OTD Banking

Consider a multi-region economy lasting for three dates $t = 0, 1, 2$. There is no discounting and the risk-free rate is normalised to zero. The economy is populated by depositors, banks, money market funds, and noninstitutional borrowers. Projects undertaken by the latter are the only source of return in the model.

Each region $i = 1, ..., n$ is endowed at the initial date with one unit of liquidity, proportionally owned by a continuum of depositors, and with a piece of relevant information that is shared by the latter and their regional banks. There is no informational asymmetry among players in a region, whereas the fact that each region has its own information determines n optimal investment strategies.

3.1. Players

a) Depositors

Among depositors there are no "early diers", to use the jargon of the previous literature. Preferences are homogeneous and described by the utility function

$$U(c^t) = \min(c^1, c^2)$$

where c^t is the amount of liquidity a depositor can use for consumption at date t. Thus, there is no shock to the demand for liquidity, and depositors strictly prefer smoothing consumption over time.

Depositors have the opportunity to place their endowment in a regional bank, in order to access its lending technology and afford a consumption schedule that yields the highest intertemporal utility. Since they face no preference shock and prefer smoothing consumption over time, a contract that specifies $c^1 = c^2$ is optimal. Such a contract is akin to limited convertibility, therefore depositors never run their bank.

b) Noninstitutional borrowers

Noninstitutional borrowers face a long-term investment and borrow from banks the cash they need to undertake it.[10] Differently from depositors, borrowers deal with banks from any region. They do not care about the information set of their lender, as long as the loan is granted under the most favourable terms. Loans are granted at the initial date, and their repayment at the final date depends on the outcome of the project.

Borrowers are homogeneous and need exerting no effort to affect a project probability of success, nor can they misrepresent the project return. Thus, there are no agency problems between banks and noninstitutional borrowers. The latter are a risky black–box banks can access remunerative projects through.

c) Banks

Each regional banking industry is perfectly competitive. In order to outperform its competitors, a bank offers deposit contracts that maximise its depositors' utility.

Banks lend depositors' liquidity to noninstitutional borrowers. Each bank takes its lending decision on the basis of the repayment it expects at the final date, given its region-specific signal on the realization of exogenous factors that affect loans return. Such decision consists of what share of deposits to invest

[10] Noninstitutional borrowers can be consumers who want to buy a home, to account for the category of borrowers the 2007-2009 crisis generated from, as well as agents involved in entrepreneurial ventures.

in loans to noninstitutional borrowers and what to be held as cash, together with a commitment on the periodic payment c^t to be paid to depositors at dates $t = 1, 2$. Every bank chooses its portfolio allocation and commitment with depositors under a solvency constraint. It goes bankrupt if, at any point in time, the liquidity it has available is lower than its commitments.

The liquidity banks in each region can use to finance entrepreneurial projects goes beyond the one unit they collect from depositors: banks can turn to the money market and raise additional liquidity they can invest in noninstitutional loans.

d) Wholesale lenders (ABCP issuers and money market funds)

Money market funds are awash with liquidity and can only invest it in short-term asset securities such as ABCP and repos. They lack specialized skills necessary to gather any regional signal on entrepreneurial projects backing the securities. Their lending decision is thus based on common knowledge. Funds are subject to a "VaR equal zero" constraint.[11] A fund exits the market if it breaches its VaR constraint.

3.2. Investment technologies

a) Storage

The storage technology is cash, that is a 1-period investment yielding no return.

b) Deposits

The deposit technology is a 2-period fixed commitment that may differ among regions. It entitles depositors in region i to receive payments c_i^t at dates $t = 1, 2$ from a regional bank.

c) Entrepreneurial projects

Entrepreneurial projects are homogeneous and illiquid 2-period ventures undertaken by noninstitutional investors. The statistical distribution of their return can be inferred, at the initial date, from information available to all players – *e.g.* rating and historical data.

d) Noninstitutional loans

The lending technology is available to banks only. It involves granting liquidity to noninstitutional borrowers at the initial date, in exchange for its repayment with interests two periods later.

Since the return on entrepreneurial projects is random, noninstitutional borrowers' ability to repay the loan is stochastic. It is assumed that the repayment of each unit of lent liquidity follows a Normal distribution $\tilde{s} \sim N(\bar{s}, \sigma_s^2)$, conditioned to all public information. All players share such prior belief, whereas banks and depositors in each region i receive region-specific signals and form a posterior beliefs $\bar{s}_i$.

[11] See Adrian and Shin (2012) for the derivation of VaR as optimal risk constraint in collateralized loans. For a discussion on the optimality of "VaR equal zero" contract, see Fostel and Geanakoplos (2012).

Loans can be securitized by banks and used as collateral, either in repos or to issue ABCP. Without any loss of generality and to keep the model as simple as possible, I assume each unit of liquidity a bank lends is securitized into one unit of MBS.

e) Wholesale funding

The wholesale funding technology is a 1-period agreement whereby banks use their pool of loans to borrow liquidity from money market funds. Banks sell their MBSs in exchange for cash either to money market funds or to conduits issuing ABCP. Banks commit to repay their debt one period later and buy back the MBSs.

