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Emanuele Bacchiega, Elias Carroni, Alessandro Fedele

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Emanuele BACCHIEGA†1, Elias CARRONI‡2, and Alessandro FEDELE§3

[1]University of Bologna, Department of Computer Science and Engineering
[2]University of Bologna, Department of Economics
[3]Free University of Bolzano, Faculty of Economics and Management

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Abstract

The Hotelling model is the workhorse for analyzing platform competition in two-sided markets. In this setup, the degree of platform differentiation must be high relative to cross-group benefits to eschew the alleged non-existence of Nash equilibria. The present paper shows instead that Nash equilibria exist even for relatively low differentiation; at such equilibria, platforms avoid competition by replicating a collusive outcome. This result widens our knowledge of the two-sided Hotelling model and is relevant for a large array of markets –especially digital ones– where platforms operate in relatively low-differentiation environments.

Keywords Hotelling model; Platform competition; Two-sided markets; Equilibrium existence.

JEL Codes: L13, C72, D21.

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†emanuele.bacchiega@unibo.it.
‡elias.carroni@unibo.it
§Corresponding author: alessandro.fedele@unibz.it.

1 Introduction

In a growing number of markets, competing platforms mediate interactions among different groups of agents, also called market sides. Since the seminal paper by Armstrong (2006), the Hotelling duopoly has become the workhorse for analyzing competition between horizontally differentiated platforms in two-sided markets.

Crucially, Armstrong (2006) shows that the degree of differentiation between platforms must be high relative to the (positive) value an agent in a group attributes to interacting with an agent in the other group (i.e., the cross-group benefit) to ensure that the platforms' equilibrium profits are positive and, consequently, Nash price equilibria exist: see condition (8) on p. 674 of his paper. If this condition is not met, the lure to expand own market shares by stealing business from the rival leads to cut-throat competition, which jeopardizes the duopolistic structure of the market.[1]

However, several real-world instances exist where the assumption of relatively high differentiation is difficult to uphold. A case in point is digital markets, where streaming, marketplace, ride-sharing, and online travel agency platforms, among others, are active. Finding the desired content, products, and services on those platforms (i.e., the cross-group benefit) is generally no less important than the idiosyncratic preference for one platform (i.e., the degree of horizontal differentiation). One might therefore be tempted to conclude that the Hotelling model is less suited to analyze most digital markets.

In this paper, we show that Nash equilibria actually exist in two-sided Hotelling duopoly setups even when platform differentiation is low relative to the cross-group benefit. At these equilibria, platforms aim at serving as many consumers as possible, but at the same time they avoid business stealing; such equilibrium behavior mimics a collusive outcome, which allows platforms to defuse ruthless competition.

Our result is relevant for two reasons. First, characterizing the Nash equilibria without any restrictive assumption on the parameter space enriches our understanding of the two-sided Hotelling model. Second, most two-sided markets are nowadays digital markets, in which the low-differentiation scenario we focus on cannot be easily ruled out.

In the remainder of the paper, the ensuing Section presents the model and formalizes the above-described issue due to relatively low differentiation. To introduce the reader to the analysis of the Hotelling price game in the unrestricted parameter space, Section 3 focuses on the standard, one-sided, Hotelling model. Section 4 extends the analysis to the two-sided framework and derives the main result. Section 5 concludes.

[1] As Caillaud and Jullien (2003) point out, strong cross-group benefits imply "highly contestable market structures, where all potential profits are eroded in order to protect a monopoly position".

2 Model

Two platforms, indexed $i \in \{0, 1\}$ and located at the endpoints of a Hotelling segment of unit length, mediate interactions between two sets of agents, referred to as consumers and producers, and set prices only on the consumer side.[2] The platforms' production costs are normalized to zero.