When a bank buys back its securities, it pays a price that is set at the initial date and equals the MBS pledgeable value plus interests. The pledgeable value of a MBS – *i.e.* the wholesale loan principal – is its market value at inception minus the margin ABCP issuers and repo lenders charge against market risk. For the sake of simplicity, I assume the interest rate and haircut on wholesale lending are the same independently on whether the loan occurs through ABCP or repos.[12] Funds earn interests as remuneration for the risk they undertake. Parties in a repo agreement are price takers with respect to the remuneration financial markets assign to risk. Therefore, the interest rate on repo lending is considered exogenous throughout the model.

3.3. Information

Accounting for the aggregation of information in this framework entails technical difficulties because of endogenous haircuts, the shape of consumers' utility, and the nonlinearity of payoffs on collateralized loans. The issue of price formation does not enrich the understanding of the contagion channel. For any market clearing equilibrium price p^0 at the initial date, there exist a set of banks' private signals that causes contagion in the model.[13]

The heterogeneity of regional signals on the future value of ABSs is sufficient to spread insolvency throughout the industry as a consequence of wholesale lenders' default. Heterogeneous signals may be tought of as proprietary pricing models used by a bank and deemed valid by some depositors, who choose then to belong to its same informational region. Since every class of players in the model has homogeneous risk preferences, banks who choose safer (riskier) strategies do so on the basis of relatively bearish (bullish) signals. Thus, they commit to a lower (higher) return to depositors. The reason why some depositors may accept a lower return is that they share the information of a particular bank.[14]

[12]The main difference between ABCP and repos is that the latter are bankruptcy-remote. Since in ABCP the issuer guarantees for the repayment of loans, neither form of collateral is allocated to other creditors in case the borrowing bank defaults.

[13]Price formation is the focus of a companion paper, "Price Formation of Pledgeable Security"where margins are set by a central clearing house and specific assumptions on preferences and distributions are made to simplify information aggregation.

[14]Depositors who share the signal of a regional bank have no incentive to shop around for a second bank/region to get a higher return. The combination of such higher return with the risk the second bank undertakes is in fact suboptimal for the depositor from a different region, given the difference among information sets.

Formally, each region $i = 1, ..., n$ receives an informative signal $\tilde{f}_i = s + \tilde{\epsilon_i}$ on the final repayment s of the lending technology. To ensure banks/depositors do not make systematic errors, it is assumed that $\tilde{\epsilon_i} \sim N(0, \sigma_\epsilon^2)$, $\tilde{s}$ and $\tilde{\epsilon_i}$ are independent, and errors are independent across banks. Since money market funds cannot observe regional signals, their uninformed expectation $\bar{s}_p = E(\tilde{s}|p^0)$ generally differ from the informed expectations $\bar{s}_i = E_i\left(\tilde{s}|p^0, f_i\right)$ made by each bank in region $i = 1, ..., n$.

Since banks in any region are perfectly competitive and share the same information and technologies, they are all alike and take the same investment decision. Thus, with no loss of generality and to simplify notation, one representative bank is considered for every region. Banks are then labelled according to their signals, from the most bullish to the most bearish one: $\bar{s}_1 \geq \bar{s}_2 \geq ... \geq \bar{s}_n$.

At the interim date, the true repayment on loans is known to all banks, and the latter agree on the value of MBSs.

3.4. Time structure

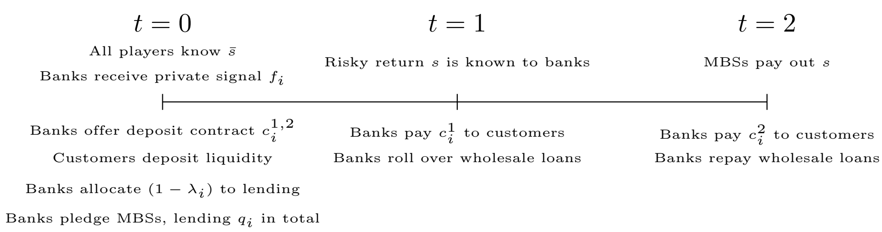
Initial date: Bank i observes its signal f_i and forms posterior belief $\bar{s}_i$. It receives from depositors one unit of liquidity and chooses what portion λ_i of to invest in the storage technology in order to remunerate depositors at $t = 1$. The remaining $(1 - \lambda_i)$ is invested in noninstitutional loans to earn the risky return $\frac{\tilde{s}}{p^0}$ at $t = 2$.

Each bank securitizes its loans to noninstitutional borrowers and use them to borrow liquidity on the money market, with a common interest rate r and a bank-specific haircut h_i. Wholesale loans allow banks to leverage and grant (issue) a total amount q_i of loans (MBS) and to commit to periodic payments c_i^t.

Interim date: Banks discover that the repayment of noninstitutional loans will be s. Each bank i must pay depositors the sum c_i^1 it specified in the contract at the initial date. The wholesale loan need to be rolled over. Solvent banks may raise an amount of liquidity that equals MBS pledgeable value given the current market price. When a bank defaults on its wholesale loans MBSs are kept by the ABCP issuers or repo lender, who exit the market if the new MBS price is not sufficient to cover losses.

Final date: Entrepreneurial projects pay out. Banks receive the repayment s from noninstitutional borrowers, repay their debt, and give depositors the sum c_i^2 they committed to at the initial date.

For the sake of clarity, the following timeline summarises the time-structure of the model:



3.5. MBS pledgeability in the money market

MBS pledgeability is endogenously determined by the need for banks' counterparties to limit the VaR of their loans. Wholesale loans are provided so that expected losses are lower than the VaR limit, with a probability $(1-\alpha)$ set by the regulator.

Since banks can invest deposits in either risky loans or safe cash, the usual trade-off between leverage and credit worthiness affects lending terms. The probability that a lender recoups the full value of its credit decreases with the risky investment undertaken by its counterparty. Under the VaR=0 constraint, bank i faces a haircut h_i such that:

$$\lambda_i + q_i p^t(1-v) \geq q_i p^t(1-h_i)(1+r), \tag{1}$$

where λ_i is the portion of deposits bank i invested in the storage technology, q_i is the number of MBSs it issued, r is the wholesale loan rate, p^t is the security market value at date t, and v is its price fall over one period in the $\alpha\%$ worst scenario, according to the unconditional distribution of $\tilde{s}$ adopted for risk management purposes. On the left hand side of the inequality is the sum between the liquidity the bank decided to hold at the interim date and the α-specific market value of its pledged assets. On the right hand side is the cash flow of the second loan, in case of roll-over at the interim date.

The solution to inequality (1) gives a lower bound for the haircut h_i that depends on r, v, p^t, and bank i portfolio allocation $(\lambda_i; 1-\lambda_i)$:

$$h_i^{min} = \frac{r + v - \frac{\lambda_i}{q_i p^t}}{1+r}. \tag{2}$$

Wholesale lenders face a trade-off between applying a higher haircut to hedge risk and earning interests on additional principal amount. Since funds are risk neutral, the lowest admissible value derived in equality (2) is optimal.

3.6. Investment and contract design

Each competitive bank chooses its optimal investment in the lending technology, at the initial date, to maximise depositors' intertemporal utility.

Omitting the index i to simplify notation, the budget constraint of any bank at the initial date is

$$q \quad \leq \quad (1-\lambda) + qp^0(1-h)$$

where q are the units of liquidity a bank that levers through wholesale loans may lend to noninstitutional borrowers, $(1-\lambda)$ is the fraction of deposits invested in loans, and $qp^0(1-h)$ is the amount of liquidity the bank borrows by pledging the MBSs it originates. The maximum amount of MBSs a bank can originate by allocating $(1-\lambda)$ of its deposits to noninstitutional lending is therefore

$$q^{max} = \frac{1-\lambda}{1-p^0(1-h)}, \tag{3}$$

that is also the liquidity a bank is ultimately able to lend to noninstitutional borrowers, given the initial allocation λ and the leverage allowed by the wholesale lending haircut h.

Ultimate MBS origination q depends on the wholesale lending technology – given a bank optimal allocation – whereas the margin h is imposed by risk-constrained liquidity provider. The two values are determined through the simultaneous solution of equalities (2) and (3). Given any initial portfolio allocation $(\lambda, 1-\lambda)$, a bank lends/originates an amount

$$q = \frac{1+r(1-\lambda)}{(1+r)-p^0(1-v)} \tag{4}$$

and faces the wholesale loan margin

$$h = \frac{p^0(1-\lambda)(r+v)+\lambda(p^0-1)}{p^0(1+r(1-\lambda))}. \tag{5}$$

When claims on 2-period noninstitutional loans are offered as collateral in the money market, each bank i can borrow an amount of liquidity specified by the pledgeability function

$$\begin{aligned} B_i^t(\lambda_i, r, v, p^t) &= (1-h_i)q_i p^t \\ &= \left(\frac{1-v+\frac{\lambda_i}{q_i p^t}}{1+r}\right) q_i p^t. \end{aligned} \tag{6}$$

Lemma 1. *As a bank risky investment $(1-\lambda_i)$ increases, a lower percentage of its collateral value is pledgeable.*

Proof. See Appendix A □

In order for banks portfolio allocation to be non-trivial, providers of liquidity must be concerned that the haircut level fulfils their VaR constraint. Otherwise, banks would borrow an infinite amount of liquidity to invest in the lending technology. Liquidity invested in the purchase of MBSs must be expected to yield a return that, in the $\alpha\%$ worst scenario, is lower than the cost paid for the same liquidity in the money market:

$$(1+r) > p^0(1-v). \tag{7}$$

Based upon the regional signal f_i, bank i chooses its portfolio allocation to maximize depositors' expected utility.