A unit mass of consumers is uniformly distributed along the line and incurs linear transportation costs. Consumers decide whether not to be active in the market (zero-home) or to join one platform (single-home); in the former case, their utility is zero; in the latter, the utility function of a consumer located at $x \in [0, 1]$ and joining platform i is

$$\mathcal{U}(x, p_i, \mathbb{E}(n_i)) = v + \alpha \cdot \mathbb{E}(n_i) - td_i(x) - p_i. \quad (1)$$

The term $v \geq 0$ denotes the stand-alone value, that is, the utility the consumer enjoys when joining a platform with no producers; $\alpha\mathbb{E}(n_i)$ captures the cross-group benefits enjoyed by the consumer on platform i, $\alpha \in (0, 1]$ being the value of the interaction between the consumer and a producer, and $\mathbb{E}(n_i)$ the mass of producers the consumer expects to be active on platform i; $t > 0$ is the marginal transportation cost and $d_i(x)$ is the Euclidean distance of the consumer from platform i; finally, p_i is the price charged by platform i on each consumer.

On the other side of the market, there is a unit mass of producers and each one chooses whether not to be active in the market (zero-home), or to join one platform (single-home), or to join both platforms (multi-home). Producers bear heterogeneous setup costs to operate in either platform; these costs, denoted f, are uniformly distributed over the interval $[0, 1]$. Each producer enjoys the cross-group benefit $\gamma \in (0, 1]$ when interacting with a consumer. For simplicity, we assume that a producer does not pay any fee to join a platform. Accordingly, a cost-f producer is willing to join platform i if and only if $\gamma \cdot \mathbb{E}(D_i) \geq f$, where $\mathbb{E}(D_i)$ is the mass of consumers the producer expects to be active on platform i; the resulting mass of producers joining platform i is given by $n_i = Prob(\gamma \cdot \mathbb{E}(D_i) \geq f)$. Under the assumption of uniform distribution of setup costs, we get $n_i = \gamma \cdot \mathbb{E}(D_i)$.

Platforms, consumers, and producers play the following game. First, the platforms simultaneously set prices p_i to maximize profits, $\pi_i = p_i D_i$. Second, consumers and producers simultaneously make their joining decisions.

In the literature, the above game is solved as follows. Equation $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = \mathcal{U}(x, p_1, \mathbb{E}(n_1))$

[2]We build on Armstrong (2006) for the consumer side and on Hagiu (2006) for the producer side; a similar framework is developed by, e.g., Rasch and Wenzel (2013).

is used to obtain the location of the consumer indifferent between joining platform 0 or 1:

$$\hat{x} = \frac{1}{2} + \frac{p_1 - p_0 + \alpha(\mathbb{E}(n_0) - \mathbb{E}(n_1))}{2t}. \tag{2}$$

Producers and consumers are assumed to have correct expectations about the other side's participation, thus $\mathbb{E}(D_0) = \hat{x}$, $\mathbb{E}(D_1) = (1 - \hat{x})$, and $\mathbb{E}(n_i) = \gamma\mathbb{E}(D_i)$. Solving the system of producers' and consumers' expectations yields $\mathbb{E}(n_0) = \gamma\hat{x}$ and $\mathbb{E}(n_1) = \gamma(1 - \hat{x})$. These values are then plugged into (2),

$$x_I = \frac{1}{2} - \frac{p_0 - p_1}{2(t - \alpha\gamma)}. \tag{3}$$

Platforms 0 and 1 simultaneously solve the problems $\max_{p_0} p_0 x_I$ and $\max_{p_1} p_1(1 - x_I)$, respectively. The symmetric equilibrium price is $t - \alpha\gamma$, and the equilibrium profits are $\frac{t-\alpha\gamma}{2}$; both values are positive if and only if $t > \alpha\gamma$, that is, if and only if the transportation cost parameter is larger than the product of the marginal cross-group benefits. This condition is akin to (8) in Armstrong (2006) and is deemed to be necessary and sufficient for equilibrium existence.