When a bank solvency constraint does not bind, the optimal portfolio allocation is trivial: every risk neutral bank invests either everything in the risky lending technology – when its posterior belief $\bar{s}_i$ offsets the costs of funding r and h – and it keeps all deposits liquid otherwise. To ensure that the solvency constraint binds, I shall assume throughout the paper that no bank receive either a "too" low or a "too" high signal that induces the choice of a corner solution.

The optimal decision is taken by backward induction and requires knowing the sequence of actions determining the final outcome for any initial allocation $(\lambda_i, 1-\lambda_i)$. Loans and MBS origination depends on λ_i through equality (8). MBSs allow a bank to borrow on the money market an amount of liquidity $q_i p^0(1-h_i)$, that is lent to noninstitutional borrowers together with $(1-\lambda_i)$. Bank i is left with liquid deposits λ_i it carries on to period 1, when $q_i p^0(1-h_i)(1+r)$ has to be paid to roll over the wholesale loan. Since the decision over λ_i is taken at the initial date, each bank establishes its optimal portfolio allocation upon the expectation that MBSs appreciate at the interim date by

$$\delta_i = \frac{\bar{s}_i}{p^0} - 1. \tag{8}$$

Given its posterior belief $\bar{s}_i$, bank i expects to raise

$$q_i p^0(1+\delta_i)(1-h_i)$$

on the money market when its loan is rolled over.

The liquidity bank i has available to pay its depositors the first periodic instalment c_i^1 amounts to the sum of the deposits λ_i it stored at the initial date and the net cash flow occurring at the interim date:

$$\lambda_i - q_i p^0(1+r)(1-h_i) + q_i p^0(1+\delta_i)(1-h_i).$$

At the interim date, after depositors receive their first payment, bank i holds an amount of liquidity

$$\lambda_i + q_i p^0(\delta_i - r)(1-h_i) - c_i^1. \tag{9}$$

At the final date, entrepreneurial projects yield their return and each MBS pays out s. The bank must pay $q_i p^0(1+\delta_i)(1-h_i)(1+r)$ to money market funds, and depositors must be paid the second instalment c_i^2. Given any belief $\bar{s}_i$ at the initial date, the residual liquidity bank i expects to hold at the end of date 2 is therefore

$$\lambda_i - q_i p^0(1-h_i)(1+r(2+\delta_i)) + q_i \bar{s}_i - c_i^1 - c_i^2. \tag{10}$$

Competitive banks offer depositors the highest possible periodic smooth payment $c_i^{1,2}$. This induces banks to select their optimal strategy leaving no spare liquidity at the end of the deposit contract. Imposing such condition at the final date allows to identify by backward induction the bank's objective function, that is the expected per period payment c_i:

$$c_i \equiv \frac{\lambda_i + q_i p^0(\bar{s}_i - 1 - r(2+\delta_i))(1-h_i)}{2} \tag{11}$$

The bank's optimization problem at the initial date is therefore:

$$\max_{\lambda_i} : c_i \tag{12}$$

$$s.t. \;\; c_i \leq \lambda_i + q_i p^0(\delta_i - r)(1-h_i) \tag{13}$$

$$q_i = \frac{1 + r(1-\lambda_i)}{(1+r) - p^0(1-v)} \tag{8}$$

$$h_i = \frac{p^0(1-\lambda_i)(r+v) + \lambda_i(p^0 - 1)}{p^0(1 + r(1-\lambda_i))} \tag{5}$$

Objective function (12) is derived from (10), imposing the final condition that a bank does not expect to hold spare liquidity after depositors are paid. Inequality (13) is the solvency constraint at the interim date, when the periodic payment c_i has to be lower than the available liquidity that was specified in (9). Equalities (8) and (5) were previously determined as simultaneous solutions to the initial budget constraint and to money market funds VaR limit.

When the bank's solvency constraint (13) does not bind, optimal portfolio allocation is trivial: every risk-neutral bank invests either everything in the risky lending technology – when its posterior belief $\bar{s}_i$ offsets the costs of funding r and h – and it keeps all deposits liquid otherwise.

In what follows I assume the value of parameters in the model are such that the budget constraint binds for some banks, so that the solution of the optimization problem is the non-trivial interior one. Some banks receive signals that are neither too low nor too high and thus, given their information sets, corner solutions are suboptimal.

Lemma 2. *When the solvency constraint binds, the optimal portfolio allocation is:*

$$\lambda_i^* = \frac{p^0\left\{(1+r)\left[p^0(1-v) + \bar{s}_i v\right] - \bar{s}_i(1-v)\right\}}{p^0(p^0 r - \bar{s}_i)(1-v) + \bar{s}_i\left[(1+r)(1+p^0 v) - (p^0 - 1)\right]}. \tag{14}$$

The number of MBSs optimally issued by bank i is

$$q_i^* = \frac{2+r}{p^0 r v + (2+r) - (2 - p^0 \bar{s}_i^{-1} r) p^0 (1-v)} \tag{15}$$

Proof. The result follows directly from the maximization of (12) under the budget constraints (8,5,13) □

Lemma 3. *The higher a bank's posterior belief $\bar{s}_i$, the larger its lending and the payment it commits to pay depositors.*

Proof. See Appendix A $\square$

4. Effect of the systemic shock

At the interim date, the risky return s is known to banks in every region. Since the latter are also involved in the spot market, such information affects the MBS spot price. Label the MBS price change following interim information on loans repayment as

$$\delta_s = \frac{s}{p^0} - 1.$$

The liquidity bank i has available given new information is not sufficient to fulfill its commitments and causes insolvency when $\bar{s}_i > s$. In such a case, $\delta_s < \delta_i$ and the available liquidity c_i^s at the final date is lower than the commitment with depositors.

$$c_i^s = \frac{\lambda_i^* + q_i^* p^0(\delta_s - r)(1-h_i) + q_i^*(s - p^0(1+\delta_s)(1-h_i)(1+r))}{2} < c_i^*. \tag{16}$$

If instead bank i underestimates loans repayment at the initial date, the fair value of its assets exceeds its commitments.

Scenario 1: considerate banking industry

The first scenario arises when all banks estimate the realization of $\tilde{s}$ conservatively. They hold enough cash at the initial date to avoid bankruptcy when interim information is released: $c_1^s \geq c_1$.

Scenario 2: inconsiderate banking industry

The second scenario arises if all banks have access to signals that overestimate the realization of $\tilde{s}$: $c_n^s < c_n$.

If this is the case, new information trail the whole industry to default. Every bank goes bankrupt as a consequence of its wrong investment in the risky technology. There is no contagion channel involved in the crisis, whereas public intervention is necessary to keep banks afloat.

Scenario 3: idiosyncratic effect of systemic shock

This is the scenario the rest of the chapter focuses on: one or more banks overestimate the repayment on loans, whereas others make a conservative estimate:

$c_1^s < c_1, \ \ c_n^s \geq c_n$.

The systemic shock has an idiosyncratic effect. Banks which overvalued loans return is unable to fulfill its commitment with depositors and to roll over its repos at the interim date. The banking industry may have enough aggregate liquidity to cope with the default, depending on the relative sizes of upward and downward misvaluations.

4.1. Contagion

In scenario 3, the pledgeable value of some banks' collateral at the interim date is in excess of what is necessary to fulfill its commitment. This is true in so far as assets are priced efficiently.

When the MBS value conveyed by banks interim information is below a threshold level, lenders of defaulting banks breach their VaR limit. This happens if new information on MBSs liquidation value is such that $s < p^0(1-v)$. If this is the case, wholesale liquidity providers "break the buck" and exit the money market.

A defaulting bank cannot roll-over its debt at the interim date. Nevertheless, its lender still offers the dispossessed collateral to try and recoup its loss. Asset supply does not change, thus there is no fire sale.[15] However, the money market is less liquid and may exhibit cash-in-the-market pricing.[16] Overall MBS supply is:

$$Q = \sum_{i=1}^{n} q_i,$$

where q_i was specified in (15) and varies according to the realization of regional signals. Only lenders of the $n-k$ less optimistic regions fulfill their VaR=0 constraint. Thus, available liquidity L in the buy side of the market is given by the repayment funds receive from solvent banks at the end of the first wholesale loan:

$$L = p^0 \sum_{i=k+1}^{n} q_i$$

where k is the number of insolvent banks. Let p^{AG} denote the maximum price money market investors are able to pay for an MBS at the interim date in the case of cash-in-the-market pricing á la Allen and Gale (1994):

$$p^{AG} = \frac{L}{Q} = p^0 \frac{\sum_{i=k+1}^{n} q_i}{\sum_{i=1}^{n} q_i}. \tag{17}$$

[15] Since prior to the Subprime crisis a substantial share of bank repos and ABCP were rolled over daily, it makes sense to assume that both the amount of MBSs originated by banks and the liquidity supplied by solvent funds does not vary at the interim date. If defaulting banks had incentive to hold MBSs or owned any other security, their sudden sale would constitute a fire sale like in Cifuentes et al. (2005), among others.

[16] For a comprehensive treatment of cash-in-the-market pricing see Allen and Gale (1994).

Such price is lower than the MBS fundamental value s when

$$\sum_{i=1}^{n} q_i > \frac{p^0}{s} \sum_{i=k+1}^{n} q_i. \tag{18}$$

In line with the too-big-to-fail doctrine, the size of the insolvent institutions portfolio determines the impact their wrong evaluation has on the industry.

Lemma 4. *The difference between MBS fair value and its price at the interim date increases with the position held by overleveraged banks.*

Proof. See Appendix A □

Over-leveraged banks are those who hold larger positions in the pledgeable asset. Thus, their default is likely to hamper money market ability to provide solvent banks enough liquidity to fulfill their commitments. The consequence of funds breaching their VaR constraint is akin to that of fire sales, even though there is no increase in asset supply.