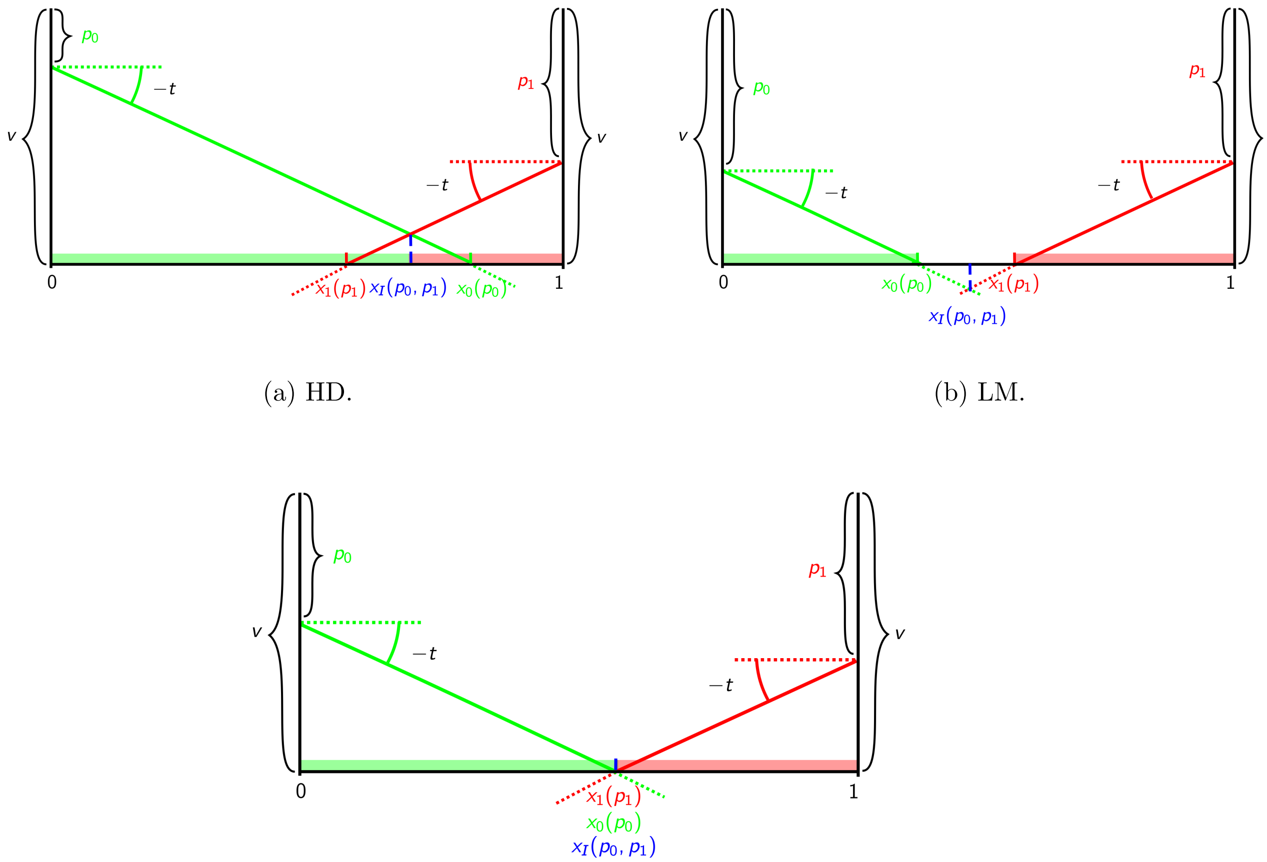
In the remainder of the analysis, we show that Nash equilibria of the above game exist also if t is lower than $\alpha\gamma$. To this aim, we proceed in two steps. (i) In Section 3, we describe the Nash equilibria in the standard, one-sided Hotelling framework, by letting t take any positive value. (ii) In Section 4, we get back to the two-sided framework and derive its Nash equilibria for any $t > 0$.

3 One-sided Framework

We consider a one-sided version of the above framework, where two producers, indexed $i \in \{0, 1\}$, are located at the endpoints of the unit segment and sell goods to consumers, whose gross utility from unit consumption is v. In this setup, at most three alternative price responses are available to producer i, for any given price $p_j \in (0, v]$ set by the rival $j = 1, 0$, if t can take any positive value. To illustrate them, we focus on producer 0 and introduce Figure 1 where: p_i is the price of producer i; the green (respectively red) line is the consumers' utility if they purchase from producer 0 (resp. 1) as a function of their location x; $x_i(p_i)$ is the (location of the) consumer indifferent between purchasing from producer i and not purchasing; $x_I(p_0, p_1)$ is the consumer indifferent between patronizing either producer; finally, the green (resp. red) portion of the unit segment represents producer 0's (resp. producer 1's) demand.

(i) In panel (a), producer 0 sets a relatively low price with the aim of stealing customers from the rival, so that $x_0(p_0) > x_1(p_1)$: this is the standard *Hotelling Duopoly* scenario (HD), where the indifferent consumer x_I obtains positive utility and the market is fully covered.

(ii) In panel (b), producer 0 sets a relatively high price to avoid strategic interaction and



(a) HD.

(b) LM.

(c) MD.

Figure 1: Possible price responses in the one-sided Hotelling game.

business stealing, so that $x_0(p_0) < x_1(p_1)$: this is the *Local Monopoly* scenario (LM), in which the indifferent consumer prefers not to buy and the market is partially covered.

(iii) In panel (c), producer 0 sets the price with the aim of serving all the consumers that would enjoy negative utility if they patronized producer 1, and *them only*. In symbols, $x_0(p_0) = x_1(p_1)$: the indifferent consumer obtains zero utility and the market is fully covered. We label this configuration *Monopolistic Duopoly* (MD) because it combines the characterizing features of the other two scenarios: there is full market coverage, but no business stealing.

The solution to the game where producers simultaneously set prices involves three mutually exclusive types of Nash Equilibria: HD equilibrium when t is low relative v; LM equilibrium when t is high relative v; MD equilibrium if t has intermediate values relative to v.[3]

While HD and LM equilibria have been extensively analyzed in the Hotelling literature, the

[3]We refer to Mérel and Sexton (2010) and Thépot (2007) for a characterization. To our knowledge, the only other papers that discuss these three types of equilibria are Salop (1979); Cowan and Yin (2008); Rey and Salant (2012); Fedele and Depedri (2016); and Rey and Tirole (2019).

MD one has been overlooked, besides few sparse contributions.[4] At this type of equilibrium, producers strategically interact to keep the market covered, hence their prices are interdependent, but do not directly compete with one another to steal any consumer from the rival. This results in counter-intuitive comparative statics: prices are strategic substitutes and decrease (along with profits) with the transportation cost parameter t. In addition, the MD equilibrium prices coincide with those of a multi-product monopolist that serves the entire market. This outcome is akin to, but distinct from, collusion because the MD prices maximize the industry profits, on one hand, but they are a Nash equilibrium of a one-shot game, on the other hand; we refer to it as *quasi-collusion*.

4 Results

In the following, we consider the two-sided framework of Section 2 and solve the game played by platforms, consumers, and producers for any positive value of t, given v, α, and γ; we define $e \equiv \alpha\gamma$. Our main goal is showing that MD is the only equilibrium type when $t \leq e$.[5]

Proposition 1. *Four alternative equilibrium configurations arise depending on the level of t relative to v and e:*

(i) if $t \leq e$, MD with prices $p^{MD} \equiv v - \frac{t}{2} + \frac{e}{2}$;

(ii) if $e < t < e + \frac{2}{3}v$, HD with prices $p^{HD} \equiv t - e$;

(iii) if $e + \frac{2}{3}v \leq t \leq e + v$, MD with prices $p^{MD} \equiv v - \frac{t}{2} + \frac{e}{2}$;

(iv) if $t > e + v$, LM with prices $p^{LM} \equiv \frac{v}{2}$.

Proof. We first prove points (ii)-(iv) and then move to point (i).

Point (ii). At the HD equilibrium, which we derived in Section 2, the indifferent consumer obtains positive utility, as illustrated in Figure 1: denoting $p^{HD} \equiv t - e$ the equilibrium price, the parametric interval in which HD arises is $\mathcal{U}(x_I(p^{HD}), p^{HD}, \mathbb{E}(n_0(p^{HD})) = v + \frac{e}{2} - (t-e) - \frac{t}{2} > 0 \Leftrightarrow (e <)t < e + \frac{2}{3}v$.