Although the exit by some money market funds drains funding opportunities, the overall pledgeable value in the industry does not need to fall. Banks who acted conservatively are solvent, but they suffer from the temporary lack of liquidity in the money market.

Proposition 5. *When lenders breach their VaR constraint and MBS cash-in-the-market price falls below the initial expectation of a solvent bank, the latter goes bankrupt for lack of liquidity.*

Proof. See Appendix A □

With a low roll-over frequency, the model does not exhibit contagion. This happens for two reasons: (1) liquidity providers have time to enter the money market to exploit any mispricing and avoid the cash-in-the-market pricing determined by limited participation; (2) banks funding liquidity is ensured throughout the maturity of their deposit contract.

Proposition 6. *Without interim roll-over, the money market does not act as channel for contagion.*

Proof. See Appendix A □

In such a case, overleveraged banks still default because the return on loans is lower than they expected. Default makes the bank which took excessive risk bear the cost of its wrong evaluation – so to avoid excessive risk taking and moral hazard – whereas considerate banks are unaffected.

5. Policy implications

Both the Subprime and the ensuing Eurozone crises confirmed that the natural reaction by governments facing systemically important defaulting banks is to bail them out. Contagion to otherwise fundamentally solvent banks, such as those using conservative pricing models in the present paper, would entail in fact a welfare loss which justifies public intervention as explained by Freixas et al. (2000).

Public bailouts imply different costs and taxpayers' reactions depending on the type of intervention and the responsibilities of the target bank. The nationalization of banks involve high costs in terms of upfront payment and public debt, possibly inducing the diabolic loop outlined by Brunnermeier et al. (2011). Purchases of banks' asset, like the Troubled Asset Relief Program adopted in the US in 2008 or the Restructured Banks Asset Management Company created in Spain in 2012, is affected by information asymmetry and may entail future losses. Injections of liquidity incur the risks outlined by Acharya et al. (2012) and Carlin et al. (2007), who find an incentive for solvent banks to hoard liquidity or adopt a vulture-like strategy. The result of the present model is reassuring in this respect.

Conservative banks hold collateral in excess of their liquidity needs at the interim date and face a strategic decision. On the one hand, in the case of default by a competitor, they have the opportunity to use their own liquidity and that injected by public intervention to buy distressed collateral from wholesale lenders at a price p^{AG} below its liquidation value. On the other hand, if the price they pay is too low, their portfolio of MBSs does not provide access to enough liquidity and they default.

Consider an economy where the most leveraged bank 1 is not able to fulfill its commitments, and assume its exposure to wholesale lenders is sufficient to trigger the contagion pointed out in Proposition 5. In the setting of the present model a nationalization or market operations are needlessly expensive to the government, as private bailouts are feasible either with or without government's intervention – depending on the amount of spare liquidity held by solvent banks.

The government may facilitate a private bailout of bank 1 by according concessional loans to other banks. Assume the government picks solvent bank 2 and offer the following scheme: if bank 1 defaults on its loans, the excess supply of MBSs will lower prices and make bank 2 go bankrupt for lack of liquidity. If bank 2 agrees to buy bank 1's MBSs, the government is ready to grant a loan under concessional rate r^G and haircut h^G.[17]

The solvent bank faces a tradeoff: MBS price must be sufficiently high to allow borrowing enough liquidity against its initial holding q_2, but a high price implies the payment of additional interests. The threat of contagion induces bank 2 to use government's money to buy bank 1's collateral, as long as a bailout price p^b that prevents contagion exists.

[17] The scheme can be generalized to a coalition of many solvent banks. However, that woud imply bargaining among banks and goes beyond the scope of the model.

Proposition 7. *The bailout price p^b that is compatible with the prevention of a crisis is:*

$$\frac{(1-h_2)q_2 s_2}{(1-h_2)q_2 - h^G q_1} \leq p^b \leq \frac{q_2 s_2[(1-h_2)r - h_2] + s(q_1+q_2)}{(1-h_2)q_2(1+r) + (1-h^G)q_1(1+r^G)} \tag{19}$$

That exists whenever

$$h^G < \frac{q_2(1-h_2)}{q_1}. \tag{20}$$

Proof. See Appendix A □

Remark 1. *The haircut applied by the government who injects liquidity through repos has to be lower the higher the insolvent institutions' portfolio of assets q_1. However, it is always feasible as the upper threshold in (2.20) is strictly positive.*

A government may even refrain from intervening, if the amount of spare liquidity in the industry is large enough – *i.e.*, if banks adopted on average a prudent pricing model relatively to the realization of the MBS fundamental value s.