[4] The literature on MD has not yet reached a commonly accepted nomenclature. Thépot (2007) labels our MD "Captive Duopoly". Cowan and Yin (2008) refer to HD as "Competitive Equilibrium" and MD as "Touching Equilibria". Mérel and Sexton (2010) use "Competitive", "Kinked-Demand" and "Monopoly" equilibria for HD, MD and LM, respectively. We believe that our terminology is more evocative of the behavioral differences among the three scenarios.

[5] We restrict our attention to the unique symmetric MD equilibrium. A continuum of asymmetric MD equilibria exist in the one-sided setup. These equilibria share the same characteristics as the symmetric one, except for the lower total industry profit they generate.

Point (iv). Exploiting symmetry, we analyze platform 0 only. We solve equation $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = 0$ for x to get the location of the consumer indifferent between joining platform 0 and not joining, $x_0 = \frac{v+\alpha\mathbb{E}(n_0)-p_0}{t}$. Given correct expectations, we can derive platform 0's demand, $D_0 = \frac{v-p_0}{t-e}$. Accordingly, platform 0 solves problem $\max_{p_0} p_0 \left(\frac{v-p_0}{t-e}\right)$. We get $p^{LM} \equiv \frac{v}{2}$, with $x_0(p^{LM}) = \frac{v}{2(t-e)}$. At the LM equilibrium, the indifferent consumer gets negative utility, so $\mathcal{U}(x_I(p^{LM}), p^{LM}, \mathbb{E}(n_0)(p^{LM})) = v + e\frac{v}{2(t-e)} - \frac{v}{2} - \frac{t}{2} < 0 \Leftrightarrow t > e + v$.

Point (iii). We focus on the interval $e+\frac{2}{3}v \leq t \leq e+v$ and look for MD equilibria. To this aim, we substitute (3) into (1) and get the indifferent consumer's utility, $v + (e-t)\left(\frac{1}{2} - \frac{p_0-p_1}{2(t-e)}\right) - p_0$. This value is zero at the MD equilibria, hence

$$p_0 + p_1 = 2v - t + e. \tag{4}$$

At the candidate symmetric equilibrium, we get $p^{MD} \equiv \frac{2v-t+e}{2}$. To check this is an equilibrium, we exploit symmetry and investigate deviations by platform 0, given that platform 1 sets $p_1 = p^{MD}$.

Deviation to HD. Plugging $p_1 = (p^{MD} \equiv)\frac{2v-t+e}{2}$ into (2) yields the indifferent consumer's location for any p_0,

$$\frac{2v+t+e}{4t} - \frac{p_0}{2t} + \frac{\alpha\left(\mathbb{E}(n_0) - \mathbb{E}(n_0)\right)}{2t}. \tag{5}$$

We assume the deviation by platform 0 leads to HD, that is, the deviation price is lower than p^{MD}; in this case, the deviation profits are maximized at $p_0^{D,HD} = \frac{2v+t+e+2\alpha(\mathbb{E}(n_0)-\mathbb{E}(n_1))}{4}$. This value is lower than p^{MD}, hence the deviation is feasible, iff

$$\mathbb{E}(n_0) - \mathbb{E}(n_1) < \frac{2v - 3t - 3e}{2\alpha}. \tag{6}$$

To investigate whether (6) is fulfilled, we proceed as follows: among all possible consumers' expectations that can be considered off the equilibrium path, we take those such that $\mathbb{E}(n_0) - \mathbb{E}(n_1) \in [0, \gamma]$ and observe that the RHS (6) is non-positive in the interval of interest. This proves there exist expectations off the equilibrium path such that the deviation to HD is not feasible.

Deviation to LM. We assume the deviation by platform 0 leads to LM, that is, the deviation price is higher than p^{MD}; in this case, the deviation profits are maximized at $p^{LM} \equiv \frac{v}{2}$. However, this value is weakly lower than p^{MD} in the interval of interest, hence a deviation to LM is not feasible.

Point (i). When $t \leq e$, the demand of platform i as in (3) would increase with p_i, which is implausible. To compute the indifferent consumer's location, we hence proceed as follows. At the MD symmetric equilibrium, producers' correct expectations are such that each platform attracts half of the consumers, $\mathbb{E}(D_i) = \frac{1}{2}$, which implies $n_i = \frac{\gamma}{2}$. Therefore, consumers' correct expectations require $\mathbb{E}(n_i) = \frac{\gamma}{2}$. Plugging $\frac{\gamma}{2}$ into (1) and solving $\mathcal{U}(x, p_0, \mathbb{E}(n_0)) = \mathcal{U}(x, p_1, \mathbb{E}(n_1))$

for x gives $x_I = \frac{1}{2} + \frac{p_1 - p_0}{2t}$. This indifferent consumer gets zero utility, that is, $v + \frac{e}{2} - p_0 - t\left(\frac{1}{2} + \frac{p_1 - p_0}{2t}\right) = 0$ when (4) holds true. As a result, the candidate MD symmetric equilibrium price is as above, $p^{MD} \equiv \frac{2v-t+e}{2}$.

Deviation to HD. A deviation to HD is feasible iff (6) holds true. Again, we consider off-the-equilibrium expectations such that $\mathbb{E}(n_0) - \mathbb{E}(n_1) \in [0, \gamma]$ and observe that the RHS of (6) is non-positive iff $v \leq \frac{3t-e}{2}$, in which case the deviation to HD is not feasible. At higher v, we check that γ is larger than the (positive) RHS of (6) iff $v < \frac{3t+2e}{2}$, in which case, there exist off-the-equilibrium expectations such that the deviation to HD is not feasible.

Deviation to LM. To prove there is no feasible deviation to LM, it is sufficient to observe that the deviation price, $p^{LM} \equiv \frac{v}{2}$, is lower than p^{MD} in the interval of interest. □

Before commenting on the foregoing proposition, we remark that interval (ii) describes the (HD) equilibrium that we derived in Section 2 and that the two-sided markets literature focuses on. The message conveyed by Proposition 1 is twofold. On the one hand, it extends the equilibrium taxonomy known for the one-sided Hotelling game to its two-sided counterpart. It is indeed easy to ascertain that Points (ii)-(iv) parallel those of Proposition 1 in Mérel and Sexton (2010), or of Section 3 in Thépot (2007).[6] On the other hand, Point (i) shows that a significant difference emerges: when t is arbitrarily close to 0, the HD equilibrium arises in one-sided markets, while the equilibrium interaction is in the form of MD in the two-sided environment.[7]

What is the intuition behind this novel result? In two-sided markets with positive cross-group effects, a relatively low t would trigger a fierce price war if platforms' goal were to steal consumers from one another, as is the case at the HD (candidate) equilibrium: this would lead to negative prices and profits. Platforms then avoid cut-throat competition by shifting to a MD equilibrium: instead of undercutting the rival, they set prices so as to attract all the consumers that do not patronize the rival and *them only*; this quasi-collusive behavior allows platforms to earn positive profits.

5 Conclusion

In this paper, we have derived a novel result concerning the Hotelling game's Nash equilibria in two-sided markets. When horizontal differentiation between platforms is low relative to cross-group benefits, the HD equilibrium fails to exist, but an equilibrium arises in the form of MD.

Our result shows that, contrary to what is commonly accepted in the literature, the Hotelling model is robust enough to describe the equilibrium behavior of platforms when cross-group

[6] See also Section 3 in Salop (1979).

[7] Here, as shown in the proof, a sufficient condition for MD to arise is $v \leq e$. This condition, similarly to $t \leq e$, is likely to hold in most digital markets, where cross-group benefits are no less important to consumers than the stand-alone value of platforms.

benefits are at least as important as the degree of differentiation; interestingly, this scenario is likely to be common in digital markets. The reason lies in the intriguing economic interpretation of the MD equilibrium: by behaving as monopolistic duopolists, platforms replicate a collusive outcome at the one-shot Nash equilibrium and circumvent the fierce competition triggered by relatively low differentiation. This provides an entirely novel insight to interpret the strategic behavior of platforms, especially of those operating in digital markets.

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