Proposition 8. *A private bailout is sufficient to avoid contagion when the liquidation value of the security is above the following threshold:*

$$s > \frac{q_2^2 s_2(1 - 3h_2 + 2h_2^2(1+r) + 2r - 4h_2 r) + q_1 q_2 s_2[(1+r)h - r]}{q_1^2 - q_2^2 + q_2 h_2(q_1+q_2)}. \tag{21}$$

Proof. See Appendix A □

Proposition 8 shows the relationship between banks' pricing errors and the feasibility of a private bailout. The heterogeneity of pricing models is not bad *per se*, as the mistake by an overly conservative bank increases the probability that the true value of the security is sustainable for the industry. The fact that the fundamental value of solvent institutions' portfolio is higher than they expected when their investment strategy was chosen allows them to prevent a liquidity crisis. This happens when the MBS final realization is sufficiently high relatively to banks' estimates.[18]

6. Concluding remarks

The model of contagion developed in the present paper is consistent with four stylized facts of the Subprime crisis: (1) banks-SPVs relied on opaque MBSs to increase their leverage through collateralized borrowing with money market funds; (2) a shock on the cash flow MBSs were expected to pay made overleveraged banks unable to raise the short-term financing they needed to pursue their business strategy; (3) Some

[18] The opaqueness of structured securities allows banks to bias their pricing models towards optimistic scenario and to increase expected returns in a setting with risk-shifting. However, that is outside the scope of the present model.

wholesale lenders faced heavy losses; and (4) the market for short-term financing froze, preventing banks that did not overestimate MBSs value from finding the liquidity they needed to fulfill their commitments.

Far from solving the debate on what allowed the shock in the US subprime mortgage delinquency rate in 2007 to spread worldwide with such a terrific effect, this paper accounts for the possibility that excessive leverage by few institutions precipitates a systemic liquidity crisis. Neglecting the issue of early diers, the model focuses on the misvaluation of opaque financial derivatives as a source of the crisis. Since limited convertibility at the interim date is optimal to depositors who prefer smoothing consumption, there are no bank runs in the model. Moreover, because the model rules out crossed claims among banks, there is no room for domino-contagion.[19]

The main result of the paper is that a market providing liquidity against collateral may act as a channel for contagion. The composite effect of opaque assets and short term collateralized borrowing is able to propagate the negative effect of asset mispricing from an institution to the whole banking sector. Such a channel adds to the domino effect and to the change in haircut that are well known in the extant literature.

Differently from what was found in previous papers on predatory liquidity hoarding, banks have no incentive to adopt vulture-like strategies. The existence of a market for collateralized borrowing introduces a strategic interdependence that limits the advantage solvent banks take from the bankruptcy of their competitors. When banks rely on the money market for their liquidity needs, the additional return they get from cheap collateral may be offset by the amount of liquidity they must give up on the money market. A policy maker can exploit the individualistic motive of liquid banks to prevent contagion in the first place. When the industry is not biased towards overly positive estimates of the future collateral value, the banking industry shall be left the burden of dealing with the failure by one or more inconsiderate banks. If financial institutions as a whole overestimated the value of collateral and committed to repayments that exceed the liquidity availabe at the industry level, public intervention is necessary. However, this comes at lower cost if the policy maker recognizes that liquid institutions have incentives to accomodate a bailout.

[19]Banks are indirectly linked one to each other through the money market. Thus, differently from what happens in domino models, a bank is unable to protect from contagion by means of an appropriate choice of its counterparty.

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A. Proofs

Proof of Lemma 1. The derivative of the pledgeable value per unit of collateral value with respect to the risky investment is

$$\frac{\partial(1-h_i)}{\partial(1-\lambda_i)} = -\frac{1+r-p(1-v)}{p(1+r(1-\lambda_i))^2} < 0.$$

$\square$

Proof of Lemma 3. Bank i optimal investment in the storage technology is lower, the higher is the expected return on the risky investment:

$$\frac{\partial\lambda_i^*}{\partial\bar{s}_i} = -\frac{(p^0)^2(2+r)(1-v)((1+r)-p^0(1-v))}{[(2+r)\bar{s}_i+(p^0)^2r(1-v)+p\bar{s}_i((2+r)v-2)]^2} < 0 \tag{22}$$

The amount of liquidity c_i bank i commits to pay its depositors at each date is given by equality (12), once the optimal portfolio allocation λ_i^* is determined. The periodic payment bank i commits to pay its depositors increases with the optimal risky investment:

$$\frac{\partial c_i^*}{\partial\lambda_i} = \frac{r((p^0)^2(1-v)-s_i(1+p^0v))}{2p^0((1+r)-p^0(1-v))} < 0. \tag{23}$$

Since the optimal risky investment increases with its expected return, banks with higher expectation $\bar{s}_i$ commit to higher periodic payments to their depositors. $\square$

Proof of Lemma 4. From (17), the level of underpricing at the interim date is

$$\frac{s-p^{AG}}{s} \quad = \quad 1-\frac{p^0}{s}\frac{\sum_{i=k+1}^{n}q_i}{\sum_{i=1}^{n}q_i}-1 \tag{24}$$

Hence, the larger the MBS position $\sum_{i=1}^{k}q_i$ of the k relatively inconsiderate banks, the higher is the difference between the fundamental value of the asset and its cash-in-the-market price. $\square$

Proof of Proposition 5. From equation (13), bank $i' \in [k+1;n]$ commits to a periodic payment

$$c_{i'} = \lambda_{i'} + q_{i'}p^0(\delta_{i'}-r)(1-h_{i'})$$

whereas, defining by

$$\delta_{AG} = \frac{p^{AG}}{p^0}-1 \tag{25}$$

the price change of the pledgeable asset in case of cash-in-the-market pricing, the liquidity it has available at the interim date is

$$L_{i'}^1 = \lambda_{i'} + q_{i'}p^0(\delta_{AG}-r)(1-h_{i'})$$

The solvent bank becomes unable to fulfil its commitments – that is, $L^1_{i'} < c^1_{i'}$ – whenever $\delta_{AG} < \delta_{i'}$. Equalities (8) and (25) show that the amount of liquidity $c^1_{i'}$ considerate bank i' has available at $t = 1$ is lower than needed to fulfil depositors' and lenders' claims whenever $p^1 < \bar{s_{i'}}$ □

Proof of Proposition 6. If banks do not roll-over their loan until the end of the deposit contract, optimal portfolio allocation at the initial date is the solution to (12)–(13), but with $\delta_i = 0$. In such case with no roll-over, labelled NR, the initial optimal allocation is

$$\lambda_i^{NR} = \frac{p^0(1-v) - \bar{s}_i}{2p^0(1-v) - (2 + r + r\bar{s}_i)},$$

and MBS origination is

$$q_i^{NR} = \frac{2 + r}{2p^0(1-v) - (2 + r + r\bar{s}_i)}.$$

With fair pricing, the impact of new information at the interim date is for bank i an excess liquidity equal to

$$G_i^{NR} = q_i^*(1 - h_i)(s - p^0), \tag{26}$$

that is positive whenever the bank had a conservative estimate on loans repayment. Overleveraged banks may endanger their wholesale lenders. However, differently from the case of cash-in-the-market pricing, this has no negative externality on conservative banks funding. □

Proof of Proposition 7. Bank 2 commitment with depositors at the interim date is

$$c_2^1 = \lambda_2 + q_2\bar{s}_2(1 - h_2) - q_2p^0(1 - h_2)(1 + r), \tag{27}$$

whereas the liquidity it has available after buying assets of the insolvent banks at a bailout price p^b is

$$\lambda_2 + q_2p^b(1 - h_2) - q_2p^0(1 - h_2)(1 + r) - q_1p^b + q_1p^b(1 - h^G). \tag{28}$$

The bank avoids contagion if (27)≤(28). This happens when it pays a sufficiently high price

$$p^b \geq \frac{(1 - h_2)q_2s_2}{(1 - h_2)q_2 - h^Gq_1}. \tag{29}$$

Nonetheless, an increase in the value of assets pledged at the interim date translates into higher interests to be paid at the final date. The commitment of solvent bank 2 with depositors at the final date is

$$q_2\bar{s}_2 - q_2\bar{s}_2(1 - h_2)(1 + r), \tag{30}$$

whereas it has available liquidity that, after repurchasing assets pledged with the policy maker, amounts to

$$(q_1 + q_2)s - q_2 p^b(1 - h_2)(1 + r) - q_1 p^b(1 - h^G)(1 + r^G). \tag{31}$$

Similarly to the interim date, the bank is solvent if (30)≤(31). This yields an additional necessary condition on p^b:

$$p^b \leq \frac{q_2 s_2[(1 - h_2)r - h_2] + s(q_1 + q_2)}{(1 - h_2)q_2(1 + r) + (1 - h^G)q_1(1 + r^G)} \tag{32}$$

The two conditions (29) and (32) are simultaneously satisfied, when $r^G \leq r$, if and only if

$$h^G < \frac{q_2(1 - h_2)}{q_1}.$$

□

Proof of Proposition 8. The proof goes along the same line as that of Proposition 7. The only difference is the amount of liquidity bailing out banks have at the interim and the final date. When the policy maker does not intervene, bank 1 available liquidity at the interim date is

$$p^b(1 - h_2)q_2 - (1 - h_2)pq_2(1 + r) - p^b q_1. \tag{33}$$

At the final date, the bank holds an amount of liquidity

$$(q_1 + q_2)s - p^b(1 - h_2)q_2(1 + r). \tag{34}$$

The conditions on p^b that allow a bailout with no public intervention are therefore

$$p^b \geq \frac{(1 - h_2)q_2 s_2}{q_1 - (1 - h_2)q_2}, \tag{35}$$

$$p^b \leq \frac{-h_2 q_2(r + 1)s_2 + s(q_1 + q_2) + q_2 r s_2}{(1 - h_2)q_2(r + 1)}. \tag{36}$$

The set defined by such conditions is empty unless

$$s > \frac{q_2^2 s_2(1 - 3h_2 + 2h_2^2(1 + r) + 2r - 4h_2 r) + q_1 q_2 s_2[(1 + r)h2 - r]}{q_1^2 - q_2^2 + q_2 h_2(q_1 + q_2)}. \tag{37}$$

□